Available online at www.sciencedirect.com
ScienceDirect
automatica
Automatica 43 (2007) 1565–1572
www.elsevier.com/locate/automatica

Brief paper

Analysis of two particular iterative learning control schemes in frequency and time domains☆

Abdelhamid Tayebi*

Department of Electrical Engineering, Lakehead University, Thunder Bay, Ont., Canada P7B 5E1

Received 15 February 2006; received in revised form 28 September 2006; accepted 15 January 2007

Abstract

This paper deals with iterative learning control design for multiple-input multiple-output (MIMO), linear time-invariant (LTI) systems. Two particular ILC schemes are considered and analyzed in both frequency and time domains. Some remarks on the convergence, implementation, robustness with respect to disturbances and reinitialization errors, as well as positive realness issues related to both schemes are provided.

Keywords: Iterative learning control; Linear systems; Positive real

1. Introduction

Iterative learning control (ILC) is an attractive technique when it comes to dealing with systems that execute the same task repeatedly over a finite time-interval. The key feature of this technique is to use information from the previous (and/or current) operation (or iteration) in order to enable the controlled system to perform progressively better from operation to operation. This technique has been the center of interest of many researchers over the last two decades (see, for instance, Arimoto, Kawamura, & Miyazaki, 1984; Bien & Xu, 1998; Chen & Wen, 1999; Moore, 1993, 1999; Xu & Tan, 2003).

Some interesting results on ILC design, related to the strict positive realness property, have been discussed in Arimoto and Naniwa (2000, 2001) and Kuc and Lee (1996). In fact, one of the results of Arimoto and Naniwa (2000, 2001) shows the existence of a convergent P-type ILC scheme, using only the tracking error from the previous iteration with a small learning gain, for strictly positive real (SPR) systems. In Kuc and Lee (1996), it is shown that a P-type ILC scheme, using only the tracking error from the current iteration, with a positive learning gain guarantees the convergence of the tracking error to zero for SPR systems. Note that, initially, ILC techniques where based upon the use of the tracking errors from the previous iterations. The benefits of introducing the current-cycle tracking error in ILC algorithms has been shown later on in several papers such as Xu, Wang, and Heng (1995), Kuc and Lee (1996) and Chen, Wen, and Sun (1997). In the recent paper (Norrlöf & Gunnarsson, 2005), dealing with discrete-time ILC, it has been shown that causal ILC algorithms, using only the current-iteration tracking error, do not guarantee a monotonic convergence of the tracking error to zero. In French, Munde, Rogers, and Owens (1999), a high gain feedback based adaptive ILC scheme has been proposed for single-input single-output–linear time-invariant (SISO–LTI) systems with relative degree one assuming the knowledge of the sign of the first Markov parameter. In Owens, Li, and Banks (2004), a multiple-input multiple-output (MIMO) repetitive control problem is considered for periodic references and disturbances, and some stability results related to the positive realness property have been provided. For the design of output-based ILC (i.e., using just the output measurements) for systems with arbitrary relative degree, see for instance Chien and Yao (2004), French and Rogers (2000), Sun and Wang (2001) and Tayebi (2006a).

☆ This research was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC). Preliminary results of this paper have been presented in Tayebi (2006b). This paper was recommended for publication in revised form by Associate Editor Raúl Ordóñez under the director of Editor Miroslav Krstic.
* Tel.: +1 807 343 8597; fax: +1 807 343 8928.
E-mail address: atayebi@lakeheadu.ca.

