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Formation Control of VTOL UAVs Without Linear-Velocity Measurements

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Abstract— **We address the formation control problem without linear-velocity measurements for a group of Vertical Take-Off and Landing (VTOL) Unmanned Aerial Vehicles (UAV) with a fixed and undirected communication topology. The vehicles among the team are required to track a desired reference linear-velocity and maintain a desired formation. Our control design is achieved in two main steps. First, an intermediary control input is designed for the translational dynamics, from which we extract the desired system attitude and thrust achieving the formation objective. Then, the torque input for the rotational dynamics of each vehicle is designed to drive the actual attitudes to the desired ones. To obviate the need for linear-velocity, we use instrumental auxiliary variables in each step of the control design. The stability of the overall closed loop system is rigorously established. Simulation results are provided to show the effectiveness of the proposed control scheme.**

I. Introduction

The attitude control problem of flying vehicles has been the focus of many researcher over the past years, resulting in several successful attitude controllers, see for instance, [1], [2]. The attitude synchronization of rigid bodies has also been extensively dealt with in the recent years, see for instance [3], [4] and references therein. However, the position control of VTOL UAVs in $SE(3)$ is a more challenging problem due to the *under-actuated* nature of the system and global stability results are difficult to achieve. Several methods dealing with this problem have been reported in the literature [5]-[10]. The authors in [8] and [10] proposed a hierarchical controller for the stabilization of hovering VTOLs, which is composed of a high level position control and a low level attitude control. In [11], a similar control architecture is applied to solve the trajectory tracking problem, where the angular velocity is used as an intermediate variable instead of the orientation, and a high gain controller is used to determine the torque signals capable of tracking the requested angular velocity. The difficulty with the latter design is to prove the stability of the global cascaded system. The authors in [12] proposed a backstepping design for the trajectory tracking problem of a class of underactuated systems, including VTOL vehicles, where the states are guaranteed to converge to a ball near the origin. In our recent work [13], we have proposed a control design methodology for the tracking and formation control of a group of VTOL UAVs and global stability results are obtained. The novelty in this work, with respect to the existing literature, is the use of a singularity-free unit-quaternion for the orientation representation. Indeed, we use an extraction algorithm for the desired direction of the vehicles thrust, which always provides a realizable solution under the condition that the translational control input is upper bounded by a well defined quantity. A more general extraction algorithm has been developed in [14] and an adaptive trajectory tracking control scheme is proposed for the class of VTOL UAVs in the presence of constant unknown disturbances.

An important assumption in the above works is the availability of the full state information for feedback. For flying vehicles, velocity estimations can be obtained via approximate derivation of the successive measurements from GPS sensors. For fast moving vehicles, the standard procedure is integrating the acceleration, and coupling this result to the derivative of GPS measurements [15]. This estimation method suffers from several problems, namely the fact that errors induced by a GPS system may reach many meters, and in practice, numerical integration along with measurement noise induces a very fast growing velocity measurement error. There are several technical solutions to overcome these problems, such as using high quality sensors, which are extremely expensive. However, for indoor/urban applications for example, GPS cannot be a reliable sensing device since satellite signals are shaded by the urban structures. Another solution to this is to use observers to estimate the missing states, as done in [16], where the trajectory tracking problem of a *planar*-VTOL is treated, and a full order observer is designed using the available positions and attitudes.

In this paper, we consider the formation control of a group of VTOL UAVs without linear-velocity measurements. To the best of our knowledge, this work is the first that considers formation control of this class of under-actuated systems without linear-velocity measurements. To achieve our control objective, we use the extraction method considered in [13] and [14]. An intermediary force input is first designed for the translational dynamics from which the required thrust and the desired system orientation are extracted. Thereafter, a torque input is designed to achieve tracking of the desired orientation. As will be clear throughout the paper, this design methodology with the lack of the linear-velocity of the vehicles make rise of several challenging problems. First, the linear-velocity-free intermediary translational control must be bounded by a predefined value. Second, with the adopted extraction method, the torque input design will rely on the first and second time derivatives of the translational control.

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These problems are solved by the introduction of new control variables and dynamic auxiliary systems for the translational and rotational dynamics.

II. System Model and Problem Formulation

In this paper, we consider $n-$aircraft modeled as rigid bodies. Let $\mathcal{F}_i \triangleq \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ denote the inertial frame, and $\mathcal{F}_j \triangleq \{\hat{e}_{1j}, \hat{e}_{2j}, \hat{e}_{3j}\}$ denote the body-fixed frame of the j^{th} aircraft. Let the position and linear velocity of the j^{th} aircraft expressed in the inertial frame, $\mathcal{F}_i$, be denoted respectively by $p_j \in \mathbb{R}^3$ and $v_j \in \mathbb{R}^3$, and let its angular velocity be expressed in the j^{th} body-fixed frame, $\mathcal{F}_j$, and is denoted by $\omega_j \in \mathbb{R}^3$. The equations of motion of the j^{th} aircraft are described by

$$(\Sigma_{1_j}) : \begin{cases} \dot{p}_j = v_j, \\ \dot{v}_j = g\hat{e}_3 - \frac{\mathcal{T}_j}{m_j} R(\mathbf{q}_j)^T \hat{e}_3, \end{cases} \tag{1}$$

$$(\Sigma_{2_j}) : \begin{cases} \dot{\mathbf{q}}_j = \frac{1}{2} \begin{pmatrix} \eta_j \ I_3 + S(q_j) \\ -q_j^T \end{pmatrix} \omega_j, \\ I_{f_j} \dot{\omega}_j = \tau_j - S(\omega_j) I_{f_j} \omega_j, \end{cases} \tag{2}$$

