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Formation control of VTOL Unmanned Aerial Vehicles with communication delays[☆]

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ABSTRACT

The formation control problem of a team of Vertical Take-Off and Landing (VTOL) Unmanned Aerial Vehicles (UAVs) with communication delays is addressed. Based on the extraction algorithm presented in Abdessameud and Tayebi (2010a), we propose a new design methodology that simplifies the design of formation control laws with delayed communication for this class of under-actuated systems. Three control schemes are presented that provide delay-dependent and delay-independent results with constant and time-varying communication delays. The stability of the overall closed loop system in each scheme is established using Lyapunov–Krasovskii functionals. The proposed design methodology achieves global results in terms of the position and removes the requirement of the linear-velocity measurements. Simulation results are provided to show the effectiveness of the proposed control schemes.

1. Introduction

Formation control of multiple autonomous vehicles has recently received an increasing interest in the control community. This interest is motivated by their potential applications in areas such as search and rescue missions, reconnaissance operations, forest fire detection, and surveillance. Work in this area is generally inspired by the recent results in the coordinated control of multi-agent systems. Related research topics include flocking of mobile autonomous agents (Fax & Murray, 2004; Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, 2006; Tanner, Jadbabaie, & Pappas, 2007) and consensus problems (Olfati-Saber, Fax, & Murray, 2007; Ren, Beard, & Atkins, 2007). The idea in these works is to design control schemes for a group of vehicles based on local information exchange to achieve a common objective in a coordinated manner.

In practical situations, the information transmission between vehicles is often delayed. The effects of communication delays in multi-agent systems with first and second order dynamics have been studied, respectively, in Olfati-Saber and Murray (2004), Sun and Wang (2009), Wang and Slotine (2006) and Hong-Yong, Xun-Lin, and Si-Ying (2010), Meng, Yu, and Ren (2010), Münz, Papachristodoulou, and Allgöwer (2008), Seuret, Dimarogonas, and Johansson (2009), Tian and Liu (2009) to cite a few, and sufficient conditions have been derived to achieve the stability of the system. In Münz et al. (2008) and Seuret et al. (2009) for example, the authors consider the Rendezvous problem of multi agents and provide different delay dependent conditions using Lyapunov–Krasovskii functionals. The authors in Hong-Yong et al. (2010) use the Nyquist stability criterion in the analysis of leader-following consensus algorithms in the presence of input and communication delays and when the velocity of the leader is constant. A particular case of this last problem (zero leader's velocity) has been discussed in Meng et al. (2010), where the authors show that Lyapunov–Krasovskii functionals can provide sufficient conditions based on the solution of an LMI. The output consensus problem of higher order linear single-input single-output systems has been discussed in Münz, Papachristodoulou, and Allgöwer (2010) using the generalized Nyquist criterion. One of the essential assumptions to use the above analysis tools is that the coupling between vehicles is linear. The case of linear multi-agent systems with nonlinear coupling has been discussed in Münz, Papachristodoulou, and Allgöwer (2009) using Lyapunov–Razumikhin functions.

The communication delays in nonlinear systems have also been considered to solve the spacecraft formation control problem (Chung, Ahsun, & Slotine, 2009) and the synchronization of bilateral teleoperators (Chopra, Spong, & Lozano, 2008; Polushin, Tayebi, & Marquez, 2006) and Euler–Lagrange systems (Nuño,

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Ortega, Basañez, & Hill, 2011). However, only a few work has been done for nonlinear systems with nonlinear coupling that may arise when control saturations are considered for example. In this context, the work of Chopra and Spong (2006) presents an output synchronization scheme for passive nonlinear systems with nonlinear coupling. The authors use the scattering variables formulation and show that output synchronization is achieved for arbitrary time delays between communicating members of the team. An important assumption in the above papers, however is that the full state vector is available for feedback.

In spite of the interesting results cited above, much work remains to be done to develop control algorithms for a group of vehicles with complex dynamics in the presence of communication delays and take into consideration the systems' input constraints in the full and partial state information cases. These difficulties are specially challenging for the class of under-actuated Vertical Take-Off and Landing (VTOL) Unmanned Aerial Vehicles (UAVs) since, as will become clear throughout the paper, the aircraft input is subject to some constraints and some of the system's states are not generally available or precisely measured.

The position control of a single VTOL UAV is a challenging problem especially when it is desirable to achieve global or semi-global results (see for instance Aguiar & Hespanha, 2007; Frazzoli, Dahleh, & Feron, 2000; Hamel, Mahony, Lozano, & Ostrowski, 2002; Hua, Hamel, Morin, & Samson, 2009; Koo & Sastry, 1998; Pflimlin, Soures, & Hamel, 2007). The main difficulty resides on the under-actuated nature of these systems. In Abdessameud and Tayebi (2009), we proposed a solution to the tracking and formation control of a group of VTOL UAVs providing global stability results in terms of the position. The proposed scheme is based on a new control design methodology for this class of under-actuated systems, which relies on a singularity-free extraction algorithm (in terms of unit-quaternion) and provides the necessary thrust and desired orientation of the aircraft from an intermediary design of the translational control. The extracted thrust input is used to drive the translational dynamics of the aircraft, and the desired orientation is considered as a time-varying reference attitude to the rotational dynamics. A similar method, with a more general formulation of the extraction algorithm, has been used in Roberts and Tayebi (2011) to solve the trajectory tracking of the class of under-actuated systems under study with external disturbances. In Abdessameud and Tayebi (2010a), we applied this control design methodology to solve the global trajectory tracking problem of a single VTOL UAV in the case where the linear-velocity of aircraft is not available for feedback. This problem is interesting from a practical point of view since good estimates of aircraft linear-velocities are generally obtained from the fusion of available measurements from accelerometers and high-quality GPS sensors. However, the GPS signal is not available in indoor and urban applications (structure/bridge inspection for example) due to signal blockage and attenuation. In addition, the implementation of a redundant velocity-free control scheme in aircraft equipped with GPS will enhance the reliability of the system to sensors failure.

The main contribution of the present paper is to provide formation stabilization schemes for a group of VTOL UAVs in the presence of communication delays. These control schemes are based on the extraction algorithm presented in Abdessameud and Tayebi (2010a). As reported in this paper, this algorithm is applicable only under some condition on the intermediary translational control input, which can be easily satisfied if this input is guaranteed to be *a priori* bounded. Furthermore, the first and second time-derivatives of the intermediary input are needed in the input torque design and must be explicitly computed using available signals. To satisfy these requirements with delayed communication, we propose a particular control structure for the intermediary translational input. The main idea is to implement two auxiliary systems to each aircraft in the team. The states of the auxiliary systems are used in the intermediary control law through smooth saturation functions, and the inputs of the auxiliary systems are constructed based on aircraft states to achieve the formation objective with delayed communication.

Based on this approach, we propose first a formation control law that uses the relative position information between neighboring aircraft in the presence of time-varying communication delays, and guarantees our control objectives under sufficient delay-dependent conditions. Next, we show that the inclusion of the relative linear-velocities in the design of the auxiliary input plays an important role to achieve formation with arbitrary constant communication delays. Finally, we propose a formation control scheme with delayed communication that removes the requirement of the linear-velocity measurement. In this scheme, the second auxiliary system describes the dynamics of a virtual vehicle, and the auxiliary input objective is to first guarantee that all virtual vehicles converge to the predefined formation in the presence of communication delays. Thereafter, each aircraft is forced to track its corresponding virtual vehicle without velocity measurements achieving hence our original objectives.

2. System model and preliminaries

2.1. System model

Consider n-aircraft modeled as rigid bodies. Let $\mathcal{F}_0 \triangleq \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ denote the inertial frame, and $\mathcal{F}_i \triangleq \{\hat{e}_{1i}, \hat{e}_{2i}, \hat{e}_{3i}\}$ denote the body-fixed frame of the ith aircraft. Let the position and linear-velocity of the ith aircraft expressed in the inertial frame, $\mathcal{F}_0$, be denoted, respectively, by $\mathbf{p}_i \in \mathbb{R}^3$ and $\mathbf{v}_i \in \mathbb{R}^3$, and let its angular velocity be expressed in $\mathcal{F}_i$, and is denoted by $\boldsymbol{\omega}_i \in \mathbb{R}^3$. The orientation (attitude) of the ith aircraft is represented using the four-element unit quaternion $\mathbf{Q}_i = (\mathbf{q}_i^\top, \eta_i)^\top$, composed of a vector component $\mathbf{q}_i \in \mathbb{R}^3$ and a scalar component $\eta_i \in \mathbb{R}$, which are subject to the unity constraint: $\mathbf{q}_i^\top \mathbf{q}_i + \eta_i^2 = 1$. The rotation matrix $\mathbf{R}(\mathbf{Q}_i)$, related to the unit-quaternion $\mathbf{Q}_i$, that brings the inertial frame into the body frame, can be obtained through the Rodriguez formula as: $\mathbf{R}(\mathbf{Q}_i) = (\eta_i^2 - \mathbf{q}_i^\top \mathbf{q}_i)\mathbf{I}_3 + 2\mathbf{q}_i\mathbf{q}_i^\top - 2\eta_i \mathbf{S}(\mathbf{q}_i)$, where $\mathbf{I}_3$ is the 3-by-3 identity matrix and the matrix $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $\mathbf{S}(\mathbf{x}_1)\mathbf{x}_2 = \mathbf{x}_1 \times \mathbf{x}_2$ for any vectors $\mathbf{x}_1 \in \mathbb{R}^3$ and $\mathbf{x}_2 \in \mathbb{R}^3$, where '$\times$' denotes the vector cross product. The quaternion multiplication between two unit quaternion, $\mathbf{Q}_1 = (\mathbf{q}_1^\top, \eta_1)^\top$ and $\mathbf{Q}_2 = (\mathbf{q}_2^\top, \eta_2)^\top$, is defined by the following non-commutative operation; $\mathbf{Q}_1 \odot \mathbf{Q}_2 = \big((\eta_1 \mathbf{q}_2 + \eta_2 \mathbf{q}_1 + S(\mathbf{q}_1)\mathbf{q}_2)^\top, \eta_1\eta_2 - \mathbf{q}_1^\top \mathbf{q}_2\big)^\top$. The inverse or conjugate of a unit quaternion is defined by, $\mathbf{Q}_i^{-1} = (-\mathbf{q}_i^\top, \eta_i)^\top$, with the quaternion identity given by $(0, 0, 0, 1)^\top$ (Shuster, 1993).

The equations of motion of aircraft are described by

$$(\Sigma_{1_i}) : \begin{cases} \dot{\mathbf{p}}_i = \mathbf{v}_i, \\ \dot{\mathbf{v}}_i = g\hat{e}_3 - \dfrac{\mathcal{T}_i}{m_i}\mathbf{R}(\mathbf{Q}_i)^\top \hat{e}_3, \end{cases} \tag{1}$$

$$(\Sigma_{2_i}) : \begin{cases} \dot{\mathbf{Q}}_i = \dfrac{1}{2}\begin{pmatrix} \eta_i \mathbf{I}_3 + \mathbf{S}(\mathbf{q}_i) \\ -\mathbf{q}_i^\top \end{pmatrix}\boldsymbol{\omega}_i, \\ \mathbf{I}_{f_i}\dot{\boldsymbol{\omega}}_i = \boldsymbol{\Gamma}_i - \mathbf{S}(\boldsymbol{\omega}_i)\mathbf{I}_{f_i}\boldsymbol{\omega}_i, \end{cases} \tag{2}$$

for $i \in \mathcal{N} \triangleq \{1, \ldots, n\}$. m_i and g are, respectively, the mass of the ith aircraft and the gravitational acceleration, $\mathbf{I}_{f_i} \in \mathbb{R}^{3\times 3}$ is the symmetric positive definite constant inertia matrix of the ith aircraft with respect to $\mathcal{F}_i$. The scalar $\mathcal{T}_i$ and the vector $\boldsymbol{\Gamma}_i$ represent, respectively, the magnitude of the thrust applied to the ith vehicle in the direction of $\hat{e}_{3i}$, and the external torque applied to the system expressed in $\mathcal{F}_i$.