0005-1098/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.01.026

In the present paper, we consider two particular P-type[1] ILC schemes for MIMO–LTI systems, and we discuss the issues of convergence, implementation, robustness with respect to disturbances and reinitialization errors, as well as the positive realness properties, related to both schemes. The two quite general ILC schemes—from which the most common P-type ILC schemes in the literature can be retrieved (see, for instance, Phan, Longman, & Moore, 2000; Xu, Lee, & Zhang, 2004)—considered here are analyzed in the frequency and time domains. Both ILC schemes are based on feedback and feedforward actions. Two different updating laws for the feedforward action are discussed: the first one is based on the use of the tracking error from the current cycle, and involves m iterative variables,[2] where m is the number of system inputs. The second updating law is based on the tracking error from the previous cycle and therefore, requires more iterative variables than the first one, namely, $p+m$, where p is the dimension of the output vector. A convergence condition related to the notion of extended strictly positive real (ESPR) property, in the frequency domain, is derived and is shown, under a particular choice of the learning filters, to be the same for both ILC schemes. Furthermore, we provide some sufficient conditions for both schemes, in the time-domain, for the existence of constant ILC filters leading to zero tracking-error for a class of MIMO–LTI systems (not necessarily SPR).

2. ILC schemes description

Let us consider the following MIMO system:

$$Y_k(s) = G(s)U_k(s), \tag{1}$$

where $k \in \mathbb{Z}_+$ denotes the operation or iteration number and $G(s)$ is a $p \times m$ transfer function matrix belonging to the field of real rational functions of s.

Our objective is to design an iterative rule to generate the control input $u_k(t)$, such that the system output $y_k(t)$ converges to the desired output $y_d(t)$ when k goes to infinity for all t within the finite time-interval $[0, T]$.

Throughout this paper, we assume that the initial resetting condition is satisfied, i.e., $y_k(0)=y_d(0)$, and without any loss of generality, we consider that $y_k(0)=y_d(0)=0$. In what follows, the Laplace variable s will be omitted when this does not lead to any confusion.

There are several definitions for positive real (PR) systems available in the literature. In this paper we will adopt the following definition (Sun, Khargonekar, & Shim, 1994):

[1] Note that P-type ILC is the most looked after ILC scheme for its simplicity since it does not require the use of the output-time derivatives. However, the design of P-type ILC schemes is a challenging problem for systems with relative degree greater than or equal to one (Arimoto, 1996; Chien & Liu, 1996; Kuc, Lee, & Nam, 1992; Saab, 1994).

[2] By iterative variables, we mean the variables to be saved in the memory at each sampling time.

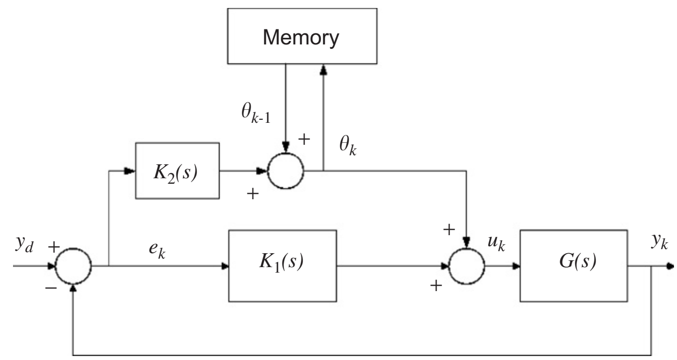

Fig. 1. ILC scheme 1: using only the tracking error from the current iteration.

Definition 1. A square transfer function matrix $H(s)$ is said to be

(1) PR if it is analytic in $\mathrm{Re}(s) > 0$ and satisfies $H(s) + H^*(s) \geqslant 0$ for $\mathrm{Re}(s) > 0$.
(2) SPR if it is analytic in $\mathrm{Re}(s) \geqslant 0$ and satisfies $H(\mathrm{j}\omega) + H^*(\mathrm{j}w) > 0$ for $\omega \in [0, \infty)$.
(3) ESPR if it is SPR and satisfies $H(\mathrm{j}\infty) + H^*(\mathrm{j}\infty) > 0$.

where $H^*(s)$ denotes the conjugate transpose of the matrix $H(s)$. Throughout this paper, for Hermitian positive definite matrices P_1 and P_2, the notation $P_1 > P_2$ (respectively, $P_1 \geqslant P_2$) means that $P_1 - P_2$ is positive definite and $P_2 - P_1$ is negative definite (respectively, $P_1 - P_2$ is positive semi-definite and $P_2 - P_1$ is negative semi-definite).