for $j \in \mathcal{N} \triangleq \{1, ..., n\}$. m_j and g are respectively the mass of the j^{th} aircraft and the gravitational acceleration, $I_{f_j} \in \mathbb{R}^{3\times3}$ is the symmetric positive definite constant inertia matrix of the j^{th} aircraft with respect to $\mathcal{F}_j$ and $\bar{\omega}_j = (\omega_j^T, 0)^T$. The scalar $\mathcal{T}_j$ and the vector τ_j represent respectively the magnitude of the thrust applied to the j^{th} vehicle in the direction of $\hat{e}_{3j}$, and the external torque applied to the system expressed in $\mathcal{F}_j$. The unit quaternion $\mathbf{q}_j = (q_j^T, \eta_j)^T$, composed of a vector component $q_j \in \mathbb{R}^3$ and a scalar component $\eta_j \in \mathbb{R}$, represents the orientation of the vehicle's body frame, $\mathcal{F}_j$, with respect to the inertial frame, $\mathcal{F}_i$, and are subject to the constraint: $q_j^T q_j + \eta_j^2 = 1$. The rotation matrix related to the unit-quaternion $\mathbf{q}_j$, that brings the inertial frame into the body frame, can be obtained through the Rodriguez formula as: $R(\mathbf{q}_j) = (\eta_j^2 - q_j^T q_j) I_3 + 2q_j q_j^T - 2\eta_j S(q_j)$, where I_3 is the 3-by-3 identity matrix and the matrix $S(\mathbf{x})$ is the skew-symmetric matrix such that $S(\mathbf{x})V = \mathbf{x} \times V$ for any vectors $\mathbf{x} \in \mathbb{R}^3$ and $V \in \mathbb{R}^3$, where $'\times'$ is the vector cross product.

The quaternion multiplication between two unit quaternion, $\mathbf{q} = (q^T, \eta)$ and $\mathbf{p} = (p^T, \epsilon)$, is defined by the following non-commutative operation: $\mathbf{q} \odot \mathbf{p} = \left(\eta p + \epsilon q + S(q)p \ , \ \eta\epsilon - q^T p\right)$. The inverse or conjugate of a unit quaternion is defined by, $\mathbf{q}_j^{-1} = (-q_j^T, \eta_j)^T$, with the quaternion identity given by $(0, 0, 0, 1)^T$, [17].

Our objective in this work is to design the thrust and torque inputs for each VTOL aircraft in the team, without linear-velocity measurements, to guarantee that all vehicles track a reference linear-velocity $v_d(t)$ and maintain a prescribed formation, *i.e,* maintain fixed desired relative distances between neighbors in the team. In other words, our objective is attained if one can guarantee that

$$v_j(t) \to v_d(t) \quad \text{and} \quad p_j - p_k \to \delta_{jk} \tag{3}$$

for $j, k \in \mathcal{N}$, where $\delta_{jk} \in \mathbb{R}^3$ defines the desired offset between the j^{th} and k^{th} aircraft, and hence defines the formation pattern.

To achieve our objective, we assume that the information flow between aircraft is fixed and undirected, and each aircraft can communicate with at least one other aircraft in the team, *i.e,* the communication flow is "connected". Also, we assume that the reference velocity $v_d(t)$ is bounded as well as its first, second and third derivatives, and is available to all aircraft in the team. In addition, we assume that the linear-velocity vectors are not available for feedback. This case corresponds to the practical use of a VTOL UAV equipped with sufficient sensors that provide measurements on the system orientation (attitude), angular velocities and measurement of the position of the vehicle.

III. Extraction Method

Consider the system model in (1). We can rewrite the equations of subsystem (Σ_{1_j}) as

$$(\Sigma_{1_j}) : \begin{cases} \dot{p}_j = v_j, \\ \dot{v}_j = F_j - \frac{\mathcal{T}_j}{m_j} f(\mathbf{q}_j, \mathbf{q}_{d_j}), \end{cases} \tag{4}$$

with $f(\mathbf{q}_j, \mathbf{q}_{d_j}) = \left(R(\mathbf{q}_j)^T - R(\mathbf{q}_{d_j})^T\right) \hat{e}_3$, and

$$F_j = g\hat{e}_3 - \frac{\mathcal{T}_j}{m_j} R(\mathbf{q}_{d_j})^T \hat{e}_3 \tag{5}$$

where the variable F_j is the "intermediary" control input to the translational dynamics.

The main idea in our work is to exploit the cascaded nature of the system (4) with (2), and first design the intermediary control input to the translational dynamics of each vehicle, given in (4), from which we can extract the magnitude and direction of the necessary thrust input for each vehicle. The magnitude of the thrust, $\mathcal{T}_j$, will be the input to the translational dynamics (Σ_{1_j}), and its direction will define a time-varying desired attitude for each aerial vehicle, namely $\mathbf{q}_{d_j}(t)$, to be tracked by the rotational dynamics with an appropriate design of the torque input for each subsystem (Σ_{2_j}). In the following, we will present an extraction algorithm of the thrust direction and magnitude from the expression of the intermediary control input, which is free from singularities if the intermediary control input satisfies some conditions. Since this procedure applies for all VTOL vehicles in the formation, $j \in \mathcal{N}$, we will omit the subscript "j" in the following result for clarity of presentation.