2.2. Attitude error dynamics

Let the unit quaternion $\mathbf{Q}_{d_i} = (\mathbf{q}_{d_i}^\top, \eta_{d_i})^\top$ represent a desired attitude for the ith aircraft, to be determined later through the control design. We define the attitude tracking error, describing the discrepancy between the vehicle's attitude and its desired attitude, namely $\tilde{\mathbf{Q}}_i \triangleq (\tilde{\mathbf{q}}_i^\top, \tilde{\eta}_i)^\top$, as;

$$\tilde{\mathbf{Q}}_i = \mathbf{Q}_{d_i}^{-1} \odot \mathbf{Q}_i, \tag{3}$$

and is governed by the unit-quaternion dynamics

$$\begin{cases} \dot{\tilde{\mathbf{q}}}_i = \frac{1}{2}(\tilde{\eta}_i \mathbf{I}_3 + \mathbf{S}(\tilde{\mathbf{q}}_i))\tilde{\boldsymbol{\omega}}_i, \qquad \dot{\tilde{\eta}}_i = -\frac{1}{2}\tilde{\mathbf{q}}_i^\top \tilde{\boldsymbol{\omega}}_i, \\ \tilde{\boldsymbol{\omega}}_i = \boldsymbol{\omega}_i - \mathbf{R}(\tilde{\mathbf{Q}}_i)\boldsymbol{\omega}_{d_i}, \end{cases} \tag{4}$$

where $\tilde{\boldsymbol{\omega}}_i$ is the angular velocity error vector and $\boldsymbol{\omega}_{d_i}$ is the desired angular velocity of the aircraft, which is related to $\mathbf{Q}_{d_i}$ by

$$\boldsymbol{\omega}_{d_i} = 2\begin{pmatrix} \eta_{d_i}\mathbf{I}_3 + \mathbf{S}(\mathbf{q}_{d_i}) \\ -\mathbf{q}_{d_i}^\top \end{pmatrix}^\top \dot{\mathbf{Q}}_{d_i}. \tag{5}$$

Matrix $\mathbf{R}(\tilde{\mathbf{Q}}_i)$ is the rotation matrix related to $\tilde{\mathbf{Q}}_i$, and is given by $\mathbf{R}(\tilde{\mathbf{Q}}_i) = \mathbf{R}(\mathbf{Q}_i)\mathbf{R}(\mathbf{Q}_{d_i})^\top$ (Shuster, 1993). We can see that attitude tracking is achieved when $\mathbf{Q}_i$ coincides with $\mathbf{Q}_{d_i}$, or $\tilde{\mathbf{Q}}_i = (0, 0, 0, \pm 1)^\top$. Note that due to the inherent redundancy of the quaternion representation, $\mathbf{Q}_i$ and $-\mathbf{Q}_i$ represent the same physical orientation however, one is rotated 2π relative to the other about an arbitrary axis. Accordingly, $\tilde{\mathbf{Q}}_i = (0, 0, 0, \pm 1)^\top$ correspond to the same physical point.

Using the above definitions, we can show that

$$\left(\mathbf{R}(\mathbf{Q}_i)^\top - \mathbf{R}(\mathbf{Q}_{d_i})^\top\right)\hat{e}_3 = \boldsymbol{\Psi}_i \bar{\mathbf{q}}_i, \tag{6}$$

with the matrix $\boldsymbol{\Psi}_i = 2\mathbf{R}(\mathbf{Q}_i)^\top \mathbf{S}(\bar{\mathbf{q}}_i)$, $\tilde{\mathbf{q}}_i = (\tilde{q}_{1i}, \tilde{q}_{2i}, \tilde{q}_{3i})^\top$ and $\bar{\mathbf{q}}_i = (\tilde{q}_{2i}, -\tilde{q}_{1i}, -\tilde{\eta}_i)^\top$.

2.3. Notation and definitions

Throughout the paper, we use the notation $\|\mathbf{x}\|$ to denote the Euclidean norm of the vector $\mathbf{x} \in \mathbb{R}^m$. For sake of clarity of presentation, the argument of all time-dependent signals will be omitted [e.g. $\mathbf{p} \equiv \mathbf{p}(t)$], except for those which are time delayed [e.g. $\mathbf{p}(t-\tau)$ for a constant delay and $\mathbf{p}(t-\tau(t))$ for time-varying delay]. Accordingly, the argument of the signals inside the integrals is omitted, which is assumed to be equal to the variable on the differential, unless otherwise stated [e.g. $\int_0^t \dot{\boldsymbol{\alpha}} ds \equiv \int_0^t \dot{\boldsymbol{\alpha}}(s) ds$]. Also, the limit of a signal at infinity is replaced by an arrow [e.g. $\mathbf{p} \to 0 \equiv \lim_{t\to\infty} \mathbf{p}(t) = 0$, and $\mathbf{p} \to \mathbf{q} \equiv \lim_{t\to\infty} \mathbf{p}(t) = \lim_{t\to\infty} \mathbf{q}(t)$].

We define for any vector $\mathbf{x} = (x^1, x^2, x^3)^\top \in \mathbb{R}^3$ the function

$$\chi(\mathbf{x}) = \mathrm{col}[\sigma(x^k)] \in \mathbb{R}^3, \quad \text{for } k = 1, 2, 3, \tag{7}$$

with $\sigma : \mathbb{R} \to \mathbb{R}$, is a strictly increasing continuously differentiable function satisfying the following properties:

P1. $\sigma(0) = 0$ and $x\sigma(x) > 0$ for $x \neq 0$,
P2. $|\sigma(x)| \leq \sigma_b$, for σ_b is a strictly positive constant.
P3. $\frac{\partial \sigma(x)}{\partial x}$ is bounded.

Note that property P3 can be verified from P1 and P2. Examples of the function $\sigma(x)$ include: $\tanh(x)$ and $\frac{x}{\sqrt{1+x^2}}$.

We state in the following lemma a preliminary result that will be used in the proof of our results.

Lemma 1. *Consider the second order system*

$$\ddot{\boldsymbol{\theta}}_i = -k_i^p \chi(\boldsymbol{\theta}_i) - k_i^d \chi(\dot{\boldsymbol{\theta}}_i) + \boldsymbol{\varepsilon}_i, \tag{8}$$

where $\boldsymbol{\theta}_i \in \mathbb{R}^3$, $\chi(\boldsymbol{\theta}_i)$ is defined in (7), k_i^p and k_i^d are positive scalars. If $\boldsymbol{\varepsilon}_i$ is bounded for all time and $\boldsymbol{\varepsilon}_i \to 0$, then $\boldsymbol{\theta}_i$ and $\dot{\boldsymbol{\theta}}_i$ are bounded and $\boldsymbol{\theta}_i \to \dot{\boldsymbol{\theta}}_i \to 0$.

Proof. See Abdessameud and Tayebi (2010b) for a similar proof with $\sigma(x) = \tanh(x)$. □

3. Problem formulation

To design formation control schemes, aircraft in the team must share some of their state information through local information exchange. We assume that the information flow between members of the team is fixed and undirected, and is described using weighted graphs. An undirected graph, $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{K})$, consists of a set of nodes $\mathcal{N}$, describing the set of vehicles in the team, a set of edges $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$, and a weighted adjacency matrix $\mathcal{K} = [k_{ij}] \in \mathbb{R}^{n\times n}$. An edge (i, j) indicates that vehicles i and j are neighbors and can obtain information from one another. The weighted adjacency matrix of a weighted undirected graph is defined such that $k_{ij} = k_{ji} > 0$ for $(i, j) \in \mathcal{E}$, and $k_{ij} = 0$ if $(i, j) \notin \mathcal{E}$. If there is a path between any two distinct nodes of a weighted undirected graph $\mathcal{G}$, then $\mathcal{G}$ is said to be connected. For more details on graph properties, the reader is referred to Jungnickel (2005). Furthermore, we assume that each aircraft can sense its state with no delay, and the communication between two neighboring aircraft, the ith and jth aircraft, is delayed by τ_{ij}, with τ_{ij} not necessarily equal to τ_{ji}.

With the above assumptions, our objective in this work is to design formation control schemes for each VTOL aircraft in the team such that the vehicles converge to a prescribed stationary formation in the presence of communication delays. More formally, our objective is to guarantee that

$$\mathbf{v}_i \to 0 \quad \text{and} \quad (\mathbf{p}_i - \mathbf{p}_j) \to \boldsymbol{\delta}_{ij}, \tag{9}$$

for $i, j \in \mathcal{N}$, where $\boldsymbol{\delta}_{ij} \in \mathbb{R}^3$, satisfying $\boldsymbol{\delta}_{ij} = -\boldsymbol{\delta}_{ji}$, defines the desired constant offset between the ith and jth aircraft, and hence defines the formation pattern. We first consider this problem when the full state vector is available for feedback, and then we extend our results to remove the requirement of linear-velocity measurements.

To design a thrust and torque input for the class of under-actuated VTOL UAVs, we have presented in Abdessameud and Tayebi (2010a) a control design method that relies on a non-singular unit-quaternion-based extraction algorithm. First, let the translational dynamics of each aircraft, (Σ_{1_i}) in (1), be rewritten as

$$(\Sigma_{1_i}) : \begin{cases} \dot{\mathbf{p}}_i = \mathbf{v}_i, \\ \dot{\mathbf{v}}_i = \mathbf{F}_i - \frac{\mathcal{T}_i}{m_i}(\mathbf{R}(\mathbf{Q}_i)^\top - \mathbf{R}(\mathbf{Q}_{d_i})^\top)\hat{e}_3, \end{cases} \tag{10}$$

with

$$\mathbf{F}_i \triangleq g\hat{e}_3 - \frac{\mathcal{T}_i}{m_i}\mathbf{R}(\mathbf{Q}_{d_i})^\top \hat{e}_3, \tag{11}$$

where the variable $\mathbf{F}_i \in \mathbb{R}^3$ is an "*intermediary*" control input to the translational dynamics, to be designed later, and $\mathbf{Q}_{d_i} = (\mathbf{q}_{d_i}^\top, \eta_{d_i})^\top$ is the unit quaternion representing the desired attitude of the ith aircraft. The extraction algorithm presented in Abdessameud and Tayebi (2010a) provides a non-singular solution to the thrust input, $\mathcal{T}_i$, and the desired attitude for each aircraft, $\mathbf{Q}_{d_i}$, from Eq. (11), if the intermediary control $\mathbf{F}_i$ is designed such that

$$\mathbf{F}_i \neq (0, 0, x_i)^\top, \quad \text{for } x_i \geq g. \tag{12}$$

The extracted values of $\mathcal{T}_i$ and $\mathbf{Q}_{d_i}$ are given in Lemma 4 in Appendix A for completeness. This extraction algorithm suggests a comprehensive design procedure that provides an almost separate control design for the translational and rotational dynamics for the class of under-actuated VTOL UAVs (Abdessameud & Tayebi, 2010a; Roberts & Tayebi, 2011).

The main difficulty in using this extraction algorithm in this paper resides on the design of the intermediary control $\mathbf{F}_i$ that achieves formation along with communication delays. In fact, to satisfy condition (12), it is sufficient to ensure that the third element of the intermediary control $\mathbf{F}_i$ is *a priori* bounded. In addition, we can see from (10) and the extracted value of the thrust, given in (A.1), that the design of a bounded intermediary control guarantees that the term $\frac{\mathcal{T}_i}{m_i}\left(\mathbf{R}(\mathbf{Q}_i)^\top - \mathbf{R}(\mathbf{Q}_{d_i})^\top\right)\hat{e}_3 = \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i$ is bounded. Note that this term can be regarded as a perturbation term to the translational dynamics in (10).

To satisfy the above requirement, one may for example consider a "bounded version" of a standard formation stabilization control law with communication delays in the full state information case

$$\mathbf{F}_i = -k_i^v\chi(\mathbf{v}_i) - \sum_{j=1}^{n} k_{ij}\chi(\mathbf{p}_i - \mathbf{p}_j(t-\tau_{ij}) - \boldsymbol{\delta}_{ij}), \tag{13}$$

where the function χ is a saturation function defined in (7), k_i^v is a positive scalar gain and $k_{ij} \geq 0$ is the (i,j)th entry of the weighted adjacency matrix $\mathcal{K}$ of the communication graph, $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{K})$, characterizing the information flow between aircraft. It is easy to verify that an upper bound of this control law can be determined *a priori* as: $\|\mathbf{F}_i\| \leq \sqrt{3}\sigma_b\left(k_i^v + \sum_{j=1}^n k_{ij}\right)$, which depends on the number of neighbors of each aircraft. As a result, if the communication topology between aircraft is known in advance, we can satisfy condition (12) and use the extraction algorithm in Lemma 4. The extracted value of the thrust will then be used as the real input of the translational dynamics of each aircraft and the desired attitude will be considered as a reference input for the rotational dynamics.

Note from Lemma 4 that the obtained desired attitude is time-varying. Therefore, to design an attitude tracking control law for the rotational dynamics, we need to derive explicit expressions of the desired angular velocity, $\boldsymbol{\omega}_{d_i}$, and its first time-derivative, $\dot{\boldsymbol{\omega}}_{d_i}$. From Eq. (A.3), we know that $\dot{\boldsymbol{\omega}}_{d_i}$ can be derived using the expressions of $\dot{\mathbf{F}}_i$ and $\ddot{\mathbf{F}}_i$. Consequently, using the intermediary control law in (13), the expressions of $\boldsymbol{\omega}_{d_i}$ and $\dot{\boldsymbol{\omega}}_{d_i}$ will be function of the aircraft linear-accelerations with their time-derivatives and the relative linear-accelerations between neighboring aircraft, which are not generally measured.