In this paper, the following two ILC schemes are considered:

- ILC Scheme 1: The first ILC scheme using the tracking error from the current iteration in the parametric updating law, shown in Fig. 1, is given by

$$U_k(s) = K_1(s)E_k(s) + \Theta_k(s), \tag{2}$$

$$\Theta_k(s) = \Theta_{k-1}(s) + K_2(s)E_k(s), \tag{3}$$

with $\Theta_{-1}(s) = 0$.
- ILC Scheme 2: The second ILC scheme using the tracking error from the previous iteration in the parametric updating law, shown in Fig. 2, is given by

$$U_k(s) = K_1(s)E_k(s) + \Theta_k(s), \tag{4}$$

$$\Theta_k(s) = \Theta_{k-1}(s) + K_2(s)E_{k-1}(s), \tag{5}$$

with $\Theta_{-1}(s) = 0$ and $E_{-1}(s) = 0$.

In both ILC schemes, $K_1(s)$ and $K_2(s)$ are $m \times p$ matrices, $\Theta_k(s)$ is an $m \times 1$ vector, $E_k(s) = Y_d(s) - Y_k(s) = \mathscr{L}[e_k(t)]$, $Y_k(s) = \mathscr{L}[y_k(t)]$, $Y_d(s) = \mathscr{L}[y_d(t)]$, $U_k(s) = \mathscr{L}[u_k(t)]$ and $\Theta_k(s) = \mathscr{L}[\theta_k(t)]$. The variable $e_k = y_d - y_k$ denotes the tracking error at the iteration k.

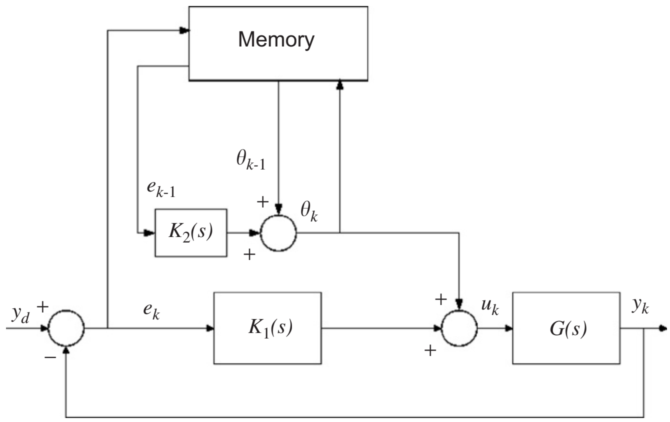

Fig. 2. ILC scheme 2: using the tracking errors from the current and the previous iteration.

3. ILC design in the frequency domain

First of all, let us consider the ILC scheme 1 and try to derive some conditions under which the stability and convergence of the iterative process is guaranteed. Towards this end, let us subtract the tracking errors at the iteration k and the iteration $k-1$, to get

$$\begin{aligned} E_k - E_{k-1} &= -(Y_k - Y_{k-1}) = -G(U_k - U_{k-1}) \\ &= -GK_1(E_k - E_{k-1}) - GK_2E_k, \end{aligned} \tag{6}$$

which leads to

$$E_k = ((I+GK)^{-1}(I+GK_1))E_{k-1}, \tag{7}$$

where $K(s) = K_1(s) + K_2(s)$. From (7), using the fact that $\|E_k(s)\|_2 = \|e_k(t)\|_2$, it is clear that

$$\|e_k(t)\|_2 \leqslant \|(I+GK)^{-1}(I+GK_1)\|_\infty \|e_{k-1}(t)\|_2, \tag{8}$$

and hence

$$\begin{aligned} \|e_k(t)\|_2 &\leqslant \|(I+GK)^{-1}(I+GK_1)\|_\infty^k \|e_0(t)\|_2 \\ &= \|I-(I+GK)^{-1}GK_2\|_\infty^k \|e_0(t)\|_2. \end{aligned} \tag{9}$$

