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Full paper

Adaptive controller for non-holonomic mobile robots with matched uncertainties

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Abstract—This paper deals with the backstepping approach for the design of adaptive discontinuous time-invariant controllers for the point-stabilization of mobile robots with matched uncertainties. First of all, we derive a control law in the disturbance-free case guaranteeing exponential convergence for a unicycle-like mobile robot. Furthermore, an adaptive version of the previous control law is proposed when the mobile robot is subjected to input disturbances. Finally, simulation results are presented.

Keywords: Backstepping method; Lyapunov function; adaptive control; mobile robot.

1. INTRODUCTION AND PROBLEM FORMULATION

Feedback stabilization of non-holonomic systems to a specified configuration has recently enjoyed great attention in the automatic control community. The challenge of this problem is due to the fact that it is not possible to find any smooth time-invariant stabilizing feedback for this class of systems [1]. To overcome this difficulty, several directions of research have evolved. Among the proposed solutions are smooth time-varying controllers, which involve periodic functions depending explicitly on an exogenous time variable, leading to low rates of convergence, and to non-smooth and oscillating trajectories (see, e.g. [2, 3]). An alternative to time-dependant smooth controllers is the discontinuous or piecewise-continuous time-invariant controllers, often leading to exponential convergence and generating non-oscillating trajectories (see [4–10]). To our knowledge, there are few works in the literature dealing with uncertain non-holonomic systems. Among these works one can distinguish [7, 11] and [12]. In fact, in [7], the authors have proposed a quasi-continuous adaptive controller for a third-order non-holonomic system by means of the invariant manifold approach, assuming that only one of the two inputs is subjected to disturbances. In papers [11] and [12], the authors have proposed a backstepping-based time-varying adaptive scheme for a special class of uncertain non-holonomic chained systems.

In [13] we have proposed a discontinuous time-invariant state feedback for the stabilization of n-dimensional non-holonomic chained systems, in the disturbance-free case, by means of the backstepping approach [14]. In this paper, using the same approach, we propose a time-invariant adaptive controller guaranteeing exponential convergence for a unicycle-like mobile robot subjected to input disturbances. Our controller is smooth and well-defined everywhere except on the manifold described by a null initial orientation (i.e. $\theta(0) = 0$) of the vehicle. In this case we have just to drive away the mobile robot from this configuration using an arbitrary open-loop control for a small period of time and then switch to the feedback controller. In other words, our time-invariant feedback controller is smooth and well-defined provided that $(\theta(0) \neq 0)$.

The unicycle-like mobile robot, in the disturbance-free case, is described by the following kinematics model:

$$\begin{aligned} \dot{x} &= v \cos\theta, \\ \dot{y} &= v \sin\theta, \\ \dot{\theta} &= \omega, \end{aligned} \tag{1}$$

which can be transformed into the following third-order chained form:

$$\begin{aligned} \dot{x}_1 &= u_1, \\ \dot{x}_2 &= u_2, \\ \dot{x}_3 &= x_2 u_1, \end{aligned} \tag{2}$$

using the following input and coordinate transformations:

$$\begin{aligned} x_1 &= \theta, \\ x_2 &= x\cos\theta + y\sin\theta, \\ x_3 &= x\sin\theta - y\cos\theta, \end{aligned}$$

and

$$\begin{aligned} u_1 &= \omega, \\ u_2 &= v - x_3\omega. \end{aligned}$$

When the mobile robot is subjected to input disturbances, system (1) becomes:

$$\begin{aligned} \dot{x} &= (v + \xi_1)\cos\theta, \\ \dot{y} &= (v + \xi_1)\sin\theta, \\ \dot{\theta} &= \omega + \xi_2 \quad , \end{aligned} \tag{3}$$

where v and ω are the control variables and ξ_1 and ξ_2 are constant inputs disturbances.

Applying the same transformations used for system (1) leads to:

$$\begin{aligned}\dot{x}_1 &= u_1 + \xi_2,\\ \dot{x}_2 &= u_2 + \xi_1 - x_3\xi_2,\\ \dot{x}_3 &= x_2 u_1 + x_2\xi_2.\end{aligned} \tag{4}$$

From (4), it is clear that if $\xi_1 = \xi_2 = 0$, we obtain the chained form (2).

The paper is organized as follows: In Section 2, we present a procedure for the design of a discontinuous time-invariant stabilizing controller for a unicycle-like vehicle (1). In Section 3, a discontinuous time-invariant adaptive controller is derived for the stabilization of the unicycle-like vehicle subjected to matched uncertainties (3). In Section 4, simulation results are given to demonstrate the effectiveness of our study. Finally, some concluding remarks end the paper.