Lemma 1: [14]
Consider equation (5) and let the intermediary control input $F \triangleq (\mu_1, \mu_2, \mu_3)^T$. It is always possible to extract the thrust magnitude and direction from (5) as

$$\mathcal{T} = m\left((g - \mu_3)^2 + \mu_1^2 + \mu_2^2\right)^{1/2} \tag{6}$$

$$q_d = \frac{m}{2\mathcal{T}\eta_d} \begin{pmatrix} \mu_2 \\ -\mu_1 \\ 0 \end{pmatrix} \quad , \quad \eta_d = \sqrt{\frac{1}{2} + \frac{m(g - \mu_3)}{2\mathcal{T}}} \tag{7}$$

under the condition that the elements of F satisfy

$$(\mu_1, \mu_2, \mu_3) \neq (0, 0, x), \quad \text{for} \quad x \geq g \tag{8}$$

In addition, we can write the desired angular velocity of each aircraft in terms of the intermediary control, F, as

$$\omega_d = \Xi(F)\dot{F}, \tag{9}$$

$$\Xi(F) = \frac{1}{\gamma_1^2\gamma_2}\begin{pmatrix} -\mu_1\mu_2 & -\mu_2^2+\gamma_1\gamma_2 & \mu_2\gamma_2 \\ \mu_1^2-\gamma_1\gamma_2 & \mu_1\mu_2 & -\mu_1\gamma_2 \\ \mu_2\gamma_1 & -\mu_1\gamma_1 & 0 \end{pmatrix},$$

with $\gamma_1 = (\mathcal{T}/m)$ and $\gamma_2 = \gamma_1 + (g - \mu_3)$.

Proof: See [14]. ■

IV. Position Control

In this section, we first consider the translational dynamics and design an intermediary control law for each aircraft, F_j in (4), without linear-velocity measurements. It is important to notice that for condition (8) to be satisfied, it is sufficient to guarantee that the third element of the control input F_j is *a priori* bounded. In addition, we can see from (6) that the design of an *a priori* bounded intermediary control input is necessary to guarantee that the term $\frac{\mathcal{T}_j}{m_j}f(\mathbf{q}_j, \mathbf{q}_{d_j})$, which constitutes a perturbation to (4), is bounded. Furthermore, we can notice from the expression of ω_{d_j} in (9) that $\dot{\omega}_{d_j}$ is derived from the expression of $\ddot{F}_j$. Note that to implement a trajectory tracking attitude controller, that necessarily requires the knowledge of $\omega_{d_j}(t)$ and $\dot{\omega}_{d_j}(t)$, we need to ensure that these vectors are bounded and they depend on available signals.

In order to achieve our control objective and solve the above problems, we define the velocity tracking error as; $\tilde{v}_j = (v_j - v_d)$, and introduce the following variables for the j^{th} aircraft

$$\xi_j := p_j - \theta_j, \qquad z_j := \tilde{v}_j - \dot{\theta}_j \tag{10}$$

where $\theta_j \in \mathbb{R}^3$ is a design variable to be determined later. With these definitions, we can easily verify that

$$\dot{\xi}_j = z_j + v_d \tag{11}$$

Exploiting the dynamics (4), we can write

$$\dot{z}_j = F_j - \dot{v}_d - \ddot{\theta}_j - \frac{\mathcal{T}_j}{m_j}f(\mathbf{q}_j, \mathbf{q}_{d_j}) \tag{12}$$

The idea behind the introduction of the new variables θ_j is to design a control scheme that first guarantees that $\xi_j - \xi_k \rightarrow \delta_{jk}$ and $z_j \rightarrow 0$. Once this is achieved, the auxiliary variable θ_j and its time derivative are forced to converge to zero asymptotically achieving hence our original objective, *i.e,* $p_j - p_k \rightarrow \delta_{jk}$ and $\tilde{v}_j \rightarrow 0$. We propose the following linear-velocity free intermediary control input for each VTOL aircraft

$$\begin{cases} F_j = \dot{v}_d - \Theta_j \\ \ddot{\theta}_j = -\Theta_j + k_{d_j}(\xi_j - \psi_j) + \sum_{k=1}^{n} k_{jk}(\xi_{jk} - \delta_{jk}) \end{cases} \tag{13}$$

with $\Theta_j = \left(k_{\theta_{1_j}} \tanh(\theta_j) + k_{\theta_{2_j}} \tanh(\dot{\theta}_j)\right)$, and

$$\dot{\psi}_j = v_d + \lambda(\xi_j - \psi_j) \tag{14}$$

where $\xi_{jk} = (\xi_j - \xi_k)$. The scalar gains k_{d_j}, $k_{\theta_{1_j}}$, $k_{\theta_{2_j}}$ and λ are strictly positive. The gains k_{jk} are the formation-keeping gains defined such that $k_{jk} = k_{kj} > 0$ if aircraft j and k are neighbors, and $k_{jk} = 0$ otherwise. We say that two aircraft are "neighbors" if they can communicate with each other and share their states information. The variable ψ_j is the output of the auxiliary system (14), which plays the role of an estimator of the linear velocity at this stage of the control design, and the term $(\xi_j - \psi_j)$ is used in the control to ensure that $v_j \rightarrow v_d(t)$ without linear-velocity measurements.

It is important to mention that the first advantage of using the new auxiliary variable θ_j is that it allows the design of a bounded intermediary control input F_j. In fact, we can see from the proposed control (13) that F_j is differentiable and is guaranteed to be bounded as

$$\|F_j\| \leq \delta_d + \sqrt{3}(k_{\theta_{1_j}} + k_{\theta_{2_j}}) \tag{15}$$

with $\delta_d = \|\dot{v}_d(t)\|_\infty$, regardless of the number of neighbors of vehicle j. In addition, an upper bound of the extracted value of the thrust $\mathcal{T}_j$, in (6), can be determined *a priori* and is given as

$$\mathcal{T}_j \leq m_j\left(g + \delta_d + \sqrt{3}(k_{\theta_{1_j}} + k_{\theta_{2_j}})\right) := \mathcal{T}_j^b \tag{16}$$

with $\mathcal{T}_j^b$ a positive constant. Also, the extracted desired attitude in (7) is guaranteed to be realizable.