Of course, the aircraft linear-acceleration can be computed on line and then transmitted through the communication channels, which will increase the communication requirements between vehicles. Also, the explicit time-derivative of the linear-acceleration will result in non-available signals in the partial state information case. In addition, due to the nonlinear interaction between aircraft, through the function χ, it is generally difficult to show that the class of control schemes (13) achieves our results using Lyapunov–Krasovskii functionals, and the scattering variables formalism (Chopra & Spong, 2006) cannot be used since the time-derivatives of these variables will be required in the torque input design.

In view of the above example, our main problem is to design an intermediary control input for each aircraft that needs to: (i) be *a priori* bounded to satisfy condition (12), (ii) achieve our control objectives in the presence of communication delays, and (iii) simplify the design of the input torque for the rotational dynamics *i.e.*, its first and second time-derivatives contain only available signals. Also, an additional challenge will be to use Lyapunov–Krasovskii functionals in the stability analysis of the closed loop system in the full and partial state information cases.

4. Control design reduction

To simplify the design of the intermediary translational control and the input torque for each aircraft, we propose in this section a preliminary design of these two inputs that satisfies some of the requirements discussed in the previous section. Let associate to each aircraft the following auxiliary second-order systems

$$\ddot{\boldsymbol{\theta}}_i = \mathbf{F}_i - \mathbf{u}_i, \tag{14}$$

$$\ddot{\boldsymbol{\alpha}}_i = \mathbf{u}_i - \boldsymbol{\phi}_i, \tag{15}$$

where $\boldsymbol{\theta}_i \in \mathbb{R}^3$ and $\boldsymbol{\alpha}_i \in \mathbb{R}^3$ are auxiliary variables, $\boldsymbol{\theta}_i(0)$, $\dot{\boldsymbol{\theta}}_i(0)$, $\boldsymbol{\alpha}_i(0)$ and $\dot{\boldsymbol{\alpha}}_i(0)$ can be selected arbitrarily, $\mathbf{u}_i \in \mathbb{R}^3$ and $\boldsymbol{\phi}_i \in \mathbb{R}^3$ are additional input vectors to be designed. The role of $\boldsymbol{\theta}_i$ and $\boldsymbol{\alpha}_i$ in the control scheme will be discussed later. We propose the following intermediary control input for each aircraft

$$\mathbf{F}_i = -k_i^p\chi(\boldsymbol{\theta}_i) - k_i^d\chi(\dot{\boldsymbol{\theta}}_i), \tag{16}$$

with k_i^p and k_i^d positive scalar gains and χ is defined in (7). We can see that $\mathbf{F}_i$ in (16) does not depend *explicitly* on the system's error variables (linear-velocity vectors and relative positions) and is guaranteed to be bounded as

$$\|\mathbf{F}_i\| \leq \sigma_b\sqrt{3}(k_i^p + k_i^d), \tag{17}$$

with σ_b defined in property P2. Hence, condition (12) can be easily satisfied with an appropriate choice of the gains k_i^p and k_i^d, and without any consideration on the communication topology between aircraft. In addition, the extracted input thrust of each aircraft, given in (A.1), is guaranteed to be strictly positive and *a priori* bounded as: $\mathcal{T}_i \leq m_i(g + \sigma_b\sqrt{3}(k_i^p + k_i^d))$.

To design the input torque for the rotational dynamics, we consider the extracted value of the desired attitude $\mathbf{Q}_{d_i}$, given in (A.2), as a time-varying reference attitude. After simple computation, explicit expressions for the desired angular velocity and its time-derivative can be obtained as

$$\boldsymbol{\omega}_{d_i} = \Xi(\mathbf{F}_i)\dot{\mathbf{F}}_i, \tag{18}$$

$$\dot{\boldsymbol{\omega}}_{d_i} = \bar{\Xi}(\mathbf{F}_i, \dot{\mathbf{F}}_i)\dot{\mathbf{F}}_i + \Xi(\mathbf{F}_i)\ddot{\mathbf{F}}_i, \tag{19}$$

where $\bar{\Xi}(\mathbf{F}_i, \dot{\mathbf{F}}_i)$ is the time-derivative of $\Xi(\mathbf{F}_i)$ given in (A.4), and

$$\dot{\mathbf{F}}_i = -k_i^p h(\boldsymbol{\theta}_i)\dot{\boldsymbol{\theta}}_i - k_i^d h(\dot{\boldsymbol{\theta}}_i)(\mathbf{F}_i - \mathbf{u}_i), \tag{20}$$

$$\begin{aligned}\ddot{\mathbf{F}}_i = & -k_i^p\dot{h}(\boldsymbol{\theta}_i)\dot{\boldsymbol{\theta}}_i - \left(k_i^p h(\boldsymbol{\theta}_i) + k_i^d\dot{h}(\dot{\boldsymbol{\theta}}_i)\right)(\mathbf{F}_i - \mathbf{u}_i)\\ & - k_i^d h(\dot{\boldsymbol{\theta}}_i)(\dot{\mathbf{F}}_i - \dot{\mathbf{u}}_i),\end{aligned} \tag{21}$$

where the diagonal matrix $h(\cdot)$ is given as $h(\mathbf{x}) \triangleq \mathrm{diag}\left(\frac{\partial\sigma(x^k)}{\partial x^k}\right)$, for $\mathbf{x} = (x^1, x^2, x^3)^\top \in \mathbb{R}^3$ and $k = 1, 2, 3$, and $\dot{h}(\cdot)$ is the time-derivative of $h(\cdot)$.

We propose the following input torque for each aircraft

$$\boldsymbol{\Gamma}_i = \mathbf{H}_i(\cdot) + \mathbf{I}_{f_i}\dot{\boldsymbol{\beta}}_i - k_i^q\tilde{\mathbf{q}}_i - k_i^{\Omega}(\tilde{\boldsymbol{\omega}}_i - \boldsymbol{\beta}_i), \tag{22}$$

where k_i^q and k_i^{Ω} are positive scalar gains, $\tilde{\mathbf{Q}}_i$ is defined in (3), $\boldsymbol{\beta}_i \in \mathbb{R}^3$ is a design variable to be determined later, and $\mathbf{H}_i(\cdot) = (\mathbf{S}(\boldsymbol{\omega}_i)\mathbf{I}_{f_i}\boldsymbol{\omega}_i - \mathbf{I}_{f_i}\mathbf{S}(\tilde{\boldsymbol{\omega}}_i)\mathbf{R}(\tilde{\mathbf{Q}}_i)\boldsymbol{\omega}_{d_i} + \mathbf{I}_{f_i}\mathbf{R}(\tilde{\mathbf{Q}}_i)\dot{\boldsymbol{\omega}}_{d_i})$, with $\boldsymbol{\omega}_{d_i}$ and $\dot{\boldsymbol{\omega}}_{d_i}$ being defined in (18)–(21). Define the new error variable

$$\boldsymbol{\Omega}_i = \tilde{\boldsymbol{\omega}}_i - \boldsymbol{\beta}_i. \tag{23}$$

Exploiting the rotational dynamics (Σ_{2_i}) in (2) with the input (22), we can show that

$$\mathbf{I}_{f_i}\dot{\boldsymbol{\Omega}}_i = -k_i^q\tilde{\mathbf{q}}_i - k_i^{\Omega}\boldsymbol{\Omega}_i. \tag{24}$$

It is important to mention that with the introduction of the "auxiliary" variables $\boldsymbol{\theta}_i$ and $\boldsymbol{\alpha}_i$ with the control inputs proposed above, the control design problem is now reduced to determine appropriate input vectors $\mathbf{u}_i$ and $\boldsymbol{\phi}_i$ in (14) and (15) such that formation is achieved in the presence of communication delays. Note that the design of $\mathbf{u}_i$ and $\boldsymbol{\phi}_i$ is independent from the boundedness requirement of the intermediary control input $\mathbf{F}_i$, and therefore, they can be constructed based on linear interactions of aircraft states. However, the first time-derivative of $\mathbf{u}_i$ is required to compute $\dot{\boldsymbol{\omega}}_{d_i}$, and therefore it must contain only available signals. With this in mind, we will focus in the remaining of the paper on the design of the inputs $\mathbf{u}_i$ and $\boldsymbol{\phi}_i$ that guarantee our formation control objective in the presence of communication delays. Specifically, we will propose formation control schemes in the full and partial state information cases. Also, to guarantee the stability of the overall system, the vector $\boldsymbol{\beta}_i$ will be designed in each case.

5. Design in the full-state information case

In this section, we assume that the full state vector is available for feedback and propose first a delay-dependent formation control scheme that achieves our control objectives with time-varying communication delays. Next, this control law is modified such that formation is achieved with arbitrary constant communication delays.

5.1. Delay-dependent design

Consider the following error variables

$$\boldsymbol{\xi}_i = \mathbf{p}_i - \boldsymbol{\theta}_i - \boldsymbol{\alpha}_i, \qquad \mathbf{z}_i := \dot{\boldsymbol{\xi}}_i, \tag{25}$$

where the dynamics of $\boldsymbol{\theta}_i$ and $\boldsymbol{\alpha}_i$ are given, respectively, in (14) and (15). The dynamics of $\mathbf{z}_i$ in (25) can be obtained from (6), (10), (14) and (15) as

$$\dot{\mathbf{z}}_i = \boldsymbol{\phi}_i - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i. \tag{26}$$

In view of (14)–(16) and (26), we propose the following control inputs for the auxiliary systems (14) and (15)

$$\mathbf{u}_i = -L_i^p\boldsymbol{\alpha}_i - L_i^d\dot{\boldsymbol{\alpha}}_i, \tag{27}$$

$$\boldsymbol{\phi}_i = -k_i^v\mathbf{z}_i - \sum_{j=1}^{n} k_{ij}\boldsymbol{\xi}_{ij}, \tag{28}$$

where $\boldsymbol{\xi}_{ij} = (\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t-\tau_{ij}(t)) - \boldsymbol{\delta}_{ij})$, k_i^v, L_i^p and L_i^d are positive scalar gains and $k_{ij} \geq 0$ is the (i,j)th entry of the weighted adjacency matrix $\mathcal{K}$ of the communication graph, $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{K})$, characterizing the information flow between aircraft. Note that the time-derivative of $\mathbf{u}_i$ in (27) can be obtained as,

$$\dot{\mathbf{u}}_i = -L_i^p\dot{\boldsymbol{\alpha}}_i - L_i^d(\mathbf{u}_i - \boldsymbol{\phi}_i). \tag{29}$$

Therefore, we can see from (18)–(21) and (27)–(29) that only available signals are used to evaluate $\boldsymbol{\omega}_{d_i}$ and $\dot{\boldsymbol{\omega}}_{d_i}$ for each aircraft, and the variable $\boldsymbol{\xi}_i$ is transmitted between each pair of communicating aircraft in the team.

Consider the following positive definite function (functional)

$$V_{t_1} = \frac{1}{2}\sum_{i=1}^{n}\left(\mathbf{z}_i^\top\mathbf{z}_i + \frac{1}{2}\sum_{j=1}^{n}k_{ij}\bar{\boldsymbol{\xi}}_{ij}^\top\bar{\boldsymbol{\xi}}_{ij}\right), \tag{30}$$

$$V_{k_1} = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{k_{ij}\tau}{2\epsilon}\left(\int_{-\tau}^{0}\int_{t+s}^{t}\mathbf{z}_j(\varrho)^\top\mathbf{z}_j(\varrho)\mathrm{d}\varrho\mathrm{d}s\right) \tag{31}$$

with $\bar{\boldsymbol{\xi}}_{ij} = (\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij})$, ϵ is some strictly positive constant, and we assume that the communication delays satisfy $\tau_{ij}(t) \leq \tau$ for all $(i,j) \in \mathcal{E}$.