2. CONTROL SYNTHESIS IN THE DISTURBANCE-FREE CASE

A general procedure for the design of discontinuous time-invariant stabilizing controllers for n-dimensional non-holonomic chained systems has been proposed in [13]. In this section, we will apply this procedure for the third-order chained system (2). Although the synthesis approaches are different, our resulting controller is of the same class as those proposed in [4]. It is worth noticing that the main result in this paper is still in the disturbance-case presented in Section 3.

Firstly, let us consider system (2) under the change of coordinates $y_i = x_{3-i+1}$ for $1 \leqslant i \leqslant 3$ and the linear state feedback $u_1 = -k_3 y_3, k_3 > 0$:

$$\begin{aligned}\dot{y}_1 &= -k_3 y_2 y_3,\\ \dot{y}_2 &= u_2,\\ \dot{y}_3 &= -k_3 y_3.\end{aligned} \tag{5}$$

Step 1. Let us take the following Lyapunov candidate function for the first equation of (5):

$$V_1(y_1) = \frac{1}{2} y_1^2. \tag{6}$$

Considering y_2 as a virtual control law defined over $\Omega = \{(y_1, y_2, y_3) \in IR^3/ y_3(t) \neq 0,\ t \geqslant 0\}$ as follows:

$$\Psi_1(y_1, y_3) = \frac{k_1}{k_3}\frac{y_1}{y_3}, \tag{7}$$

the time derivative of (6) becomes:

$$\dot{V}_1(y_1) = -k_1 y_1^2, \tag{8}$$

where k_1 is a positive parameter.

Step 2. Now, let us introduce the new variable $z_2 = y_2 - \Psi_1(y_1, y_3)$ which represents the deviation between y_2 and the virtual control Ψ_1, and consider the first two equations of (5) where y_2 is substituted by $z_2 + \Psi_1(y_1, y_3)$:

$$\begin{aligned} \dot{y}_1 &= -k_3 y_3 (z_2 + \Psi_1), \\ \dot{z}_2 &= u_2 + k_1(z_2 + \Psi_1) - k_1 \frac{y_1}{y_3}. \end{aligned} \tag{9}$$

Using the following Lyapunov candidate function:

$$V_2(y_1, z_2) = V_1(y_1) + \frac{1}{2} z_2^2, \tag{10}$$

and the following control law defined over Ω:

$$u_2 = \Psi_2(y_1, y_2, y_3) \; = k_3 y_1 y_3 - k_1(z_2 + \Psi_1) - k_2 z_2 + k_1 \frac{y_1}{y_3}, \tag{11}$$

leads to:

$$\dot{V}_2(y_1, z_2) = -k_1 y_1^2 - k_2 z_2^2, \tag{12}$$

where k_2 is a positive parameter.

Now, one can easily conclude that y_1 and z_2 are bounded and tend to zero when t tends to infinity. Therefore, y_2 tends to $(k_1/k_3)(y_1/y_3)$.

To guarantee the boundedness and the convergence to zero of y_2, one must ensure the boundedness and the convergence to zero of y_1/y_3. So, from (6) and (8), one can conclude that y_1 decays to zero as $\exp(-k_1 t)$ when $t \to \infty$. Therefore, if we take $k_1 > k_3$, the boundedness and the convergence to zero of y_1/y_3 becomes obvious whenever $y_3(0) \neq 0$, since y_3 decays to zero as $\exp(-k_3 t)$.

The previous results can be summarized in the following proposition

PROPOSITION 1. *Consider the following control law defined over* $\Omega = \{(y_1, y_2, y_3) \in \Re^3 / y_3 \neq 0\}$*:*

$$\begin{aligned} u_1 &= -k_3 y_3, \\ u_2 &= k_3 y_1 y_3 - (k_2 + k_1) y_2 + k_1 \left(1 + \frac{k_2}{k_3}\right) \frac{y_1}{y_3}, \end{aligned} \tag{13}$$

with $y_i = x_{3-i+1}$, $1 \leqslant i \leqslant 3$, $k_3 > 0$, $k_2 > 0$ *and* $k_1 > k_3$, *and assume that* $y_3(0) \neq 0$. *Then, the following hold :*

(i) The whole state of the closed loop system (2)–(13) *remains in the domain* Ω,

(ii) *The whole state of the closed-loop system* (2)–(13) *is bounded and tends to zero when* t *tends to infinity*

(iii) *The control law* (13) *is bounded and well-defined for all* $t \geqslant 0$.