V. Attitude Control

In this section, we consider the rotational dynamics and design a torque input for each aircraft in order to track the desired orientation, $\mathbf{q}_{d_j}(t)$, extracted according to (7) from F_j given in (13). We define the attitude tracking error for each vehicle, namely $\tilde{\mathbf{q}}_j \triangleq (\tilde{q}_j^T, \tilde{\eta}_j)^T = \mathbf{q}_{d_j}^{-1} \odot \mathbf{q}_j$, governed by the unit-quaternion dynamics

$$\begin{cases} \dot{\tilde{q}}_j = \frac{1}{2}(\tilde{\eta}_j\ I_3 + S(\tilde{q}_j))\tilde{\omega}_j, \quad \dot{\tilde{\eta}}_j = -\ \frac{1}{2}\tilde{q}_j^T\tilde{\omega}_j, \\ \tilde{\omega}_j = \omega_j - R(\tilde{\mathbf{q}}_j)\ \omega_{d_j}, \end{cases} \tag{17}$$

where $\tilde{\omega}_j$ is the angular velocity error vector. $R(\tilde{\mathbf{q}}_j)$ is the rotation matrix, related to $\tilde{\mathbf{q}}_j$, and is given by $R(\tilde{\mathbf{q}}_j) = R(\mathbf{q}_j)R(\mathbf{q}_{d_j})^T$, [17]. The vector $\omega_{d_j}(t)$ is the desired angular velocity and is given in (9) for each aircraft. It is important to notice that from the design (13) and (9), the desired angular velocity, $\omega_{d_j}(t)$ is independent from the linear-velocity tracking error. In addition, in view of (13)-(14), the time derivative of the desired angular velocity can be expressed as

$$\dot{\omega}_{d_j} = \Psi_{1j} - \Psi_{2j}\left(k_{d_j}z_j + \sum_{k=1}^{n} k_{jk}(z_j - z_k)\right) \tag{18}$$

where the terms Ψ_{1j} and Ψ_{2j} are given in (A-1)-(A-2) in the appendix for the sake of clarity of presentation. It is important to mention that only $\dot{\omega}_{d_j}$ is function of the vectors z_j and z_k that explicitly depend on the linear-velocities of the vehicles. This is another advantage of the introduction of the variable θ_j in the translational control design method. Note that without this new variable, the use of a partial state feedback directly in the expression of F_j results in

ω_{d_j} function of the linear-velocity and $\dot{\omega}_{d_j}$ function of the linear-acceleration, which will make the control design more complicated.

From the definition of $\tilde{\mathbf{q}}_j$ and the rotation matrix, it can be easily shown that the function $f(\mathbf{q}_j, \mathbf{q}_{d_j})$ satisfies the following equation

$$R(\mathbf{q}_j)f(\mathbf{q}_j,\mathbf{q}_{d_j}) = \left(I_3 - R(\tilde{\mathbf{q}}_j)\right)\hat{e}_3 = 2\begin{pmatrix} \tilde{\eta}_j\tilde{q}_{2j} - \tilde{q}_{1j}\tilde{q}_{3j} \\ -\tilde{\eta}_j\tilde{q}_{1j} - \tilde{q}_{2j}\tilde{q}_{3j} \\ (\tilde{q}_{1j}^2 + \tilde{q}_{2j}^2) \end{pmatrix} = 2S(\bar{q}_j)\tilde{q}_j \quad (19)$$

with $\tilde{q}_j = (\tilde{q}_{1j}, \tilde{q}_{2j}, \tilde{q}_{3j})^T$ and $\bar{q}_j = (\tilde{q}_{2j}, -\tilde{q}_{1j}, -\tilde{\eta}_j)^T$. In addition, it is easy to verify that $\|R(\mathbf{q}_j)^T S(\bar{q}_j)\| \leq 1$.

We introduce the new variable for each aircraft as: $\Omega_j = \tilde{\omega}_j - \beta_j$, where β_j is a design parameter to be determined latter. Exploiting the rotational dynamics (2) with (18), we can easily show that

$$I_{f_j}\dot{\Omega}_j = \tau_j - \mathbf{H}_j(\cdot) + k_{d_j}\chi_j z_j + \chi_j\sum_{k=1}^{n} k_{jk}(z_j - z_k) \quad (20)$$

with $\chi_j = I_{f_j}R(\tilde{\mathbf{q}}_j)\Psi_{2j}$ and $\mathbf{H}_j(\cdot) = (S(\omega_j)I_{f_j}\omega_j - I_{f_j}S(\tilde{\omega}_j)R(\tilde{\mathbf{q}}_j)\omega_{d_j} + I_{f_j}R(\tilde{\mathbf{q}}_j)\Psi_{1j} + I_{f_j}\dot{\beta}_j)$. To this point, we can notice that the attitude error dynamics depend on the linear-velocity tracking error which is not available for feedback.

To design a torque input for each aircraft without linear-velocity measurements, we introduce the following dynamic system

$$\begin{cases} \hat{z}_j := & \dot{\hat{\xi}}_j = u_j - L_p\tilde{\xi}_j \\ \dot{u}_j = & \Phi_j + k_{d_j}\chi_j^T\Omega_j - L_v^2\tilde{\xi}_j \\ & + \sum_{k=1}^{n} k_{jk}\left(\chi_j^T\Omega_j - \chi_k^T\Omega_k\right) \end{cases} \quad (21)$$

for $j \in \mathcal{N}$, where L_p and L_v are strictly positive scalar gains, $\tilde{\xi}_j = (\hat{\xi}_j - \xi_j)$ and $\Phi_j = F_j - \ddot{\theta}_j - \frac{\mathcal{T}_j}{m_j}f(\mathbf{q}_j, \mathbf{q}_{d_j})$. Define the error vector; $\tilde{z}_j := \dot{\tilde{\xi}}_j = \dot{\hat{\xi}}_j - \dot{\xi}_j$, which using (11) can be rewritten as

$$\tilde{z}_j = \hat{z}_j - z_j - v_d \quad (22)$$

The dynamics of $\hat{z}_j$ can be obtained as

$$\dot{\hat{z}}_j = -L_p\tilde{z}_j - L_v^2\tilde{\xi}_j + k_{d_j}\chi_j^T\Omega_j + \sum_{k=1}^{n} k_{jk}\left(\chi_j^T\Omega_j - \chi_k^T\Omega_k\right) \quad (23)$$