Claim 2. *The time-derivative of the function $V_{t_1} + V_{k_1}$ evaluated along the error dynamics (26) with (28) can be upper bounded as*

$$\dot{V}_{t_1} + \dot{V}_{k_1} \leq -\sum_{i=1}^{n}\frac{\mathcal{T}_i}{m_i}\mathbf{z}_i^\top\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i - \sum_{i=1}^{n}\left(k_i^v - \frac{1}{2}\sum_{j=1}^{n}k_{ij}\left(\epsilon + \frac{\tau^2}{\epsilon}\right)\right)\mathbf{z}_i^\top\mathbf{z}_i. \tag{32}$$

Proof. See Appendix B. □

Note that the perturbation term in the translational dynamics appears in (32) and must be considered in the design of the variable $\boldsymbol{\beta}_i$ in (23) to ensure the stability of the overall system. To design this variable, we consider the following positive-definite function

$$V_{a_1} = \sum_{i=1}^{n}\left(\frac{1}{2}\boldsymbol{\Omega}_i^\top\mathbf{I}_{f_i}\boldsymbol{\Omega}_i + k_i^q\tilde{\mathbf{q}}_i^\top\tilde{\mathbf{q}}_i + k_i^q(1-\tilde{\eta}_i)^2\right). \tag{33}$$

The time-derivative of V_{a1} evaluated along the attitude tracking error (24) using (4) and (23) gives

$$\dot{V}_{a_1} = \sum_{i=1}^{n}\left(-k_i^\Omega\boldsymbol{\Omega}_i^\top\boldsymbol{\Omega}_i + k_i^q\tilde{\mathbf{q}}_i^\top\boldsymbol{\beta}_i\right). \tag{34}$$

In view of this last equation, we propose the following design for the variable $\boldsymbol{\beta}_i$,

$$\boldsymbol{\beta}_i = -k_i^\beta\tilde{\mathbf{q}}_i + \frac{\mathcal{T}_i}{k_i^q m_i}\boldsymbol{\Psi}_i^\top\mathbf{z}_i, \tag{35}$$

with k_i^β a positive scalar gain, $\boldsymbol{\Psi}_i$ is given in (6) and $\mathcal{T}_i$ is obtained from (16) according to (A.1).

Theorem 1. *Consider the VTOL–UAVs formation modeled as in (1)–(2). For each aircraft, let the thrust input $\mathcal{T}_i$ and the desired attitude $\mathbf{Q}_{d_i}$ be given, respectively, by (A.1) and (A.2), with $\mathbf{F}_i$ given by (16) with (14), (15), (27) and (28). Let the input torque for each aircraft be given by (22) and $\boldsymbol{\beta}_i$ be given as in (35). Let the controller gains satisfy*

$$\sqrt{3}\sigma_b\left(k_i^p + k_i^d\right) < g, \tag{36}$$

$$k_i^z = k_i^v - \frac{1}{2}\sum_{j=1}^{n}k_{ij}\left(\epsilon + \frac{\tau^2}{\epsilon}\right) > 0, \tag{37}$$

for some $\epsilon > 0$ and $\tau_{ij}(t) \leq \tau$, for all $(i,j) \in \mathcal{E}$, and assume that the communication graph $\mathcal{G}$ is connected. Then starting from any initial conditions, the signals $\mathbf{v}_i$, $(\mathbf{p}_i - \mathbf{p}_j)$ and $\tilde{\boldsymbol{\omega}}_i$ are bounded and $\mathbf{v}_i \to 0$, $(\mathbf{p}_i - \mathbf{p}_j) \to \boldsymbol{\delta}_{ij}$, $\tilde{\mathbf{q}}_i \to 0$ and $\tilde{\boldsymbol{\omega}}_i \to 0$ for all $i, j \in \mathcal{N}$.

Proof. First, we can see that if the control gains are selected according to (36), the extraction condition (12) will be always satisfied, in view of (17). Therefore, it is always possible to extract the magnitude of the thrust and the desired attitude from (A.1) and (A.2), respectively, for each VTOL vehicle.

Consider the following Lyapunov–Krasovskii functional candidate

$$V = V_{t_1} + V_{k_1} + V_{a_1}, \tag{38}$$

with V_{t_1}, V_{k_1} and V_{a_1} given in (30), (31) and (33), respectively. The time-derivative of V evaluated along the closed loop dynamics (26) and (24) using (28) and (35) can be upper bounded in view of (32) and (34) as

$$\dot{V} \leq \sum_{i=1}^{n}\left(-k_i^z\mathbf{z}_i^\top\mathbf{z}_i - k_i^\Omega\boldsymbol{\Omega}_i^\top\boldsymbol{\Omega}_i - k_i^q k_i^\beta\tilde{\mathbf{q}}_i^\top\tilde{\mathbf{q}}_i\right), \tag{39}$$

with k_i^z being given in (37), which is negative semi-definite if condition (37) is satisfied. Hence, we conclude that $\mathbf{z}_i$, $\tilde{\mathbf{q}}_i$ and $\boldsymbol{\Omega}_i$ are bounded for $i \in \mathcal{N}$ and $(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j)$ is bounded for all $(i,j) \in \mathcal{E}$. Since the communication graph is assumed connected, this last result is valid for all $i, j \in \mathcal{N}$. Now, using the relation

$$(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t - \tau_{ij}(t))) = (\boldsymbol{\xi}_i - \boldsymbol{\xi}_j) + \int_{t-\tau_{ij}(t)}^{t} \mathbf{z}_j \mathrm{d}s, \tag{40}$$

the error dynamics (26) with (28) can be rewritten as

$$\dot{\mathbf{z}}_i = -k_i^v \mathbf{z}_i - \sum_{j=1}^{n} k_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij}) - \sum_{j=1}^{n} k_{ij} \int_{t-\tau_{ij}(t)}^{t} \mathbf{z}_j \mathrm{d}s - \frac{\mathcal{T}_i}{m_i} \boldsymbol{\Psi}_i \tilde{\mathbf{q}}_i, \tag{41}$$

and we can conclude that $\dot{\mathbf{z}}_i$ is bounded for $i \in \mathcal{N}$.

From Eq. (35), we know that $\boldsymbol{\beta}_i$ is bounded since $\tilde{\mathbf{q}}_i$ and $\mathbf{z}_i$ are bounded, and consequently $\tilde{\boldsymbol{\omega}}_i$ is bounded. Therefore, we conclude that $\dot{\tilde{\mathbf{q}}}_i$ is bounded from (4). In addition, we know from (24) that $\dot{\boldsymbol{\Omega}}_i$ is bounded. Exploiting the above results together with (39), we can verify that $\mathbf{z}_i, \tilde{\mathbf{q}}_i, \boldsymbol{\Omega}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and since we have shown that $\dot{\mathbf{z}}_i, \dot{\tilde{\mathbf{q}}}_i, \dot{\boldsymbol{\Omega}}_i \in \mathcal{L}_\infty$ for $i \in \mathcal{N}$, we conclude that $\mathbf{z}_i \to 0$, $\boldsymbol{\Omega}_i \to 0$ and $\tilde{\mathbf{q}}_i \to 0$, and therefore, $\boldsymbol{\beta}_i \to 0$ and $\tilde{\boldsymbol{\omega}}_i \to 0$ for $i \in \mathcal{N}$.

Since $\mathbf{z}_i \to 0$, for $i \in \mathcal{N}$, and $\tau_{ij}(t)$ is bounded, we can verify that $\int_{t-\tau_{ij}(t)}^{t} \mathbf{z}_i \mathrm{d}s \to 0$, for $i \in \mathcal{N}$. In addition, we know that $\boldsymbol{\xi}_i$ is uniformly continuous since we have shown that $\mathbf{z}_i$ is bounded for $i \in \mathcal{N}$. Invoking the extended Barbălat Lemma, Lemma 5 given in Appendix D, we can conclude from (41), and the above results, that $\dot{\mathbf{z}}_i \to 0$ for $i \in \mathcal{N}$ and therefore, we know from (41) that

$$\sum_{j=1}^{n} k_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij}) \to 0, \quad \text{for } i \in \mathcal{N}. \tag{42}$$

Then, multiplying the above equation by $(\boldsymbol{\xi}_i - \boldsymbol{\delta}_i)$ and taking the sum over i, we can write: $\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\delta}_i)^\top(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij}) \to 0$, for $i \in \mathcal{N}$, where the constant vector $\boldsymbol{\delta}_i$ can be regarded as the desired position of the ith aircraft with respect to the center of the formation, with $\boldsymbol{\delta}_{ij} = (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)$. Using the relation $k_{ij} = k_{ji}$, this last equation can be rewritten as: $\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij})^\top(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij}) \to 0$, and consequently, we conclude that $(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j) \to \boldsymbol{\delta}_{ij}$, for all $i, j \in \mathcal{N}$, since the communication graph is connected.

To this point, the dynamics of the variable $\boldsymbol{\alpha}_i$ in (15) can be rewritten as

$$\ddot{\boldsymbol{\alpha}}_i = -L_i^p \boldsymbol{\alpha}_i - L_i^d \dot{\boldsymbol{\alpha}}_i - \boldsymbol{\phi}_i, \tag{43}$$

for $i \in \mathcal{N}$, and represents the dynamics of a double integrator with a perturbation term $\boldsymbol{\phi}_i$, which is, in view of the above results, bounded and asymptotically vanishing. Hence, it is easy to verify that $\dot{\boldsymbol{\alpha}}_i$ and $\boldsymbol{\alpha}_i$ are bounded and $\boldsymbol{\alpha}_i \to \dot{\boldsymbol{\alpha}}_i \to 0$. As a result, the dynamics of the variable $\boldsymbol{\theta}_i$ in (14) can be rewritten as in (8), with $\boldsymbol{\varepsilon}_i = -\mathbf{u}_i$. We can verify from (27) and the above results that $\boldsymbol{\varepsilon}_i$ is bounded and converges asymptotically to zero. Therefore, using the result of Lemma 1, we conclude that $\boldsymbol{\theta}_i$ and $\dot{\boldsymbol{\theta}}_i$ are bounded and $\boldsymbol{\theta}_i \to \dot{\boldsymbol{\theta}}_i \to 0$, for $i \in \mathcal{N}$. Finally, we conclude from (25) that $\mathbf{v}_i$ and $(\mathbf{p}_i - \mathbf{p}_j)$ are bounded and $\mathbf{v}_i \to 0$ and $(\mathbf{p}_i - \mathbf{p}_j) \to \boldsymbol{\delta}_{ij}$ for all $i, j \in \mathcal{N}$. □

Remark 1. Note that the time-derivative of the variable $\boldsymbol{\beta}_i$ is required in the control input (22). An explicit expression of $\dot{\boldsymbol{\beta}}_i$ can be obtained by simple computations as

$$\dot{\boldsymbol{\beta}}_i = \frac{-k_i^\beta}{2}(\tilde{\eta}_i \mathbf{I}_3 + \mathbf{S}(\tilde{\mathbf{q}}_i))\tilde{\boldsymbol{\omega}}_i + \frac{\mathcal{T}_i}{k_i^q m_i}\frac{\mathrm{d}}{\mathrm{d}t}\left(\boldsymbol{\Psi}_i^\top \mathbf{z}_i\right) + \frac{m_i}{k_i^q \mathcal{T}_i}(\mathbf{F}_i - g\hat{e}_3)^\top \dot{\mathbf{F}}_i \boldsymbol{\Psi}_i^\top \mathbf{z}_i, \tag{44}$$

with $\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\Psi}_i^\top) = 2\left(\mathbf{S}(\dot{\bar{\mathbf{q}}}_i)^\top - \mathbf{S}(\bar{\mathbf{q}}_i)^\top \mathbf{S}(\boldsymbol{\omega}_i)\right)\mathbf{R}(\mathbf{Q}_i)$. Note that $\dot{\boldsymbol{\beta}}_i$ is a function of only available signals.

It can be seen from the proof of Theorem 1 that the main role of the auxiliary variables $\boldsymbol{\theta}_i$ and $\boldsymbol{\alpha}_i$ is to change the system trajectories during the transient. In fact, instead of designing the intermediary control input $\mathbf{F}_i$ to achieve our control objective *i.e.*, $\mathbf{v}_i \to 0$ and $(\mathbf{p}_i - \mathbf{p}_j - \boldsymbol{\delta}_{ij}) \to 0$, we have first used the error signals $\boldsymbol{\xi}_i$ and $\mathbf{z}_i$, given in (25), to design the input $\boldsymbol{\phi}_i$ such that $\mathbf{v}_i \to (\dot{\boldsymbol{\theta}}_i + \dot{\boldsymbol{\alpha}}_i)$ and $(\mathbf{p}_i - \mathbf{p}_j - \boldsymbol{\delta}_{ij}) \to (\boldsymbol{\theta}_i - \boldsymbol{\theta}_j) + (\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j)$, for all $i, j \in \mathcal{N}$. Then, the states of the auxiliary system (15) are used in the design of the input $\mathbf{u}_i$, given in (27), to drive the variables $\boldsymbol{\alpha}_i$ and $\dot{\boldsymbol{\alpha}}_i$ asymptotically to zero. Once this is achieved, the intermediary control $\mathbf{F}_i$ in (10) and (14) is designed as in (16) to drive the auxiliary variables $\boldsymbol{\theta}_i$ and $\dot{\boldsymbol{\theta}}_i$ to zero asymptotically leading to our original objective. As a result, we had the facility in this section to design the *a priori* bounded intermediary control law that achieves our objectives using linear coupling between neighboring aircraft in the presence of time-varying communication delays, and Lyapunov–Krasovskii functionals have been used to prove our result.