3. ADAPTIVE CONTROL DESIGN

In this section, we will derive a discontinuous time-invariant adaptive controller for the stabilization of system (4), using the backstepping approach. To this end, let us introduce the change of coordinates $y_i = x_{3-i+1}$ for $1 \leqslant i \leqslant 3$ leading to:

$$\begin{aligned}\dot{y}_1 &= y_2 u_1 + y_2 \xi_2,\\ \dot{y}_2 &= u_2 + \xi_1 - y_1 \xi_2,\\ \dot{y}_3 &= u_1 + \xi_2.\end{aligned} \tag{14}$$

Let us first determine an adaptive control law u_1 for the stabilization of the last equation of (14):

$$\dot{y}_3 = u_1 + \xi_2. \tag{15}$$

Consider the following Lyapunov candidate function:

$$V(y_3, \tilde{\xi}_2) = \frac{1}{2} y_3^2 + \frac{1}{2\Gamma_3} \tilde{\xi}_2^2, \tag{16}$$

where Γ_3 is a positive parameter and $\tilde{\xi}_2 = \xi_2 - \hat{\xi}_2$, with ξ_2 the unknown constant parameter and $\hat{\xi}_2$ its estimated value.

In view of (15), differentiating (16) with respect to time leads to:

$$\dot{V} = y_3 \big(u_1 + \hat{\xi}_2\big) + \tilde{\xi}_2 \left(y_3 + \frac{1}{\Gamma_3} \dot{\tilde{\xi}}_2 \right). \tag{17}$$

Vanishing the second term of the right-hand side of (17) and choosing the following control law:

$$u_1 = -k_3 y_3 - \hat{\xi}_2, \tag{18}$$

leads to the following negative semi-definite function:

$$\dot{V}(y_3) = -k_3 y_3^2, \tag{19}$$

and gives the following expression for the estimation error:

$$\dot{\tilde{\xi}}_2 = -\Gamma_3 y_3. \tag{20}$$

Since ξ_2 is assumed to be constant, one has the following adaptive controller for the stabilization of y_3:

$$\begin{aligned} u_1 &= -k_3 y_3 - \hat{\xi}_2, \\ \dot{\hat{\xi}}_2 &= \Gamma_3 y_3, \end{aligned} \tag{21}$$

where k_3 is a positive parameter.

From (19), one can easily conclude that y_3 tends to zero when t tends to infinity. Using the La Salle invariance theorem, the convergence of $\tilde{\xi}_2$ to zero immediately follows from (15) and (18).

Under the dynamic control law (21), system (14) becomes:

$$\begin{aligned} \dot{y}_1 &= -k_3 y_2 y_3 + y_2 \tilde{\xi}_2, \\ \dot{y}_2 &= u_2 + \xi_1 - y_1(\hat{\xi}_2 + \tilde{\xi}_2), \\ \dot{y}_3 &= -k_3 y_3 + \tilde{\xi}_2, \\ \dot{\hat{\xi}}_2 &= \Gamma_3 y_3. \end{aligned} \tag{22}$$

Since $\tilde{\xi}_2$ tends to zero when t tends to infinity, system (22) becomes:

$$\begin{aligned} \dot{y}_1 &= -k_3 y_2 y_3, \\ \dot{y}_2 &= u_2 + \xi_1 - y_1 \hat{\xi}_2, \\ \dot{y}_3 &= -k_3 y_3, \\ \dot{\hat{\xi}}_2 &= \Gamma_3 y_3. \end{aligned} \tag{23}$$

Now, let us apply the backstepping procedure to design a discontinuous feedback u_2 for system (23). The procedure is in two steps. The first step consists in finding an adequate control Lyapunov function, for the first equation of (23), which leads to a virtual control law $y_2 = \Psi_1(y_1, y_3)$ that stabilizes y_1. The second step consists in finding the control law u_2 which stabilizes both y_1 and $(y_2 - \Psi_1(y_1, y_3))$. Finally, to guarantee the boundedness and the convergence to zero of the whole state, one must ensure the boundedness and the convergence to zero of $\Psi_1(y_1, y_3)$.