We propose the following input torque for each aircraft

$$\tau_j = \mathbf{H}_j(\cdot) - k_{q_j}\tilde{q}_j - k_{\Omega_j}\Omega_j - k_{d_j}\chi_j(\hat{z}_j + L_v\tilde{\xi}_j - v_d) - \chi_j\sum_{k=1}^{n} k_{jk}\left((\hat{z}_j + L_v\tilde{\xi}_j) - (\hat{z}_k + L_v\tilde{\xi}_k)\right) \quad (24)$$

which leads to the closed loop dynamics

$$I_{f_j}\dot{\Omega}_j = -k_{q_j}\tilde{q}_j - k_{\Omega_j}\Omega_j - k_{d_j}\chi_j(\tilde{z}_j + L_v\tilde{\xi}_j) - \chi_j\sum_{k=1}^{n} k_{jk}\left(\left(\tilde{z}_j + L_v\tilde{\xi}_j\right) - \left(\tilde{z}_k + L_v\tilde{\xi}_k\right)\right) \quad (25)$$

for $j \in \mathcal{N}$. It can be seen that since the vector $\mathbf{H}_j(\cdot)$ contains $\dot{\beta}_j$, the design parameter β_j cannot be based on ξ_j since its time derivative will give rise to z_j.

VI. Stability of the overall closed loop System

Our result is stated in the following Theorem.

Theorem 1: Consider the VTOL-UAVs formation modeled as in (1)-(2). Let the reference velocity and the strictly positive gains $k_{\theta_{1_j}}$ and $k_{\theta_{2_j}}$ satisfy

$$\delta_d + \sqrt{3}(k_{\theta_{1_j}} + k_{\theta_{2_j}}) < g \quad (26)$$

with δ_d is given in (15). Let the thrust input $\mathcal{T}_j(t)$ and the desired attitude $\mathbf{q}_{d_j}(t)$ for each aircraft be given, respectively, by (6) and (7), with the intermediary control input, F_j, given by (13)-(14). Let the torque input for each aircraft be as in (24) with

$$\beta_j = -k_{\beta_j}\tilde{q}_j + \frac{2\mathcal{T}_j}{k_{q_j}m_j}S(\bar{q}_j)^T R(\mathbf{q}_j)(u_j - v_d) \quad (27)$$

with $k_{\beta_j} > 0$, $u_j = (\hat{z}_j + L_p\tilde{\xi}_j)$ and $\bar{q}_j = (\tilde{q}_{2j}, -\tilde{q}_{1j}, -\tilde{\eta}_j)^T$. If the control gains satisfy

$$\begin{array}{c} k_{\beta_j}k_{q_j} > \frac{\mathcal{T}_j^b}{m_j}\left(\frac{1}{\sigma_{1j}} + \frac{L_p^2}{\sigma_{2j}}\right) \\ L_p - L_v > \sigma_{1j}\frac{\mathcal{T}_j^b}{m_j}, \quad L_v^3 > \sigma_{2j}\frac{\mathcal{T}_j^b}{m_j} \end{array} \quad (28)$$

for some $\sigma_{1j} > 0$, and $\sigma_{2j} > 0$ and $\mathcal{T}_j^b$ defined in (16), then all signals are bounded and $\lim_{t\to\infty}\tilde{q}_j(t) = 0$, $\lim_{t\to\infty}\tilde{\omega}_j(t) = 0$, $\lim_{t\to\infty}v_j(t) = v_d(t)$ and $\lim_{t\to\infty}(p_j(t) - p_k(t)) = \delta_{jk}$, for all $j, k \in \mathcal{N}$.

Proof: First, it is straightforward to verify that if (26) is satisfied, then $\|F_j\| < g$ and condition (8) is always satisfied, and hence it is always possible to extract the magnitude of the thrust and the desired attitude from (6) and (7) respectively for each VTOL vehicle in the team.

Consider the following Lyapunov function candidate

$$\begin{aligned} V = & \tfrac{1}{2}\textstyle\sum_{j=1}^{n}\left(z_j^T z_j + k_{d_j}(\xi_j - \psi_j)^T(\xi_j - \psi_j)\right) \\ & + \tfrac{1}{4}\textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}(\xi_{jk} - \delta_{jk})^T(\xi_{jk} - \delta_{jk}) \\ & + \tfrac{1}{2}\textstyle\sum_{j=1}^{n}(\tilde{z}_j + L_v\tilde{\xi}_j)^T(\tilde{z}_j + L_v\tilde{\xi}_j) \\ & + \tfrac{1}{2}\textstyle\sum_{j=1}^{n} L_v L_p\tilde{\xi}_j^T\tilde{\xi}_j \\ & + \tfrac{1}{2}\textstyle\sum_{j=1}^{n}\Omega_j^T I_{f_j}\Omega_j + 2\sum_{j=1}^{n} k_{q_j}(1 - \tilde{\eta}_j) \end{aligned}$$