Note that at the first iteration, i.e., for $k=0$, the variable $\theta_{-1}(t)$ is set to zero. Therefore, the ILC scheme in Fig. 1, at the first iteration, is a unity feedback system, with a feedback controller $K(s)$. Therefore, the stability of the closed-loop system $(I+G(s)K(s))^{-1}G(s)K(s)$ is required if one seeks the boundedness of $\|e_0(t)\|_2$ over an infinite time-interval. However, the boundedness of $\|e_0(t)\|_{2,[0,T]}$ over a finite time-interval $[0,T]$ does not require the stability of the closed-loop system. Consequently, over a finite time-interval $[0,T]$, one can conclude that if

$$\|(I+GK)^{-1}(I+GK_1)\|_\infty < 1, \tag{10}$$

then $\|e_k(t)\|_{2,[0,T]}$ is bounded for all $k \in \mathbb{Z}_+$ and $\lim_{k\to\infty}\|e_k(t)\|_{2,[0,T]} = 0$.

Now, if we chose $K_2 = -2K_1$, inequality (10) becomes

$$\|(I-GK_1)^{-1}(I+GK_1)\|_\infty < 1, \tag{11}$$

which is equivalent to

$$(I-GK_1)^{-1}(I+GK_1)(I+GK_1)^*(I-GK_1)^{-*} < I, \tag{12}$$

where $X^{-*} \equiv (X^*)^{-1}$ denotes the inverse of the conjugate transpose of the matrix X. Multiplying (12) by $(I-GK_1)$ from the left and $(I-GK_1)^*$ from the right, we obtain

$$(I+GK_1)(I+GK_1)^* < (I-GK_1)(I-GK_1)^*, \tag{13}$$

and finally, substituting $K_1 = -\frac{1}{2}K_2$, the previous inequality leads to

$$G(\mathrm{j}\omega)K_2(\mathrm{j}\omega) + (G(\mathrm{j}\omega)K_2(\mathrm{j}\omega))^* > 0, \quad \forall\omega \geqslant 0. \tag{14}$$

According to the previous development, one can state the following result:

Proposition 1. *Consider system* (1) *under the ILC scheme* (2)–(3) *shown in* Fig. 1 *with* $K_2(s) = -2K_1(s)$. *If* $G(s)K_2(s)$ *is ESPR, then* $\|e_k(t)\|_{2,[0,T]}$ *is bounded for all* $k \in \mathbb{Z}_+$ *and* $\lim_{k\to\infty}\|e_k(t)\|_{2,[0,T]} = 0$ (*monotonic convergence in the sense of the* $\mathscr{L}_2$-*norm*).

In the same way as in the proof of Proposition 1, one can prove the following result related to the ILC scheme 2:

Proposition 2. *Consider system* (1) *under the ILC scheme* (4)–(5) *shown in Fig.* 2 *with* $K_2(s) = 2K_1(s)$. *If* $G(s)K_2(s)$ *is ESPR, then* $\|e_k(t)\|_{2,[0,T]}$ *is bounded for all* $k \in \mathbb{Z}_+$ *and* $\lim_{k\to\infty}\|e_k(t)\|_{2,[0,T]} = 0$ (*monotonic convergence in the sense of the* $\mathscr{L}_2$-*norm*).