Step 1. Let us consider the first equation of (23):

$$\dot{y}_1 = -k_3 y_2 y_3, \tag{24}$$

with the following Lyapunov candidate function:

$$V_1(y_1) = \frac{1}{2} y_1^2. \tag{25}$$

Differentiating (25) with respect to time and considering y_2 as a virtual control law defined over $\Omega_3 = \{(y_1, y_2, y_3) \in \Re^3 / y_3 \neq 0\}$ as follows:

$$\Psi_1(y_1, y_3) = \frac{k_1}{k_3} \frac{y_1}{y_3}, \tag{26}$$

where k_1 is a positive parameter, leads to:

$$\dot{V}_1 = -k_1 y_1^2. \tag{27}$$

Step 2. Now, let us introduce a new variable $z_2 \equiv y_2 - \Psi_1(y_1, y_3) = y_2 - (k_1/k_3)(y_1/y_3)$ and consider the following subsystem obtained by substituting y_2 by $z_2 + (k_1/k_3)(y_1/y_3)$ in the first two equations of (23):

$$\begin{aligned}
\dot{y}_1 &= (-k_3 y_3)\Big(z_2 + \frac{k_1}{k_3}\frac{y_1}{y_3}\Big),\\
\dot{z}_2 &= u_2 + \xi_1 - y_1\hat{\xi}_2 + k_1\Big(z_2 + \frac{k_1}{k_3}\frac{y_1}{y_3}\Big) - k_1\frac{y_1}{y_3}.
\end{aligned} \tag{28}$$

Taking the following Lyapunov candidate function:

$$V_2(y_1, z_2, \tilde{\xi}_1) = V_1(y_1) + \frac{1}{2}z_2^2 + \frac{1}{2\Gamma_1}\tilde{\xi}_1^2, \tag{29}$$

where Γ_1 is a positive parameter and $\tilde{\xi}_1 = \xi_1 - \hat{\xi}_1$, with ξ_1 the unknown constant parameter and $\hat{\xi}_1$ its estimated value.

Differentiating (29) with respect to time yields:

$$\begin{aligned}
\dot{V}_2 = {} & y_1\left(z_2 + \frac{k_1}{k_3}\frac{y_1}{y_3}\right)(-k_3 y_3)\\
& + z_2\left(u_2 + \hat{\xi}_1 + \tilde{\xi}_1 - y_1\hat{\xi}_2 + k_1\left(z_2 + \frac{k_1}{k_3}\frac{y_1}{y_3}\right) - k_1\frac{y_1}{y_3}\right)\\
& + \frac{1}{\Gamma_1}\tilde{\xi}_1\dot{\tilde{\xi}}_1.
\end{aligned} \tag{30}$$

Taking the control law u_2 as:

$$u_2 = k_3 y_1 y_3 - (k_1 + k_2)y_2 + \left(k_1 + \frac{k_1 k_2}{k_3}\right)\frac{y_1}{y_3} - \hat{\xi}_1 + y_1\hat{\xi}_2, \tag{31}$$

where k_2 is a positive parameter, and vanishing the terms with $\tilde{\xi}_1$ we get:

$$\frac{1}{\Gamma_1}\tilde{\xi}_1\dot{\tilde{\xi}}_1 + z_2\tilde{\xi}_1 = 0, \tag{32}$$

which gives the adaptation law for the constant parameter ξ_1 as:

$$\dot{\hat{\xi}}_1 = \Gamma_1 z_2. \tag{33}$$

Equation (30) then becomes:

$$\dot{V}_2 = -k_1 y_1^2 - k_2 z_2^2. \tag{34}$$

Now, it is clear that y_1 and z_2 will be bounded and tend to zero when t tends to infinity. Since $z_2 = y_2 - (k_1/k_3)(y_1/y_3) \to 0$ when $t \to \infty$, we have just to ensure the boundedness and the convergence to zero of the ratio y_1/y_3. This allows us to ensure the boundedness of the control law and the convergence to zero of the variable y_2. Hence, one must find some conditions under which the following hold:

(i) $y_3(t)$ never crosses $y_3 = 0$ for all $t > 0$, as long as $y_3(0) \neq 0$.

(ii) y_1/y_3 bounded and $y_1/y_3 \to 0$ when $t \to \infty$.