Remark 2. It is clear that the proposed control scheme in Theorem 1 can be applied in the case of constant communication delays. However, it is important to mention that, in this case, the second auxiliary system (15) is not required and $\mathbf{u}_i$ can be designed as follows

$$\mathbf{u}_i = -k_i^v \tilde{\mathbf{z}}_i - \sum_{j=1}^{n} k_{ij} \tilde{\boldsymbol{\xi}}_{ij}, \tag{45}$$

with the control gains being defined as in Theorem 1, $\tilde{\boldsymbol{\xi}}_{ij} = (\tilde{\boldsymbol{\xi}}_i - \tilde{\boldsymbol{\xi}}_j(t - \tau_{ij}) - \boldsymbol{\delta}_{ij})$, $\tilde{\boldsymbol{\xi}}_i = \mathbf{p}_i - \boldsymbol{\theta}_i$ and $\tilde{\mathbf{z}}_i = \dot{\tilde{\boldsymbol{\xi}}}_i$. Following the same steps as in the proof of Theorem 1, we can show that our control objective is achieved with constant communication delays if the input $\boldsymbol{\beta}_i$ is designed as in (35) with $\mathbf{z}_i$ replaced by $\tilde{\mathbf{z}}_i$, and the control gains satisfy conditions (36) and (37). Note that with the above design, the first time-derivative of $\mathbf{u}_i$ can be evaluated using available signals and is given by: $\dot{\mathbf{u}}_i = -k_i^v \dot{\tilde{\mathbf{z}}}_i - \sum_{j=1}^{n} k_{ij}(\tilde{\mathbf{z}}_i - \tilde{\mathbf{z}}_j(t - \tau_{ij}))$. However, if the communication delays are time-varying, the implementation of $\dot{\mathbf{u}}_i$ will require the time-derivatives of the delays, which are not generally known.

Remark 3. It is important to mention that the authors in Münz et al. (2008) have considered a similar coordination algorithm as in (28) to solve the Rendezvous problem of multi-agent systems modeled as double integrators in the presence of constant communication delays, and delay-dependent conditions have been derived using Lyapunov–Krasovskii functionals.

5.2. Delay-independent design

We can see from the proposed control scheme presented above that the relative velocities of communicating aircraft are not used in the design of the input of the auxiliary systems. Usually, these signals are used in a formation control scheme to improve the system's response in the sense that additional damping is introduced to the closed loop system through the relative velocities. In this section, we will show that the inclusion of the relative velocities will enable the design of a formation control scheme in the presence of arbitrary constant communication delays. For this purpose, we consider the input $\mathbf{u}_i$ given in (27) and the following extension of the design of the input $\boldsymbol{\phi}_i$,

$$\boldsymbol{\phi}_i = -k_i^v \mathbf{z}_i - k_i^v \lambda \sum_{j=1}^{n} k_{ij} \boldsymbol{\xi}_{ij} - 2\lambda \sum_{j=1}^{n} k_{ij} \mathbf{z}_{ij}, \tag{46}$$

where $\boldsymbol{\xi}_{ij} = (\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t - \tau_{ij}) - \boldsymbol{\delta}_{ij})$, $\mathbf{z}_{ij} = (\mathbf{z}_i - \mathbf{z}_j(t - \tau_{ij}))$, the control gains are defined as in Theorem 1, λ is a positive scalar and the vectors $\boldsymbol{\xi}_i$ and $\mathbf{z}_i$ are defined in (25). Inspired by the work of Nuño et al. (2011), we define the new error vector

$$\mathbf{r}_i = \mathbf{z}_i + \lambda \sum_{j=1}^{n} k_{ij}\boldsymbol{\xi}_{ij}, \tag{47}$$

for $i \in \mathcal{N}$. The time-derivative of this error vector can be obtained from (26) with (46) as

$$\dot{\mathbf{r}}_i = -k_i^v \mathbf{r}_i - \lambda \sum_{j=1}^{n} k_{ij}\mathbf{z}_{ij} - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i. \tag{48}$$

It is worth mentioning that the idea of using the variable $\mathbf{r}_i$, given in (47), in the control design and analysis has been considered in Nuño et al. (2011) to solve the adaptive synchronization problem of Euler–Lagrange systems in the presence of constant communication delays.

Similarly to the previous section, we can see from (18)–(21), (27), (29) and (46) that $\boldsymbol{\omega}_{d_i}$ and $\dot{\boldsymbol{\omega}}_{d_i}$ can be evaluated using available signals and aircraft need only to communicate their variables $\boldsymbol{\xi}_i$ and $\mathbf{z}_i$. Therefore, the input torque (22) can be applied to the rotational dynamics with the vector $\boldsymbol{\beta}_i$ given in the following theorem.

Theorem 2. *Consider the VTOL–UAVs formation modeled as in (1)–(2). For each aircraft, let the thrust input $\mathcal{T}_i$ and the desired attitude $\mathbf{Q}_{d_i}$ be given, respectively, by (A.1) and (A.2), with $\mathbf{F}_i$ given by (16) with (14), (15), (27) and (46). Let the input torque for each aircraft be as in (22) with the vector $\boldsymbol{\beta}_i$ defined as*

$$\boldsymbol{\beta}_i = -k_i^{\beta}\tilde{\mathbf{q}}_i + \frac{\mathcal{T}_i}{k_i^q m_i}\boldsymbol{\Psi}_i^{\top}\mathbf{r}_i, \tag{49}$$

with the variable $\mathbf{r}_i$ defined in (47). Let the controller gains satisfy condition (36), and assume that the communication graph $\mathcal{G}$ is connected. Then, starting from any initial conditions, the signals $\mathbf{v}_i$, $(\mathbf{p}_i - \mathbf{p}_j)$ and $\tilde{\boldsymbol{\omega}}_i$ are bounded and $\mathbf{v}_i \to 0$, $(\mathbf{p}_i - \mathbf{p}_j) \to \boldsymbol{\delta}_{ij}$, $\tilde{\mathbf{q}}_i \to 0$ and $\tilde{\boldsymbol{\omega}}_i \to 0$ for all $i, j \in \mathcal{N}$.

Proof. Similar to the proof of Theorem 1, from condition (36), we can use the extraction algorithm in Lemma 4 to extract the necessary thrust and the desired attitude for each VTOL vehicle. Consider the following Lyapunov–Krasovskii functional candidate

$$V = \frac{1}{2}\sum_{i=1}^{n}\mathbf{r}_i^{\top}\mathbf{r}_i + \frac{\lambda^2}{2}\sum_{i=1}^{n}\left(\sum_{j=1}^{n}k_{ij}\boldsymbol{\xi}_{ij}\right)^{\top}\left(\sum_{j=1}^{n}k_{ij}\boldsymbol{\xi}_{ij}\right) + \frac{\lambda}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}\int_{t-\tau_{ij}}^{t}\mathbf{z}_j^{\top}\mathbf{z}_j \mathrm{d}s + V_{a_1}, \tag{50}$$

where V_{a_1} is given in (33). The time-derivative of V evaluated along the closed loop dynamics is given as

$$\begin{aligned}\dot{V} = &\sum_{i=1}^{n}\mathbf{r}_i^{\top}\left(-k_i^v\mathbf{r}_i - \lambda\sum_{j=1}^{n}k_{ij}\mathbf{z}_{ij} - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i\right)\\ &+ \frac{\lambda}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}\left(\mathbf{z}_j^{\top}\mathbf{z}_j - \mathbf{z}_j(t-\tau_{ij})^{\top}\mathbf{z}_j(t-\tau_{ij})\right)\\ &+ \sum_{i=1}^{n}\lambda^2\left(\sum_{j=1}^{n}k_{ij}\mathbf{z}_{ij}\right)^{\top}\left(\sum_{j=1}^{n}k_{ij}\boldsymbol{\xi}_{ij}\right) + \dot{V}_{a_1}.\end{aligned} \tag{51}$$

Then, using the expression of $\mathbf{r}_i$ in (47), Eq. (34) with (49), and the relation $k_{ij} = k_{ji}$, we obtain

$$\begin{aligned}\dot{V} = &-\sum_{i=1}^{n}\left(k_i^v\mathbf{r}_i^{\top}\mathbf{r}_i + k_i^{\Omega}\boldsymbol{\Omega}_i^{\top}\boldsymbol{\Omega}_i + k_i^q k_i^{\beta}\tilde{\mathbf{q}}_i^{\top}\tilde{\mathbf{q}}_i\right)\\ &- \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\lambda k_{ij}\mathbf{z}_{ij}^{\top}\mathbf{z}_{ij},\end{aligned} \tag{52}$$

which is negative semi-definite. Then, we conclude that $\mathbf{r}_i$, $\boldsymbol{\Omega}_i$, $\tilde{\mathbf{q}}_i$ and $\left(\sum_{j=1}^{n}k_{ij}\boldsymbol{\xi}_{ij}\right)$ are bounded, for $i \in \mathcal{N}$. Consequently, we know that $\mathbf{z}_i$ is bounded. Hence, from (48) and the above results, we know that $\dot{\mathbf{r}}_i$ is bounded, for $i \in \mathcal{N}$. Therefore, we conclude that $\dot{\mathbf{z}}_i$ is bounded, for $i \in \mathcal{N}$, from the time-derivative of (47) and the fact that $\mathbf{z}_i$ is bounded. On the other hand, using similar arguments as in the proof of Theorem 1, we can verify that $\dot{\boldsymbol{\Omega}}_i$ and $\dot{\tilde{\mathbf{q}}}_i$ are bounded, for $i \in \mathcal{N}$. As a result, we conclude that $\ddot{V}$ is bounded, and by Barbălat Lemma, we conclude that $\mathbf{r}_i \to 0$, $(\mathbf{z}_i - \mathbf{z}_j(t-\tau_{ij})) \to 0$, $\boldsymbol{\Omega}_i \to 0$ and $\tilde{\mathbf{q}}_i \to 0$, and therefore, $\boldsymbol{\beta}_i \to 0$ and $\tilde{\boldsymbol{\omega}}_i \to 0$ for $i \in \mathcal{N}$. Exploiting the above results, we conclude from (48) that $\dot{\mathbf{r}}_i \to 0$, for $i \in \mathcal{N}$, and therefore we know from the definition of $\mathbf{r}_i$ in (47) that $\dot{\mathbf{z}}_i \to 0$, for $i \in \mathcal{N}$. Consequently, using a similar relation to (40) in the proof of Theorem 1, we can show that $(\mathbf{z}_i - \mathbf{z}_j(t-\tau_{ij})) \to 0$ is equivalent to $(\mathbf{z}_i - \mathbf{z}_j) \to 0$.

Now, let $\tilde{\boldsymbol{\xi}}_i = (\tilde{\xi}_i^1, \tilde{\xi}_i^2, \tilde{\xi}_i^3)^{\top} := (\boldsymbol{\xi}_i - \boldsymbol{\delta}_i)$, where $\boldsymbol{\delta}_i$ is defined as in Theorem 1, and rewrite Eq. (47) as

$$\dot{\tilde{\boldsymbol{\xi}}}_i = -\lambda\tilde{\boldsymbol{\xi}}_i\sum_{j=1}^{n}k_{ij} + \lambda\sum_{j=1}^{n}k_{ij}\tilde{\boldsymbol{\xi}}_j(t-\tau_{ij}) + \mathbf{r}_i, \tag{53}$$

for $i \in \mathcal{N}$, where it is clear that $\dot{\tilde{\boldsymbol{\xi}}}_i = \mathbf{z}_i$. Motivated by the work of Nuño et al. (2011), we define $\tilde{\boldsymbol{\xi}}^k = \mathrm{col}(\tilde{\xi}_1^k, \tilde{\xi}_2^k, \dots, \tilde{\xi}_n^k) \in \mathbb{R}^n$ and $\mathbf{r}^k = \mathrm{col}(r_1^k, r_2^k, \dots, r_n^k) \in \mathbb{R}^n$, with $k \in \{1, 2, 3\}$ and $\mathbf{r}_i = (r_i^1, r_i^2, r_i^3)^{\top}$, for $i \in \mathcal{N}$. In addition, let $\mathcal{N}_i$ be the set containing the indices of all aircraft that communicate with the ith aircraft and define $m = \sum_{i=1}^{n}|\mathcal{N}_i|$ and $\tau_l = \tau_{ij}$, for $l \in \{1, \dots, m\}$ and $(i, j) \in \bar{\mathcal{E}}$, where $|\cdot|$ is used to indicate the cardinality of a set and $\bar{\mathcal{E}}$ is the set of all pairs of nodes (i, j) such that the ith aircraft receives information from the jth aircraft. It is clear that m is equal to twice the number of undirected edges in the communication graph $\mathcal{G}$. With the above definitions, the set of equations in (53) can be written as