whose time derivative along the closed loop dynamics is obtained as

$$\begin{aligned} \dot{V} = & -\textstyle\sum_{j=1}^{n}\frac{2\mathcal{T}_j}{m_j}z_j^T R(\tilde{\mathbf{q}}_j)^T S(\bar{q}_j)\tilde{q}_j - \sum_{j=1}^{n} L_v^3\tilde{\xi}_j^T\tilde{\xi}_j \\ & -\textstyle\sum_{j=1}^{n}\lambda k_{d_j}(\xi_j - \psi_j)^T(\xi_j - \psi_j) + \sum_{j=1}^{n} k_{q_j}\tilde{q}_j^T\beta_j \\ & -\textstyle\sum_{j=1}^{n}(L_p - L_v)\tilde{z}_j^T\tilde{z}_j - \sum_{j=1}^{n} k_{\Omega_j}\Omega_j^T\Omega_j \end{aligned}$$

where we have used equations (13)-(14), (19), (23) and (25) with the relations

$$\tfrac{1}{2}\textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}(\xi_{jk} - \delta_{jk})^T(z_j - z_k) = \sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}(\xi_{jk} - \delta_{jk})^T z_j$$

$$\textstyle\sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}\left(\tilde{z}_j + L_v\tilde{\xi}_j\right)^T\left(\chi_j^T\Omega_j - \chi_k^T\Omega_k\right) = \sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}\Omega_j^T\chi_j\left(\left(\tilde{z}_j + L_v\tilde{\xi}_j\right) - \left(\tilde{z}_k + L_v\tilde{\xi}_k\right)\right)$$

which can be easily verified since the communication flow is assumed indirected.

Then, using (27) with (22) yields

$$\begin{aligned}\dot{V} = & -\sum_{j=1}^{n} \lambda k_{d_j} (\xi_j - \psi_j)^T (\xi_j - \psi_j) \\ & -\sum_{j=1}^{n} L_v^3 \tilde{\xi}_j^T \tilde{\xi}_j - \sum_{j=1}^{n} (L_p - L_v) \tilde{z}_j^T \tilde{z}_j \\ & -\sum_{j=1}^{n} k_{q_j} k_{\beta_j} \tilde{q}_j^T \tilde{q}_j - \sum_{j=1}^{n} k_{\Omega_j} \Omega_j^T \Omega_j \\ & +\sum_{j=1}^{n} \frac{2\mathcal{T}_j}{m_j} \tilde{q}_j^T S(\bar{q}_j)^T R(\mathbf{q}_j) \left(\tilde{z}_j + L_p \tilde{\xi}_j \right)\end{aligned}$$

which can be upper bounded as

$$\begin{aligned}\dot{V} \leq & -\sum_{j=1}^{n} \lambda k_{d_j} \|\xi_j - \psi_j\|^2 \\ & -\sum_{j=1}^{n} (L_p - L_v - \sigma_{1j} \frac{\mathcal{T}_j^b}{m_j}) \|\tilde{z}_j\|^2 \\ & -\sum_{j=1}^{n} (L_v^3 - \sigma_{2j} \frac{\mathcal{T}_j^b}{m_j}) \|\tilde{\xi}_j\|^2 - \sum_{j=1}^{n} k_{\Omega_j} \|\Omega_j\|^2 \\ & -\sum_{j=1}^{n} \left(k_{\beta_j} k_{q_j} - \frac{\mathcal{T}_j^b}{m_j} (\frac{1}{\sigma_{1j}} + \frac{L_p^2}{\sigma_{2j}}) \right) \|\tilde{q}_j\|^2\end{aligned}$$

where we have used (16), the fact that $L_p > L_v$ from (28) and young's inequality: for any two real numbers a and b we have $2ab < \sigma a^2 + b^2/\sigma$, for some $\sigma > 0$. Hence, $\dot{V}$ is negative semi-definite if the gains satisfy conditions (28).

To this point, we can conclude that z_j, $(\xi_j - \psi_j)$, $\tilde{z}_j$, $\tilde{\xi}_j$, Ω_j and $\tilde{q}_j$, for $j \in \mathcal{N}$, and ξ_{jk}, for each pair of communicating aircraft (j,k), are bounded. Since each aircraft can communicate with at least one other aircraft in the team, we conclude that ξ_{jk} is bounded for all $j,k \in \mathcal{N}$. Consequently, we know that $\dot{z}_j$, $\dot{\tilde{z}}_j$, $\ddot{\theta}_j$ and u_j are bounded. Also, we have $\dot{\psi}_j$ and β_j are bounded for $j \in \mathcal{N}$.

Furthermore, we can see from (25) that $\dot{\Omega}_j$ is bounded. Also, since $\tilde{\omega}_j = \Omega_j + \beta_j$ is bounded, we know from (17) that $\dot{\tilde{q}}_j$ is bounded . Hence, we can conclude that $\ddot{V}$ is bounded. Invoking Barbalat's Lemma [18], we can conclude that $\lim_{t\to\infty}(\xi_j(t) - \psi_j(t)) = 0$, $\lim_{t\to\infty} \tilde{z}_j(t) = 0$, $\lim_{t\to\infty} \tilde{\xi}_j(t) = 0$, $\lim_{t\to\infty} \tilde{q}_j(t) = 0$, and $\lim_{t\to\infty} \Omega_j(t) = 0$, for $j \in \mathcal{N}$.