From the last two equations of (22) it is clear that $y_3(t)$ is given by the solution of the following differential equation:

$$\ddot{y}_3 + k_3\dot{y}_3 + \Gamma_3 y_3 = 0. \tag{35}$$

If the parameters k_3 and Γ_3 are such that $k_3^2 - 4\Gamma_3 > 0$, the solution $y_3(t)$ is given by:

$$y_3(t) = C_1 \exp(\lambda_1 t) + C_2 \exp(\lambda_2 t), \tag{36}$$

with:

$$\lambda_1 = \frac{-k_3 - \sqrt{k_3^2 - 4\Gamma_3}}{2}, \qquad \lambda_2 = \frac{-k_3 + \sqrt{k_3^2 - 4\Gamma_3}}{2},$$
$$C_1 = \frac{\tilde{\xi}_2(0) + \lambda_1 y_3(0)}{\lambda_1 - \lambda_2} \quad \text{and} \quad C_2 = \frac{\tilde{\xi}_2(0) + \lambda_2 y_3(0)}{\lambda_2 - \lambda_1}.$$

To satisfy condition (i), the following inequality must be fulfilled:

$$\frac{\tilde{\xi}_2(0) + \lambda_1 y_3(0)}{\tilde{\xi}_2(0) + \lambda_2 y_3(0)} < 1. \tag{37}$$

This means that the time for which $y_3(t) = 0$ is negative.

Now, we focus our attention on condition (ii). From (25) and (27), it is clear that y_1 decays to zero as $\exp(-k_1 t)$ when t tends to infinity. From the third equation of (23), it is clear that y_3 decays to zero as $\exp(-k_3 t)$ when t tends to infinity. Therefore, y_1/y_3 decays to zero as $\exp(-(k_1 - k_3)t)$ when t tends to infinity, as long as $k_1 > k_3$.

Finally, one can summarize the previous results in the following theorem

THEOREM 1. *Consider system* (4) *under the following adaptive controller defined over* $\Omega = \{(y_1, y_2, y_3) \in \Re^3 / y_3 \neq 0\}$:

$$\begin{aligned}
u_1 &= -k_3 y_3 - \hat{\xi}_2,\\
u_2 &= k_3 y_1 y_3 - (k_1 + k_2) y_2 + \left(k_1 + \frac{k_1 k_2}{k_3}\right)\frac{y_1}{y_3} - \hat{\xi}_1 + y_1 \hat{\xi}_2,\\
\dot{\hat{\xi}}_1 &= \Gamma_1\left(y_2 - \frac{k_1}{k_3}\frac{y_1}{y_3}\right),\\
\dot{\hat{\xi}}_2 &= \Gamma_3 y_3,
\end{aligned} \tag{38}$$

where $y_i = x_{3-i+1}$, $1 \leqslant i \leqslant 3$, $k_2 > 0$, $\Gamma_1 > 0$, $\Gamma_3 > 0$, $k_3 > 2\sqrt{\Gamma_3}$, $k_1 > k_3$ *and* $(\tilde{\xi}_2(0) + \lambda_1 y_3(0))/(\tilde{\xi}_2(0) + \lambda_2 y_3(0)) < 1$. *Assume that* $y_3(0) \neq 0$. *Then, the whole* *state of the closed-loop system* (4)–(38) *remains in* Ω *for all* $t \geqslant 0$*, and tends to zero when* t *tends to infinity.*

If the control variable ω is not subjected to any uncertainty (i.e. $\xi_2 = 0$), one can easily deduce the following theorem from the previous development.

THEOREM 2. *Consider system* (4)*, with* $\xi_2 = 0$*, under the following adaptive controller defined over* $\Omega = \{(y_1, y_2, y_3) \in \Re^3 / y_3 \neq 0\}$:

$$
\begin{aligned}
u_1 &= -k_3 y_3, \\
u_2 &= k_3 y_1 y_3 - (k_1 + k_2) y_2 + \left(k_1 + \frac{k_1 k_2}{k_3}\right)\frac{y_1}{y_3} - \hat{\xi}_1, \\
\dot{\hat{\xi}}_1 &= \Gamma_1 \left(y_2 - \frac{k_1}{k_3}\frac{y_1}{y_3}\right),
\end{aligned}
\tag{39}
$$

where $y_i = x_{3-i+1}$, $1 \leqslant i \leqslant 3$ *and* $k_2 > 0$*,* $k_3 > 0$*,* $k_1 > k_3$ *and* $\Gamma_1 > 0$.

Assume that $y_3(0) \neq 0$. Then:

(i) The whole state of the closed-loop system (4)–(39) remains in Ω for all $t \geqslant 0$.

(ii) The whole state of the closed-loop system (4)–(39) is bounded and tends to zero when t tends to infinity.

(iii) The control law is well defined and bounded for all $t \geqslant 0$.

Remark 1. *The discontinuity introduced in the control law is not very restrictive since we have just to avoid a null orientation of the mobile robot at* $t = 0$*. If necessary, one can apply an open-loop control* u_2 *for an arbitrary small period of time to make the orientation* $\theta \neq 0$ *and then switch to the feedback* (38) *or* (39).