$$\dot{\tilde{\boldsymbol{\xi}}}^k = -\lambda\mathbf{A}_0\tilde{\boldsymbol{\xi}}^k + \lambda\sum_{l=1}^{m}\mathbf{A}_l\tilde{\boldsymbol{\xi}}^k(t-\tau_l) + \mathbf{r}^k, \tag{54}$$

for $k \in \{1, 2, 3\}$, where $\mathbf{A}_0 \in \mathbb{R}^{n\times n}$ is a diagonal matrix with its (i, i)th element equal to $\sum_{j=1}^{n}k_{ij}$, and the matrices $\mathbf{A}_l \in \mathbb{R}^{n\times n}$ have all elements equal to zero except one off-diagonal element that takes one of the weights k_{ij} such that $\sum_{l=1}^{m}\mathbf{A}_l = \mathcal{K}$, with $\mathcal{K}$ being the weighted adjacency matrix of $\mathcal{G}$. Following the same steps as in the proof of Proposition 2 in Nuño et al. (2011), we can show that $\dot{\tilde{\boldsymbol{\xi}}}^k \to 0$, for $k \in \{1, 2, 3\}$, since $\mathbf{r}^k \to 0$ and the matrix $\mathbf{A}_0 - \mathcal{K}$, defining the Laplacian matrix of the connected undirected communication graph $\mathcal{G}$, has a simple zero eigenvalue and all other eigenvalues are real and positive (Ren et al., 2007). Consequently, we can conclude that $\mathbf{z}_i \to 0$, for $i \in \mathcal{N}$, which together with relation (40) implies that $(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t-\tau_{ij})) \to (\boldsymbol{\xi}_i - \boldsymbol{\xi}_j)$. As a result, we know from (47) that $\sum_{j=1}^{n}k_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j - \boldsymbol{\delta}_{ij}) \to 0$, and using the same procedure as in the proof of Theorem 1, we conclude that $(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j) \to \boldsymbol{\delta}_{ij}$, for all $i, j \in \mathcal{N}$.

To this point, we can see that $\boldsymbol{\phi}_i$ in (46) is bounded and converges asymptotically to zero. Using the same arguments as in the proof of Theorem 1 with the result of Lemma 1, we can verify that $\dot{\boldsymbol{\alpha}}_i$, $\boldsymbol{\alpha}_i$, $\boldsymbol{\theta}_i$ and $\dot{\boldsymbol{\theta}}_i$ are bounded and $\boldsymbol{\alpha}_i \to 0$, $\dot{\boldsymbol{\alpha}}_i \to 0$, $\boldsymbol{\theta}_i \to 0$ and $\dot{\boldsymbol{\theta}}_i \to 0$, for $i \in \mathcal{N}$, which leads to the results of the theorem. □

It is worth noticing that the control schemes in this section rely on the assumption that the linear-velocity vectors are available for feedback. In fact, this assumption is essential when using Lyapunov–Krasovskii functionals in the proof of our results. In the next section, we will show that the auxiliary systems can still be used to remove the linear-velocity requirements and similar analysis tools will be used.

6. Design without linear-velocity information

In this section, we exploit the advantage of the introduction of the auxiliary system (15) to each aircraft to solve the formation stabilization problem of VTOL UAVs in the presence of constant communication delays and without linear-velocity measurement. As done in the previous section, in addition to the auxiliary system (14), we associate to each aircraft the modified second-order system

$$\ddot{\boldsymbol{\alpha}}_i = \mathbf{u}_i - \boldsymbol{\phi}_i - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i, \tag{55}$$

with $\mathbf{u}_i$ and $\boldsymbol{\phi}_i$ are input vectors to be designed. The role of this system is quite different from the formation control schemes in the full-information case. System (55) in this section describes the translational dynamics of a virtual vehicle moving in space. The main idea here is to design the input $\mathbf{u}_i$ based on the virtual vehicle's linear-velocity and position, $\dot{\boldsymbol{\alpha}}_i$ and $\boldsymbol{\alpha}_i$, respectively, to guarantee that all the virtual vehicles converge to the desired formation in the presence of communication delays *i.e.*, $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j) \to \boldsymbol{\delta}_{ij}$ and $\dot{\boldsymbol{\alpha}}_i \to 0$. We propose the following input $\mathbf{u}_i$ in (14) and (55),

$$\mathbf{u}_i = -k_i^v\dot{\boldsymbol{\alpha}}_i - \sum_{j=1}^{n} k_{ij}\boldsymbol{\alpha}_{ij}, \tag{56}$$

with $\boldsymbol{\alpha}_{ij} = (\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j(t-\tau_{ij}) - \boldsymbol{\delta}_{ij})$ and k_i^v and k_{ij} are given as in Theorem 1. The design of this input is motivated by the following preliminary result proved in Appendix C.

Lemma 3. *Consider n-vehicles modeled as*

$$\ddot{\boldsymbol{\alpha}}_i = -k_i^v\dot{\boldsymbol{\alpha}}_i - \sum_{j=1}^{n} k_{ij}\boldsymbol{\alpha}_{ij} + \bar{\boldsymbol{\varepsilon}}_i, \tag{57}$$

for $i \in \mathcal{N}$, with τ_{ij} the constant communication delay between the ith and jth vehicles satisfying $\tau_{ij} \le \tau$ for all $(i,j) \in \mathcal{E}$. Let the control gains k_i^v and k_{ij} satisfy condition (37), for some $\epsilon > 0$ and assume that the communication graph $\mathcal{G}$ is connected. If the vector $\bar{\boldsymbol{\varepsilon}}_i$ converges asymptotically to zero and is bounded by an arbitrary constant $\bar{\varepsilon}_i^b$, such that $\|\bar{\boldsymbol{\varepsilon}}_i\| \le \bar{\varepsilon}_i^b$, for all $t > 0$ and $i \in \mathcal{N}$, then $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j)$ and $\dot{\boldsymbol{\alpha}}_i$ are bounded and $\dot{\boldsymbol{\alpha}}_i \to 0$, and $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j) \to \boldsymbol{\delta}_{ij}$, for all $i, j \in \mathcal{N}$.

Define the error signals for each aircraft as in (25), *i.e.*,

$$\boldsymbol{\xi}_i = \mathbf{p}_i - \boldsymbol{\theta}_i - \boldsymbol{\alpha}_i, \qquad \mathbf{z}_i := \dot{\boldsymbol{\xi}}_i. \tag{58}$$

In view of the dynamics of the auxiliary systems (14) and (55) and the results of Lemmas 1 and 3, the formation control design problem is reduced to determine the input $\boldsymbol{\phi}_i$, without linear-velocity measurements, such that each vehicle tracks the states of its corresponding virtual vehicle, *i.e.*, $\mathbf{z}_i \to 0$ and $\boldsymbol{\xi}_i \to 0$. Motivated by our recent result in Abdessameud and Tayebi (2010a), we propose the following input in (55)

$$\boldsymbol{\phi}_i = -L_i^p\boldsymbol{\xi}_i - L_i^d(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i), \tag{59}$$

$$\dot{\boldsymbol{\psi}}_i = L_i^\psi(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i), \tag{60}$$

with L_i^p, L_i^d and L_i^ψ are positive scalar gains and $\boldsymbol{\psi}_i \in \mathbb{R}^3$ is the output of the first order system (60) that can be initialized arbitrarily. The time-derivative of the vector $\mathbf{z}_i$ defined in (58) in view of (10), (14), (55) and (59) is obtained as

$$\dot{\mathbf{z}}_i = -L_i^p\boldsymbol{\xi}_i - L_i^d(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i). \tag{61}$$

To complete the design of the input torque of each vehicle, note first that the time-derivative of $\mathbf{u}_i$ in (56) can be obtained as

$$\dot{\mathbf{u}}_i = -k_i^v\left(\mathbf{u}_i - \boldsymbol{\phi}_i - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i\right) - \sum_{j=1}^{n} k_{ij}\left(\dot{\boldsymbol{\alpha}}_i - \dot{\boldsymbol{\alpha}}_j(t-\tau_{ij})\right), \tag{62}$$

and is function of available signals. Therefore, the desired angular velocity and its time-derivative given in (18)–(21) are explicitly known. However, to implement the above control scheme, neighboring aircraft must communicate the position and linear-velocity of their corresponding virtual vehicles, $\boldsymbol{\alpha}_i$ and $\dot{\boldsymbol{\alpha}}_i$. Note also that the perturbation term in the translational dynamics (10) has been compensated in the dynamics of the virtual system (55). To guarantee the stability of the overall system, we propose the following expression for the variable $\boldsymbol{\beta}_i$ in (23)

$$\boldsymbol{\beta}_i = -k_i^\beta\tilde{\mathbf{q}}_i. \tag{63}$$

Our result in this section is given in the following theorem.

Theorem 3. *Consider the VTOL–UAVs formation modeled as in (1)–(2). For each aircraft, let the thrust input $\mathcal{T}_i$ and the desired attitude $\mathbf{Q}_{d_i}$ be given, respectively, by (A.1) and (A.2), with $\mathbf{F}_i$ given by (16) with (14), (55), (56) and (59)–(60). Let the input torque for each aircraft be given by (22) and the vector $\boldsymbol{\beta}_i$ is defined in (63). Let the controller gains satisfy conditions (36) and (37) for some $\epsilon > 0$ and $\tau_{ij} \le \tau$, for all $(i,j) \in \mathcal{E}$, and assume that the communication graph is connected. Then, starting from any initial conditions, the signals $\mathbf{v}_i$, $(\mathbf{p}_i - \mathbf{p}_j)$ and $\tilde{\boldsymbol{\omega}}_i$ are bounded and $\mathbf{v}_i \to 0$, $(\mathbf{p}_i - \mathbf{p}_j) \to \boldsymbol{\delta}_{ij}$, $\tilde{\mathbf{q}}_i \to 0$ and $\tilde{\boldsymbol{\omega}}_i \to 0$ for all $i, j \in \mathcal{N}$.*

Proof. Similar to the proof of Theorem 1, the thrust input and desired attitude for each aircraft can be extracted from (11) if condition (36) is satisfied. Consider the following Lyapunov function candidate

$$V = V_{t_2} + V_{a_1}, \tag{64}$$

with V_{a_1} given in (33) and

$$V_{t_2} = \frac{1}{2}\sum_{i=1}^{n}\left(\mathbf{z}_i^\top\mathbf{z}_i + L_i^p\boldsymbol{\xi}_i^\top\boldsymbol{\xi}_i + L_i^d(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i)^\top(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i)\right). \tag{65}$$

The time-derivative of V evaluated along (61) and (24) is obtained as

$$\dot{V} = -\sum_{i=1}^{n} L_i^d\dot{\boldsymbol{\psi}}_i^\top(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i) + \dot{V}_{a_1}. \tag{66}$$

Using (60) and (63) in view of (34), we obtain

$$\dot{V} = -\sum_{i=1}^{n} L_i^d L_i^\psi(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i)^\top(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i) + \sum_{i=1}^{n}\left(-k_i^\Omega\boldsymbol{\Omega}_i^\top\boldsymbol{\Omega}_i - k_i^q k_i^\beta\tilde{\mathbf{q}}_i^\top\tilde{\mathbf{q}}_i\right). \tag{67}$$

The time-derivative of V is then negative semi-definite, and we conclude that $\mathbf{z}_i, \boldsymbol{\xi}_i, \boldsymbol{\psi}_i, \tilde{\mathbf{q}}_i$ and $\boldsymbol{\Omega}_i$ are bounded for $i \in \mathcal{N}$. In addition, we can see from (24) that $\dot{\boldsymbol{\Omega}}_i$ is bounded. Also, since $\tilde{\mathbf{q}}_i$ is bounded, we know that $\boldsymbol{\beta}_i$ is bounded and consequently $\tilde{\boldsymbol{\omega}}_i$ is bounded. Hence, we conclude that $\dot{\tilde{\mathbf{q}}}_i$ is bounded. In addition, we can see from (60) that $\dot{\boldsymbol{\psi}}_i$ is bounded. As a result, we have that $\ddot{V}$ is bounded, and invoking Barbălat Lemma, we conclude that $(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i) \to 0$, $\tilde{\mathbf{q}}_i \to 0$ and $\boldsymbol{\Omega}_i \to 0$, for $i \in \mathcal{N}$. Consequently, we conclude that $\tilde{\boldsymbol{\omega}}_i \to 0$.