Since $\lim_{t\to\infty}(\xi_j(t) - \psi_j(t)) = 0$, we know that $\lim_{t\to\infty} \dot{\psi}_j(t) = v_d(t)$. Also, we can verify that $(\ddot{\xi}_j - \ddot{\psi}_j)$ is bounded from the boundedness of $\dot{z}_j$ and $(\dot{\xi}_j - \dot{\psi}_j)$. As a result, and using Barbalat's Lemma, we conclude that $\lim_{t\to\infty}(\dot{\xi}_j(t) - \dot{\psi}_j(t)) = 0$. Consequently, we have $\lim_{t\to\infty} \dot{\xi}_j(t) = v_d(t)$ and $\lim_{t\to\infty} z_j(t) = 0$ for $j \in \mathcal{N}$. Also, since $\tilde{z}_j$ converges to zero, we have $\lim_{t\to\infty} \hat{z}_j(t) = v_d(t)$, and consequently $\lim_{t\to\infty} \beta_j(t) = 0$, since $\lim_{t\to\infty} u_j(t) = \lim_{t\to\infty} \hat{z}_j(t)$, and hence we conclude that $\lim_{t\to\infty} \tilde{\omega}_j(t) = 0$ for $j \in \mathcal{N}$.

Exploiting the fact that $\tilde{q}_j$ and $(\xi_j - \psi_j)$ converge to zero asymptotically, we can use the extended barbalat's Lemma (see for example Lemma 2 in [11]) to conclude that $\lim_{t\to\infty} \dot{z}_j(t) = 0$ and hence, we conclude that $\lim_{t\to\infty} \sum_{k=1}^{n} k_{jk}(\xi_{jk} - \delta_{jk}) = 0$, for $j \in \mathcal{N}$, which is equivalent to $\lim_{t\to\infty} \sum_{j=1}^{n} \sum_{k=1}^{n} k_{jk}(\xi_j - \delta_j)^T(\xi_j - \xi_k - \delta_{jk}) = 0$, where the constant vector δ_j can be regarded as the desired position of the j^{th} aircraft with respect to the center of the formation. It is then clear that $\delta_{jk} = (\delta_j - \delta_k)$. Then, since $k_{jk} = k_{kj}$, this last relation is equivalent to: $\lim_{t\to\infty} \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} k_{jk}(\xi_j - \xi_k - \delta_{jk})^T(\xi_j - \xi_k - \delta_{jk}) = 0$, from which we conclude that $\lim_{t\to\infty}(\xi_j(t) - \xi_k(t)) = \delta_{jk}$, for each pair of communicating aircraft (j,k). Since the communication flow is assumed connected, this last result is valid for all $j,k \in \mathcal{N}$.

TABLE I

SIMULATION PARAMETERS

$p_1(0) = (14,0,2)^T$,$p_2(0) = (10,-1,2)^T$,$p_3(0) = (6,0,-2)^T$,
$p_4(0) = (9,-4,1)^T$,$v_1(0) = 0.1(-1,9,-1)^T$,$\mathbf{q}_j(0) = (0,0,0,1)^T$,
$v_2(0) = 0.1(-5,-8,3)^T$, $v_3(0) = 0.1(-2,4,-4)^T$,
$v_4(0) = 0.1(8,-1,1)^T$, $\psi_j(0) = (0,1,-1)^T$,
$\omega_j(0) = \theta_j(0) = \dot{\theta}_j(0) = \tilde{\xi}_j(0) = u_j(0) = (0,0,0)^T$,
$\lambda = 5$, $L_p = 8$, $L_v = 3$, $g = 9.8$, $k_{d_j} = 5$, $k_{\beta_j} = 20$,
$k_{q_j} = 20$, $k_{\Omega_j} = 30$, $k_{\theta_{1_j}} = 2$, $k_{\theta_{2_j}} = 2$, for $j = 1,\ldots,4$,
$k_{jk} = 3$, for $(j,k) \in \{(1,2),(1,3),(2,3),(2,4)\}$,$\delta_1 = 2(1,1,0)^T$,
$\delta_2 = 2(-1,1,0)^T$,$\delta_3 = 2(-1,-1,0)^T$,$\delta_4 = 2(1,-1,0)^T$.

Exploiting the above results, we can see that the term; $\eta_j = \left(k_{d_j}(\xi_j - \psi_j) + \sum_{k=1}^{n} k_{jk}(\xi_{jk} - \delta_{jk})\right)$, is globally bounded and converges asymptotically to zero. Then following similar steps as in the proof of Theorem 1 in [13], we can conclude that $\lim_{t\to\infty} \theta_j(t) = 0$, $\lim_{t\to\infty} \dot{\theta}_j(t) = 0$ for all $j \in \mathcal{N}$. As a result, we conclude that $\lim_{t\to\infty} \tilde{v}_j(t) = 0$ and $\lim_{t\to\infty}(p_j(t) - p_k(t)) = \delta_{jk}$ for all $j,k \in \mathcal{N}$. ∎

Remark 1: It is important to mention that in order to implement the torque input (24), we need the time derivative of β_j, given in (27), which, in view of the expression of $\dot{u}_j$ in (21), does not depend on the linear-velocity.

VII. SIMULATION RESULTS

In order to demonstrate the effectiveness of the proposed control scheme, we present in this section simulation results obtained using SIMULINK. We consider a group of four aircraft modeled as in (1)-(2), with $m_j = 3\ kg$, I_{f_j} =diag$(0.13, 0.13, 0.04)\ kg.m^2$, for $j = 1,...,4$. The simulation parameters are shown in table 1, where the gains are selected to satisfy conditions (26) and (28). The desired trajectory is given by $v_d(t) = (\sin(0.1t), 0.5\cos(0.1t), -1)\ m/\sec$. In addition, the vectors δ_{jk} are computed according to the variables δ_j in the above table such that the desired formation pattern is a square, with $\delta_{jk} = (\delta_j - \delta_k)$. The obtained results are illustrated in Figs. 1-2. Fig. 1 illustrates the three components of the velocity tracking errors of each aircraft. It is clear from these figures that asymptotic convergence to zero is guaranteed. In order to illustrate the vehicle's formation, a 3-D plot of the positions of the vehicles in space is given in Fig. 2, where we can see that the prescribed square formation is maintained.