Remark 2. *It is worth noticing that the condition* $(\tilde{\xi}_2(0) + \lambda_1 y_3(0))/(\tilde{\xi}_2(0) + \lambda_2 y_3(0)) < 1$ *in Theorem* 1 *depends on the initial condition* $\tilde{\xi}_2(0) = \xi_2(0) - \hat{\xi}_2(0)$*, where* $\xi_2(0)$ *is unknown. However, if we assume that the unknown parameter* ξ_2 *is bounded and the bounds are known (i.e.* $|\xi_2| \leqslant \xi_{2\,\max}$*), one can choose the initial value of* $\hat{\xi}_2$ *in accordance with the bounds of* ξ_2 *to make the condition independent from* $\xi_2(0)$*. In fact, one can make the conditions of Theorem* 1 $\tilde{\xi}_2(0)$*-independent according to the following proposition.*

PROPOSITION 2. *Assuming that* $|\xi_2| \leqslant \xi_{2\,\max}$*, where* $\xi_{2\,\max}$ *is a known positive parameter, the condition* $(\tilde{\xi}_2(0) + \lambda_1 y_3(0))/(\tilde{\xi}_2(0) + \lambda_2 y_3(0)) < 1$ *is fulfilled if we take* $\hat{\xi}_2(0)$ *as one of the following expressions*:

(a) $\hat{\xi}_2(0) = -k\,\mathrm{sign}(y_3(0))\xi_{2\,\max} + \lambda_1 y_3(0)$*, with* $k > 1$.

(b) $\hat{\xi}_2(0) = -k\,\mathrm{sign}(y_3(0))\xi_{2\,\max}$*, with* $k > 1$.

Proof.

(a) Consider the $\tilde{\xi}_2(0)$-dependent condition involved in Theorem 1:

$$
\frac{\tilde{\xi}_2(0) + \lambda_1 y_3(0)}{\tilde{\xi}_2(0) + \lambda_2 y_3(0)} < 1. \tag{40}
$$

Condition (40) is fulfilled in the following two cases:

$$\text{(i)} \begin{cases} \tilde{\xi}_2(0) + \lambda_1 y_3(0) > 0, \\ \tilde{\xi}_2(0) + \lambda_2 y_3(0) > 0, \end{cases} \quad \text{for } y_3(0) > 0,$$

$$\text{(ii)} \begin{cases} \tilde{\xi}_2(0) + \lambda_1 y_3(0) < 0, \\ \tilde{\xi}_2(0) + \lambda_2 y_3(0) < 0, \end{cases} \quad \text{for } y_3(0) < 0.$$

From (i), one has:

$$\begin{cases} \lambda_1 > -\dfrac{\tilde{\xi}_2(0)}{y_3(0)}, \\ \lambda_2 > -\dfrac{\tilde{\xi}_2(0)}{y_3(0)}. \end{cases}$$

Since $\lambda_1 < \lambda_2 < 0$, it suffices to consider only the condition $\lambda_1 > -\tilde{\xi}_2(0)/y_3(0)$.

This leads to:

$$\lambda_1 + \frac{\tilde{\xi}_2(0)}{y_3(0)} > 0 \quad \Rightarrow \quad \lambda_1 + \frac{\xi_2(0)}{y_3(0)} - \frac{\hat{\xi}_2(0)}{y_3(0)} > 0, \quad \text{since } \tilde{\xi}_2 = \xi_2 - \hat{\xi}_2.$$

Taking:

$$\hat{\xi}_2(0) = -k\xi_{2\max} + \lambda_1 y_3(0), \tag{41}$$

with $k > 1$ we obtain:

$$\lambda_1 + \frac{\xi_2(0)}{y_3(0)} - \frac{\hat{\xi}_2(0)}{y_3(0)} = \frac{\xi_2(0)}{y_3(0)} + \frac{k\xi_{2\max}}{y_3(0)} > 0, \tag{42}$$

which is always satisfied since $|\xi_2| \leqslant \xi_{2\max}$.

In the same way, it is easy to show that (ii) is fulfilled by taking:

$$\hat{\xi}_2(0) = k\ \xi_{2\max} + \lambda_1 y_3(0), \tag{43}$$

with $k > 1$.