Furthermore, we can easily verify that $(\ddot{\boldsymbol{\xi}}_i - \ddot{\boldsymbol{\psi}}_i)$ is bounded from (60)–(61). Therefore, by Barbălat Lemma and since $(\boldsymbol{\xi}_i - \boldsymbol{\psi}_i) \to 0$, we conclude that $\mathbf{z}_i \to \dot{\boldsymbol{\psi}}_i$, and consequently we know that $\mathbf{z}_i \to 0$ for $i \in \mathcal{N}$. Also, we can verify from the time-derivative of (61) that $\ddot{\mathbf{z}}_i$ is bounded, and we conclude by Barbălat lemma that $\dot{\mathbf{z}}_i \to 0$, and as a result we have $\boldsymbol{\xi}_i \to 0$ for $i \in \mathcal{N}$.

From the above results, we can verify that the term $\bar{\boldsymbol{\varepsilon}}_i = \left(-\boldsymbol{\phi}_i - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i\right)$ is bounded and converges asymptotically to zero.

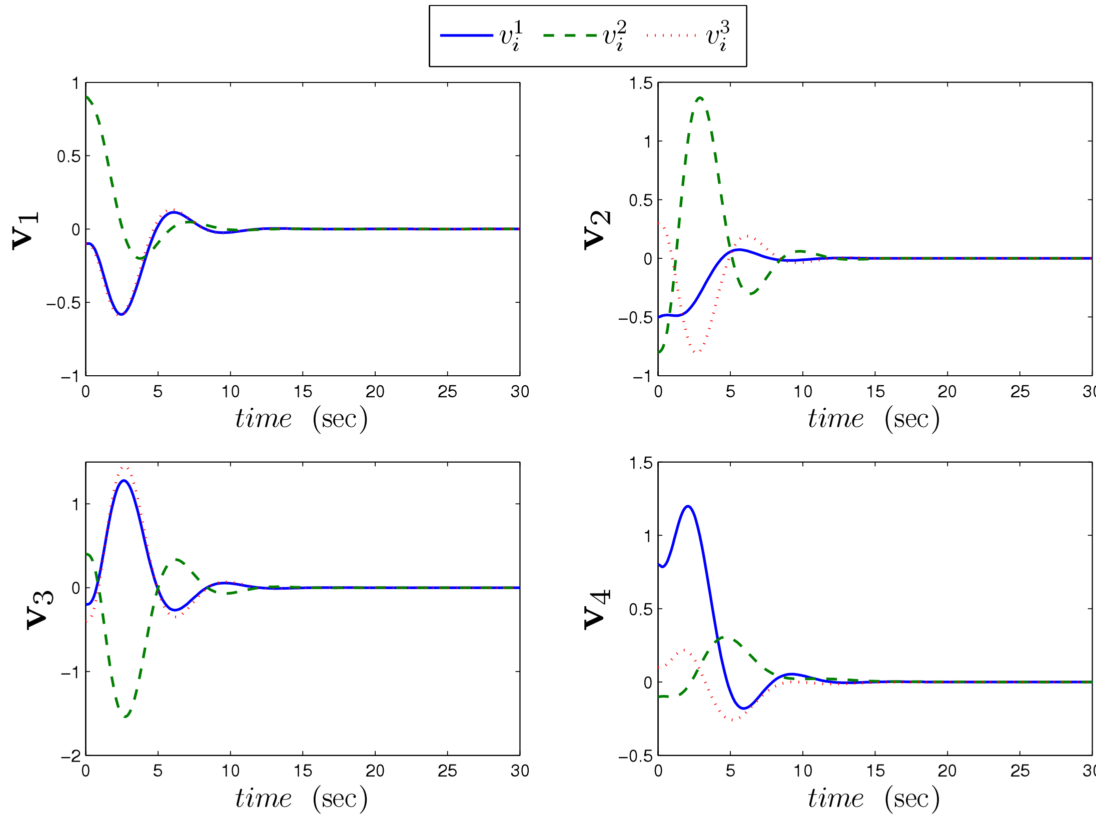

Fig. 1. Linear velocity vectors, $\mathbf{v}_i = (v_i^1, v_i^2, v_i^3)^\top$ m/s.

Therefore, the dynamics of the virtual system (55) can be rewritten as in (57), and we can conclude from the result of Lemma 3 that if the control gains satisfy condition (37), the signals $\dot{\boldsymbol{\alpha}}_i$ and $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j)$ are bounded and $\dot{\boldsymbol{\alpha}}_i \to 0$ and $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j) \to \boldsymbol{\delta}_{ij}$, for all $i, j \in \mathcal{N}$. As a result, we have the term $\boldsymbol{\varepsilon}_i = -\mathbf{u}_i$ is bounded and converges asymptotically to zero. Therefore, we conclude from (14) with (16) and the results of Lemma 1 that $\boldsymbol{\theta}_i$ and $\dot{\boldsymbol{\theta}}_i$ are bounded and $\boldsymbol{\theta}_i \to \dot{\boldsymbol{\theta}}_i \to 0$. Finally, from the error signals definition (58) and the above results, we conclude that $\mathbf{v}_i$ and $(\mathbf{p}_i - \mathbf{p}_j)$ are bounded and $\mathbf{v}_i \to 0$ and $(\mathbf{p}_i - \mathbf{p}_j) \to \boldsymbol{\delta}_{ij}$ for all $i, j \in \mathcal{N}$. $\square$

7. Simulation results

In this section, we provide simulation results to demonstrate the effectiveness of the proposed control schemes. We consider a group of four aircraft modeled as in (1)–(2), with $m_i = 3$ kg, $\mathbf{I}_{f_i} = \mathrm{diag}(0.13, 0.13, 0.04)$ kg $\cdot$ m^2, for $i \in \mathcal{N} \triangleq \{1, \ldots, 4\}$, and initial conditions: $\mathbf{p}_1(0) = (14, 0, 2)^\top$, $\mathbf{p}_2(0) = (10, -1, 2)^\top$, $\mathbf{p}_3(0) = (6, 0, -2)^\top$, $\mathbf{p}_4(0) = (9, -4, 1)^\top$, $\mathbf{v}_1(0) = (-0.1, 0.9, -0.1)^\top$, $\mathbf{v}_2(0) = (-0.5, -0.8, 0.3)^\top$, $\mathbf{v}_3(0) = (-0.2, 0.4, -0.4)^\top$, $\mathbf{v}_4(0) = (0.8, -0.1, 0.1)^\top$, $\mathbf{Q}_i(0) = (0, 0, 0, 1)^\top$, and $\boldsymbol{\omega}_i(0) = (0, 0, 0)^\top$. The control objective is to guarantee that the four aircraft maintain a pre-defined formation pattern, described by a square parallel to the universal x–y plane, with $\boldsymbol{\delta}_{ij} = (\boldsymbol{\delta}_i - \boldsymbol{\delta}_j)$, with $\boldsymbol{\delta}_1 = (2, 2, 0)^\top$, $\boldsymbol{\delta}_2 = (-2, 2, 0)^\top$, $\boldsymbol{\delta}_3 = (-2, -2, 0)^\top$, and $\boldsymbol{\delta}_4 = (2, -2, 0)^\top$. The information flow between aircraft is fixed, undirected and connected and is represented by the undirected graph having the set of edges: $\mathcal{E} = \{(1, 2), (1, 3), (2, 3), (2, 4)\}$, and the adjacency matrix $\mathcal{K} = \mathrm{col}[k_{ij}]$, with $k_{ij} = 0.5$ for $(i, j) \in \mathcal{E}$ and zero otherwise. We consider the saturation function in (7) as: $\sigma(\cdot) = \tanh(\cdot)$, with $\sigma_b = 1$.

First, we implement the control law in Theorem 1, with the control gains: $(k_i^v, k_i^p, k_i^d, L_i^p, L_i^d, k_i^\beta, k_i^q, k_i^\Omega) = (3, 1.5, 1.5, 1, 1, 50, 80, 80)$, for $i \in \mathcal{N}$, and the time-varying communication delays are taken as: $\tau_{ij}(t) = \tilde{\tau}_{ij}|\sin(0.5t)|$ s, with $\tilde{\tau}_{1i} = 0.1$, $\tilde{\tau}_{2i} = 0.15$, and $\tilde{\tau}_{3i} = \tilde{\tau}_{4i} = 0.2$, for $i \in \mathcal{N}$. It is clear that with this choice of the gains, conditions (36) and (37) are satisfied, with $\tau = 0.3$. The auxiliary systems (14) and (15) are initialized as $\boldsymbol{\theta}_i(0) = \dot{\boldsymbol{\theta}}_i(0) = \boldsymbol{\alpha}_i(0) = \dot{\boldsymbol{\alpha}}_i(0) = (0, 0, 0)^\top$. The obtained results in this case are given in Figs. 1 and 2, which illustrate, respectively, the aircraft linear velocities and positions in space. We can see from these figures that our control objective (9) is achieved in the presence of time-varying communication delays.

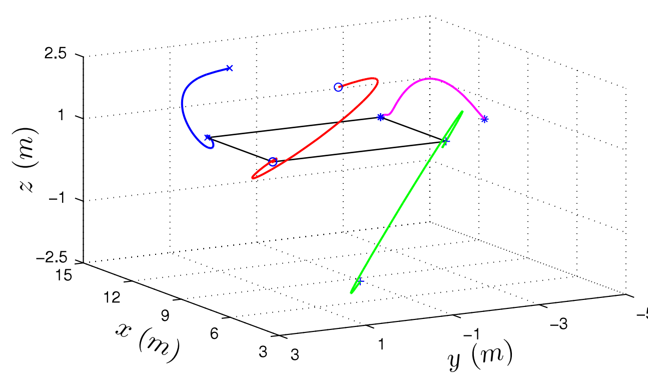

Fig. 2. VTOL formation.

Similar results have been obtained when the control scheme in Theorem 2 is implemented with arbitrary constant communication delays, and are omitted in this section due to space limitations.

Next, we consider the linear-velocity-free formation control scheme proposed in Theorem 3, with the control gains $(L_i^p, L_i^d, L_i^\psi, k_i^v, k_i^p, k_i^d, k_i^\beta, k_i^q, k_i^\Omega) = (0.5, 5, 5, 2, 1.5, 1.5, 40, 80, 80)$ for $i \in \mathcal{N}$, and the constant communication delays are selected as: $\tau_{1i} = 0.1$ s, $\tau_{2i} = 0.15$ s, and $\tau_{3i} = \tau_{4i} = 0.2$ s, for $i \in \mathcal{N}$, such that

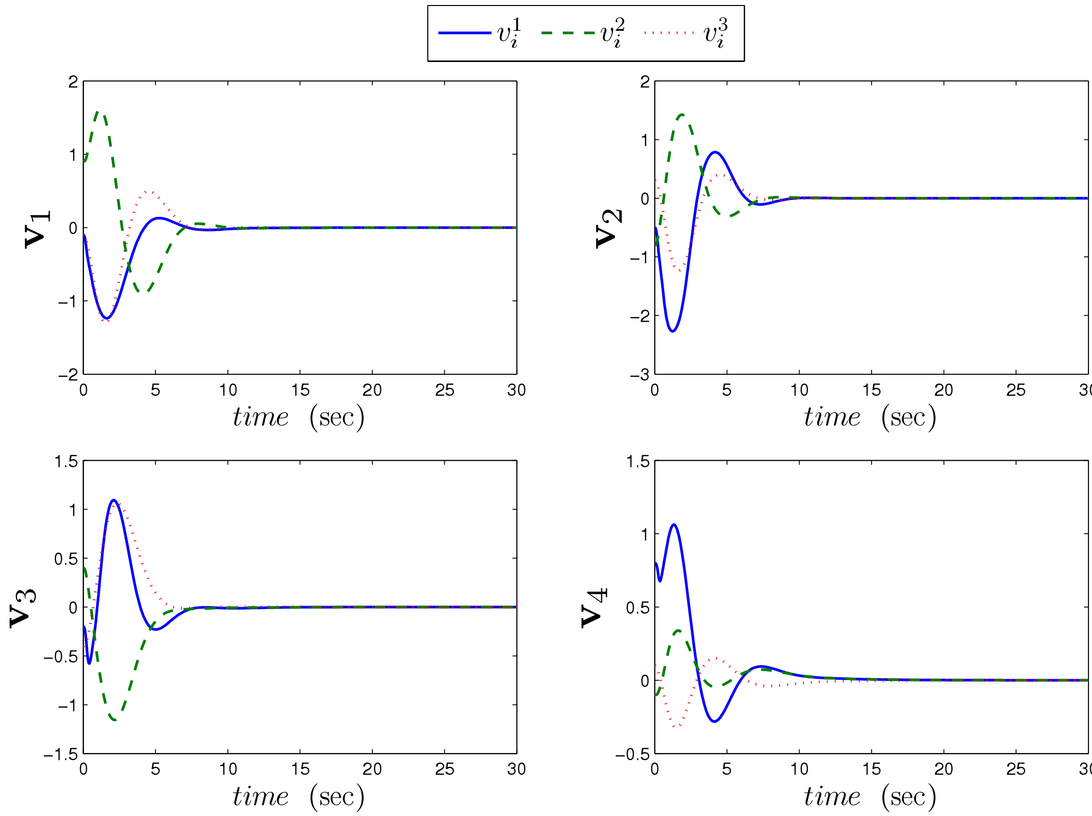

Fig. 3. Linear velocity vectors, $\mathbf{v}_i = (v_i^1, v_i^2, v_i^3)^\top$ m/s.