VIII. CONCLUSION

A formation control scheme without velocity measurements for a class of under-actuated VTOL UAVs has been presented. Our approach is based on a cascade control design for the translational and rotational dynamics guaranteeing global asymptotic stability for the overall closed-loop system. To account for the missing linear velocity measurements, an intermediary partial state feedback control scheme has been used in the first stage of the control design with the introduction of some auxiliary variables guaranteeing the boundedness (*a priori*) of the intermediary control F_j

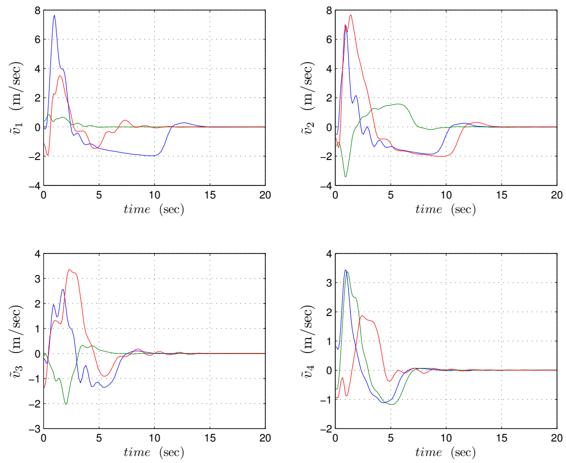

Fig. 1. Velocity tracking errors for the four aircraft

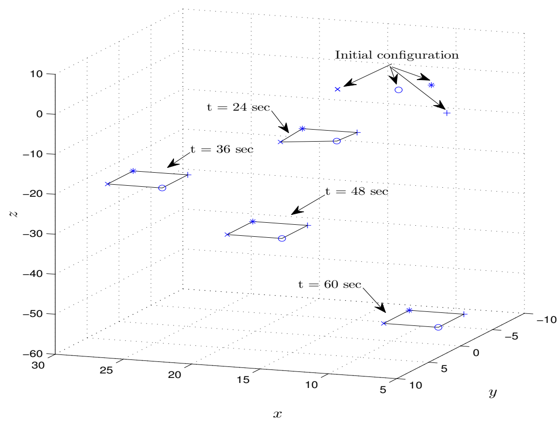

Fig. 2. VTOL aircraft formation

regardless of the number of neighbors of each aircraft. At the second stage of the control design, an additional dynamic system has been introduced to design a linear-velocity-free torque input for each aircraft and achieve the overall required control objective. Simulation results have been provided to show the effectiveness of the proposed scheme.

IX. Appendix

Explicit expressions of the terms Ψ_{1j} and Ψ_{2j}, given in (18), are derived in the following. From (9), we have; $\dot{\omega}_{d_j} = \dot{\Xi}(F_j)\dot{F}_j + \Xi(F_j)\ddot{F}_j$, where $\dot{\Xi}(F_j)$ can be directly obtained from $\dot{F}_j$. Note that ω_{d_j} and $\dot{\omega}_{d_j}$ are defined if condition (8) is satisfied. In view of the design (13), we have

$$\dot{F}_j = \ddot{v}_d - k_{\theta_{1_j}} h(\theta_j)\dot{\theta}_j - k_{\theta_{2_j}} h(\dot{\theta}_j)\ddot{\theta}_j$$

$$\begin{aligned}\ddot{F}_j = \; & v_d^{(3)} - k_{\theta_{1_j}} \bar{h}(\theta_j)\dot{\theta}_j - (k_{\theta_{1_j}} h(\theta_j) + k_{\theta_{2_j}} \bar{h}(\dot{\theta}_j))\ddot{\theta}_j \\ & - k_{\theta_{2_j}} h(\dot{\theta}_j)\{-k_{\theta_{1_j}} h(\theta_j)\dot{\theta}_j - k_{\theta_{2_j}} h(\dot{\theta}_j)\ddot{\theta}_j \\ & + k_{d_j}(v_d - \dot{\psi}_j) + k_{d_j} z_j + \textstyle\sum_{k=1}^{n} k_{jk}(z_j - z_k)\}\end{aligned}$$

where $\ddot{v}_d$ and $v_d^{(3)}$ are, respectively, the second and third derivatives of the desired velocity, which are assumed to be bounded, and for $x \in \mathbb{R}^3$, we have defined $h(x) = \text{diag}((v_1^1, v_1^2, v_1^3)^T)$, with $v_1^i = (1 - \tanh^2(x_i))$, for $i = 1,2,3$, $\bar{h}(x) = \text{diag}((v_2^1, v_2^2, v_2^3)^T)$, with $v_2^i = (-2\dot{x}_i(1 - \tanh^2(x_i))\tanh(x_i))$, for $i = 1,2,3$, and "diag" is the diagonal matrix operator. Hence we can rewrite $\dot{\omega}_{d_j}$ as in (18) with

$$\begin{aligned}\Psi_{1j} = & \dot{\Xi}(F_j)\dot{F}_j + \Xi(F_j)\{v_d^{(3)} - k_{\theta_{1_j}} \bar{h}(\theta_j)\dot{\theta}_j - (k_{\theta_{1_j}} h(\theta_j) \\ & + k_{\theta_{2_j}} \bar{h}(\dot{\theta}_j))\ddot{\theta}_j - k_{\theta_{2_j}} h(\dot{\theta}_j)\{-k_{\theta_{1_j}} h(\theta_j)\dot{\theta}_j \\ & - k_{\theta_{2_j}} h(\dot{\theta}_j)\ddot{\theta}_j + k_{d_j}(v_d - \dot{\psi}_j)\}\} \quad \text{(A-1)}\end{aligned}$$

$$\Psi_{2j} = k_{\theta_{2_j}} \Xi(F_j) h(\dot{\theta}_j). \quad \text{(A-2)}$$

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