Consequently, to satisfy both of the conditions (i) and (ii), one can take:

$$\hat{\xi}_2(0) = -k\,\mathrm{sign}(y_3(0))\xi_{2\max} + \lambda_1 y_3(0), \ \text{with } k > 1. \tag{44}$$

(b) In this case the proof is omitted since we use the same development as in the case (a), considering the following conditions:

$$\begin{cases} \tilde{\xi}_2(0) + \lambda_1 y_3(0) < 0, \\ \tilde{\xi}_2(0) + \lambda_2 y_3(0) > 0, \end{cases} \tag{45}$$

instead of (i) and (ii) to satisfy the condition (40). □

4. SIMULATION RESULTS

In this section, we present some simulation results for the stabilization of a unicycle-like mobile robot. Firstly, we consider the kinematics model with no disturbances. Secondly, we consider a mobile robot subjected to constant disturbances.

4.1. Non-adaptive controller: disturbance-free case

In this case, we aim to steer the vehicle to the origin starting from the initial configuration ($x_0 = 2, y_0 = 2, \theta_0 = \pi/2$). The control parameters we have used are: $k_1 = 2, k_2 = 1, k_3 = 1$. Figure 1 shows the time evolution of the state variables and the plot of the generated trajectory in the Cartesian plane $y(x)$. The vehicle motion, in the parking maneuver, is illustrated in Fig. 2.

4.2. Adaptive controller

In this case, our objective is to steer the vehicle, subjected to unknown disturbances, to the origin starting from the initial configuration ($x_0 = 2, y_0 = 2, \theta_0 = \pi/2$). The unknown parameters we have introduced — for simulation purposes — are $\xi_1 = \xi_2 = 2$ and the control parameters are: $k_1 = 15.8$, $k_2 = 1$, $k_3 = 7.9$, $\Gamma_1 = \Gamma_3 = 10$ and $\hat{\xi}_1(0) = 0, \hat{\xi}_2(0) = -k\,\mathrm{sign}(y_3(0))\xi_{2\,\max} + \lambda_1 y_3(0)$, with $k = 1.1$ and $\xi_{2\,\max} = 2$.

Figure 3 shows the time evolution of the state variables as well as the estimated values $\hat{\xi}_1$ and $\hat{\xi}_2$. In Fig. 4, one can see the convergence of the mobile robot to the origin under the adaptive control law.

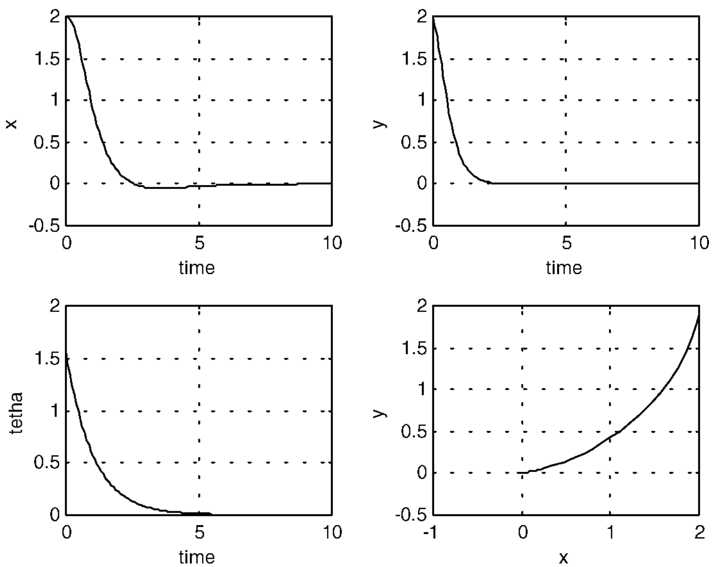

Figure 1. Disturbance-free case: state variables (x, y, θ) and the generated trajectory $y(x)$.

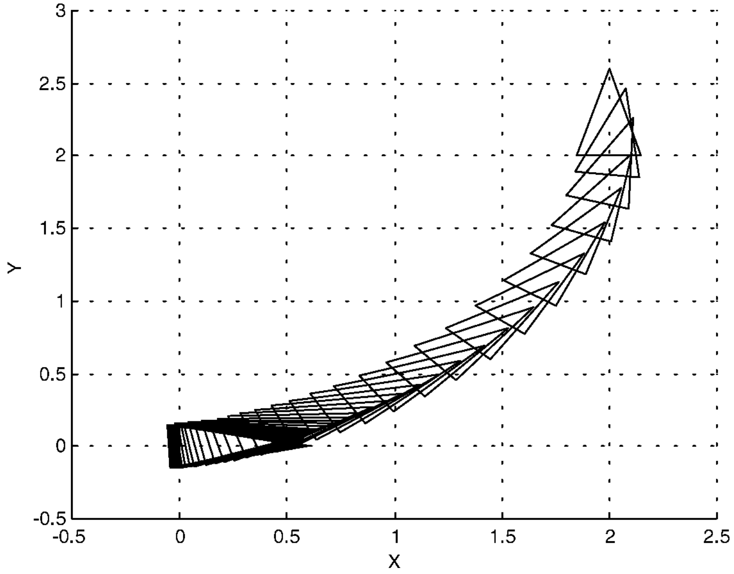

Figure 2. Disturbance-free case: parking maneuver.