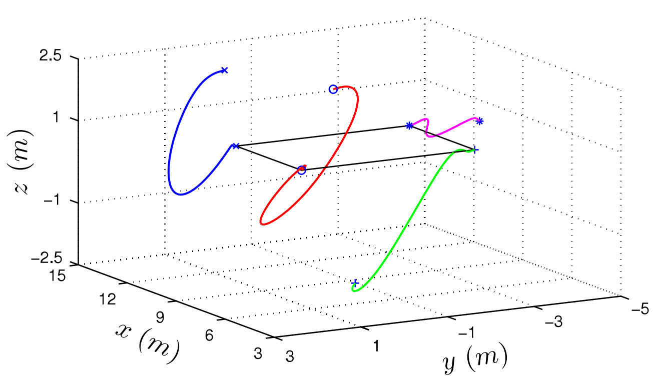

Fig. 4. VTOL formation.

conditions (36)–(37) are satisfied. The first order system (60) and system (55) are initialized as: $\boldsymbol{\psi}_i(0) = (0, 1, -1)^\top$, $\boldsymbol{\alpha}_i(0) = \mathbf{p}_i(0)$ and $\dot{\boldsymbol{\alpha}}_i(0) = (0, 0, 0)^\top$. We show the obtained results in Figs. 3 and 4 which validate the theoretical results proposed in Theorem 3.

8. Conclusion

The formation control problem of a group of VTOL aircraft with delayed communication has been addressed. The control design relies on a singularity free extraction algorithm presented in Abdessameud and Tayebi (2010a), which has enabled a separate translational and rotational control design. Instrumental auxiliary systems, leading to a suitable intermediary translational control input, have been used. Three formation control schemes, under delay-dependent and delay-independent conditions, have been proposed. To the best of our knowledge, the proposed schemes are the first solutions to the formation stabilization problem with delayed communication for the class of under-actuated systems under study in the full and partial state information cases. In addition, they can be applied in a straightforward manner to the Rendezvous problem of multi-agent systems with double integrator dynamics in the presence of delayed communication with input constraints and remove the requirements of the velocity measurements, which constitutes a new contribution in this research area.

The information exchange between aircraft is assumed to be undirected and fixed. The performance of the proposed control schemes under directed and switching communication topology is an interesting topic that will be addressed in our future work. Furthermore, a practical problem in multi-vehicles motion coordination is the collision avoidance between members of the team while converging to the desired final configuration. This problem is generally solved by the introduction of potential functions that grow unbounded if two vehicles (or more) enter a predefined collision region. The main difficulty in the application of this technique in our case is that the intermediary control input needs to be *a priori* bounded and satisfies the extraction algorithm condition.

Appendix A. Thrust and desired attitude extraction algorithm

The following lemma gives one possible singularity-free extraction algorithm that provides the necessary thrust, $\mathcal{T}_i$, and desired attitude, $\mathbf{Q}_{d_i} = (\mathbf{q}_{d_i}^\top, \eta_{d_i})^\top$, for each aircraft from a known value of the intermediary control $\mathbf{F}_i$. Since this procedure applies for all VTOL vehicles in the formation, we will omit the subscript "i" in the following result for clarity of presentation.

Lemma 4 (*Roberts & Tayebi, 2011*)**.** *Consider Eq. (11) and let the vector* $\mathbf{F} \triangleq (\mu_1, \mu_2, \mu_3)^\top$. *It is always possible to extract the thrust magnitude and the desired system's attitude from (11) as*

$$\mathcal{T} = m\|g\hat{\mathbf{e}}_3 - \mathbf{F}\|, \tag{A.1}$$

$$\eta_d = \sqrt{\frac{1}{2} + \frac{m(g-\mu_3)}{2\mathcal{T}}}, \qquad \mathbf{q}_d = \frac{m}{2\mathcal{T}\eta_d}\begin{pmatrix}\mu_2 \\ -\mu_1 \\ 0\end{pmatrix}, \tag{A.2}$$

under the condition that (12) is satisfied. In addition, under the condition that the intermediary control $\mathbf{F}$ *is differentiable, we can write the desired angular velocity of the aircraft as*

$$\boldsymbol{\omega}_d = \boldsymbol{\Xi}(\mathbf{F})\dot{\mathbf{F}}, \tag{A.3}$$

$$\boldsymbol{\Xi}(\mathbf{F}) = \frac{1}{\gamma_1^2\gamma_2}\begin{pmatrix} -\mu_1\mu_2 & -\mu_2^2+\gamma_1\gamma_2 & \mu_2\gamma_2 \\ \mu_1^2-\gamma_1\gamma_2 & \mu_1\mu_2 & -\mu_1\gamma_2 \\ \mu_2\gamma_1 & -\mu_1\gamma_1 & 0 \end{pmatrix}, \tag{A.4}$$

with $\gamma_1 = (\mathcal{T}/m)$ *and* $\gamma_2 = \gamma_1 + (g-\mu_3)$.

Proof. A similar proof can be found in Abdessameud and Tayebi (2009) and Roberts and Tayebi (2011). □

Appendix B. Proof of Claim 2

In view of Eqs. (26), (28) and (30), we have

$$\begin{aligned}\dot{V}_{t_1} &= \sum_{i=1}^{n}\mathbf{z}_i^\top\left(-k_i^v\mathbf{z}_i - \sum_{j=1}^{n}k_{ij}\boldsymbol{\xi}_{ij} - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i\right) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}(\mathbf{z}_i-\mathbf{z}_j)^\top\bar{\boldsymbol{\xi}}_{ij} \\ &= \sum_{i=1}^{n}\mathbf{z}_i^\top\left(-k_i^v\mathbf{z}_i - \sum_{j=1}^{n}k_{ij}(\boldsymbol{\xi}_j - \boldsymbol{\xi}_j(t-\tau_{ij}(t)))\right) \\ &\quad - \sum_{i=1}^{n}\frac{\mathcal{T}_i}{m_i}\mathbf{z}_i^\top\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i,\end{aligned} \tag{B.1}$$

where we have used the relation $\boldsymbol{\xi}_{ij} = (\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t-\tau_{ij}(t)) - \boldsymbol{\delta}_{ij})$ and

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}(\mathbf{z}_i-\mathbf{z}_j)^\top\bar{\boldsymbol{\xi}}_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}\mathbf{z}_i^\top\bar{\boldsymbol{\xi}}_{ij}, \tag{B.2}$$

which can be verified using $k_{ij} = k_{ji}$ and $\boldsymbol{\delta}_{ij} = -\boldsymbol{\delta}_{ji}$. From the error signals definition (25), we know that $\left(\boldsymbol{\xi}_j - \boldsymbol{\xi}_j(t-\tau_{ij}(t))\right) = \left(\int_{t-\tau_{ij}(t)}^{t}\mathbf{z}_j\mathrm{d}s\right)$. Also, using Young's inequality and Jensen's inequality (Seuret et al., 2009), we can verify that

$$2\mathbf{z}_i^\top\int_{t-\tau_{ij}(t)}^{t}\mathbf{z}_j\mathrm{d}s \le \epsilon_{ij}\mathbf{z}_i^\top\mathbf{z}_i + \frac{\tau_{ij}(t)}{\epsilon_{ij}}\int_{t-\tau_{ij}(t)}^{t}\mathbf{z}_j^\top\mathbf{z}_j\mathrm{d}s,$$

for some strictly positive ϵ_{ij}. Without loss of generality, we consider $\epsilon_{ij} = \epsilon_{ji} = \epsilon > 0$. Exploiting the above relations, an upper bound of $\dot{V}_{t_1}$ can be obtained as

$$\begin{aligned}\dot{V}_{t_1} &\le -\sum_{i=1}^{n}\frac{\mathcal{T}_i}{m_i}\mathbf{z}_i^\top\boldsymbol{\Psi}_i\tilde{\mathbf{q}}_i - \sum_{i=1}^{n}k_i^v\mathbf{z}_i^\top\mathbf{z}_i \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}\left(\epsilon\mathbf{z}_i^\top\mathbf{z}_i + \frac{\tau_{ij}(t)}{\epsilon}\int_{t-\tau_{ij}(t)}^{t}\mathbf{z}_j^\top\mathbf{z}_j\mathrm{d}s\right).\end{aligned}$$

On the other hand, the time-derivative of V_{k_1} in (31) can be obtained as

$$\dot{V}_{k_1} = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{k_{ij}\tau}{2\epsilon}\left(\tau\mathbf{z}_j^\top\mathbf{z}_j - \int_{t-\tau}^{t}\mathbf{z}_j^\top\mathbf{z}_j\mathrm{d}s\right). \tag{B.3}$$

Therefore, using the relations $k_{ij} = k_{ji}$ and

$$\tau_{ij}(t)\int_{t-\tau_{ij}(t)}^{t}\mathbf{z}_j^\top\mathbf{z}_j\mathrm{d}s \le \tau\int_{t-\tau}^{t}\mathbf{z}_j^\top\mathbf{z}_j\mathrm{d}s, \tag{B.4}$$

the result in (32) is obtained.

Appendix C. Proof of Lemma 3

Consider the Lyapunov–Krasovskii functional candidate

$$\begin{aligned}W &= \frac{1}{2}\sum_{i=1}^{n}\dot{\boldsymbol{\alpha}}_i^\top\dot{\boldsymbol{\alpha}}_i + \frac{1}{4}\sum_{i=1}^{n}\sum_{j=1}^{n}k_{ij}\bar{\boldsymbol{\alpha}}_{ij}^\top\bar{\boldsymbol{\alpha}}_{ij} \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{k_{ij}}{2\epsilon}\tau\int_{-\tau}^{0}\int_{t+s}^{t}\dot{\boldsymbol{\alpha}}_j^\top(\varrho)\dot{\boldsymbol{\alpha}}_j(\varrho)\mathrm{d}\varrho\mathrm{d}s,\end{aligned} \tag{C.1}$$

with $\bar{\boldsymbol{\alpha}}_{ij} = (\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j - \boldsymbol{\delta}_{ij})$ and $\epsilon > 0$. Following similar steps as in the proof of Claim 2, the time-derivative of W evaluated along (57) can be upper bounded as

$$\dot{W} \le -\sum_{i=1}^{n}(k_i^z\|\dot{\boldsymbol{\alpha}}_i\| - \bar{\boldsymbol{\varepsilon}}_i^b)\|\dot{\boldsymbol{\alpha}}_i\|, \tag{C.2}$$

with k_i^z given in (37). It is clear that $\dot{W} < 0$ outside the set $\bar{\mathcal{S}} = \left\{\dot{\boldsymbol{\alpha}}_i \mid \|\dot{\boldsymbol{\alpha}}_i\| \le \frac{\bar{\boldsymbol{\varepsilon}}_i^b}{k_i^z}\right\}$, and consequently $\dot{\boldsymbol{\alpha}}_i$, for $i \in \mathcal{N}$, and $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j)$, for all $(i,j) \in \mathcal{E}$, are bounded outside $\bar{\mathcal{S}}$. Since the communicate graph is connected, this last result is valid for all $i, j \in \mathcal{N}$.

It is also clear that $\dot{\boldsymbol{\alpha}}_i$ will ultimately reach the set $\bar{\mathcal{S}}$ and will be driven to zero as $\bar{\boldsymbol{\varepsilon}}_i \to 0$. Invoking Lemma 5, we can conclude from (57) and a similar relation to (40) that $\ddot{\boldsymbol{\alpha}}_i \to 0$, and (57) reduces to: $\sum_{j=1}^{n}k_{ij}(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j - \boldsymbol{\delta}_{ij}) \to 0$, for $i \in \mathcal{N}$. Following similar steps as in the proof of Theorem 1, we can conclude that $(\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j) \to \boldsymbol{\delta}_{ij}$ for all $i, j \in \mathcal{N}$.

Appendix D. Extension of Barbălat Lemma

Lemma 5 (*Hua et al., 2009*)**.** *Let* $x(t)$ *denote a solution to the differential equation:* $\dot{x} = a(t) + b(t)$, *with* $a(t)$ *a uniformly continuous function. Assume that* $\lim_{t\to+\infty}x(t) = c$ *and* $\lim_{t\to+\infty}b(t) = 0$, *with* c *a constant value. Then,* $\lim_{t\to+\infty}\dot{x}(t) = 0$.

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