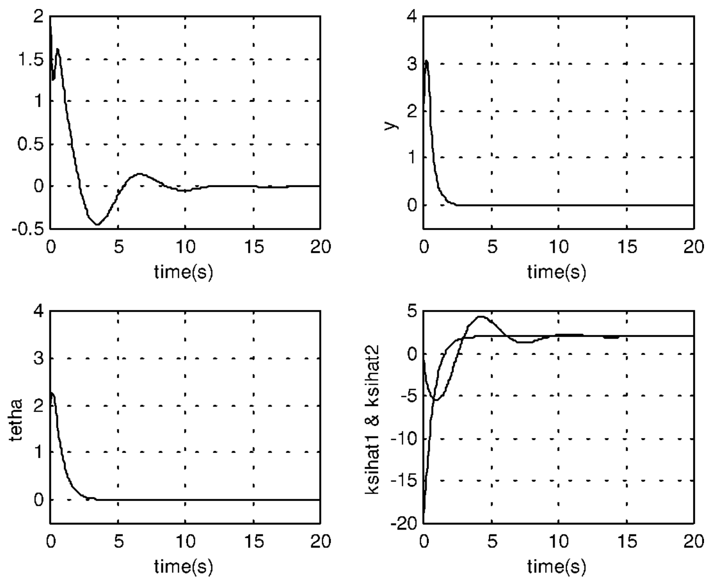

Figure 3. Adaptive case: state variables (x, y, θ) and the estimations $\hat{\xi}_1$, $\hat{\xi}_2$.

5. CONCLUSION

In this paper, we have presented a backstepping-based procedure for the design of a discontinuous time-invariant controller for the stabilization of a non-holonomic mobile robot. This approach is then applied for a mobile robot subjected to slowly varying or constant input disturbances, leading to an adaptive time-invariant

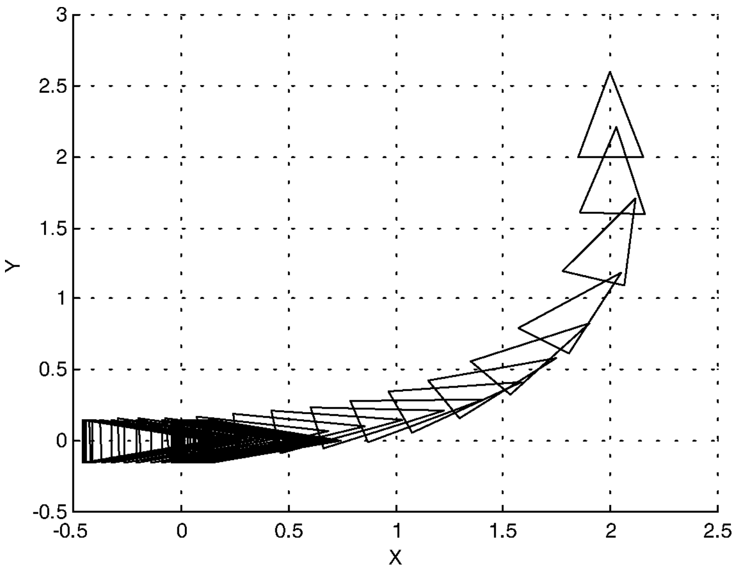

Figure 4. Adaptive case: parking maneuver.

stabilizing controller. The discontinuity introduced at $y_3(0) \equiv \theta(0) = 0$ allows us to avoid the periodic functions usually involved in the smooth time-varying controllers, which generally lead to low rates of convergence and oscillating trajectories. It is worth noticing that this discontinuity is not very restrictive since we have just to avoid it at the initial time by driving away the mobile robot from this configuration using an arbitrary open-loop control for a small period of time.

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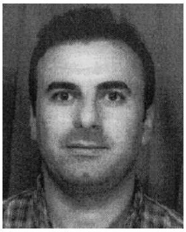

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