Now, the following remarks are in order:

Remark 1. Following the same steps of the proof of Proposition 1, one can show that if $K_1(s) = 0$ in the ILC scheme 1 and $K_1(s) = K_2(s)$ in the ILC scheme 2, the convergence condition for both schemes becomes

$$\begin{aligned} &G(\mathrm{j}\omega)K_2(\mathrm{j}\omega) + (G(\mathrm{j}\omega)K_2(\mathrm{j}\omega))^* \\ &\quad + G(\mathrm{j}\omega)K_2(\mathrm{j}\omega)K_2^*(\mathrm{j}\omega)G^*(\mathrm{j}\omega) > 0, \quad \forall\omega \geqslant 0, \end{aligned}$$

which is obviously satisfied if (14) is satisfied, and therefore the result in Proposition 1 holds also with $K_1(s) = 0$ and the result in Proposition 2 holds also with $K_1(s) = K_2(s)$.

Remark 2. If $K_1(s) = -K_2(s)$ in the ILC scheme 1 and $K_1(s) = 0$ in the ILC scheme 2, the convergence condition for both schemes becomes

$$\begin{aligned} &G(\mathrm{j}\omega)K_2(\mathrm{j}\omega) + (G(\mathrm{j}\omega)K_2(\mathrm{j}\omega))^* \\ &\quad - G(\mathrm{j}\omega)K_2(\mathrm{j}\omega)K_2^*(\mathrm{j}\omega)G^*(\mathrm{j}\omega) > 0, \quad \forall\omega \geqslant 0. \end{aligned}$$

In this case, the convergence condition is not guaranteed to be satisfied if (14) is satisfied. In fact, it could be satisfied if (14) is satisfied and $\|K_2\|$ is sufficiently small. Note that the

ILC scheme 2 with $K_1 = 0$ has been addressed in Arimoto and Naniwa (2000, 2001), in the time domain, showing the existence of a sufficiently small gain K_2 leading to the convergence of the tracking error to zero for output-dissipative (SPR plus an extra condition) systems.

Remark 3. After some simple algebraic manipulations, one can easily see that the ILC scheme (2)–(3) can be rewritten in the following form:

$$U_k = U_{k-1} + (K_1 + K_2)E_k - K_1 E_{k-1}, \tag{15}$$

which is in the same form as the ILC scheme—using the previous and the current iteration tracking errors—considered by many authors in the literature, e.g., Amman, Owens, Rogers, and Wahl (1996), Phan et al. (2000), and Xu et al. (2004). From (15), it is clear that by taking $K_1 = -K_2$ we obtain the classical P-type ILC using only the previous tracking error (Arimoto et al., 1984; Arimoto & Naniwa, 2000, 2001). On the other hand, taking $K_1 = 0$ leads to the P-type ILC using only the current iteration tracking error (Kuc & Lee, 1996; Tayebi & Zaremba, 2003).

Also, the ILC scheme (4)–(5) can bee rewritten in the following form:

$$U_k = U_{k-1} + K_1 E_k + (K_2 - K_1)E_{k-1}. \tag{16}$$

From (16), it is clear that by taking $K_1 = 0$ we end up with the classical P-type ILC using the previous iteration tracking error, and by taking $K_1 = K_2$ we obtain the P-type ILC scheme using the current iteration tracking error.

Remark 4. From Remark 3, it is clear that it is advantageous to implement (2)–(3) instead of (15). In fact, to implement (15), we need to save u_k and e_k in the memory at each sampling time, i.e., we need to save $m + p$ variables at each sampling time, while using (2)–(3) we need to save just m variables at each sampling time.

Remark 5. One can easily see that if we take $K_2 = -2K_1$ in (15) and $K_2 = 2K_1$ in (16), we end up with the same ILC law

$$U_k = U_{k-1} + \tfrac{1}{2}K_2(E_k + E_{k-1}),$$

which explains that the convergence condition $GK_2 + (GK_2)^* > 0$ is the same for both schemes. This convergence condition is related to the fact that the control input is not adjusted just according to the tracking error signal from the current iteration or the tracking error signal from the previous iteration, but it is adjusted according to the instantaneous average of the two signals.

Remark 6. It is worth noting that the ILC scheme 1 uses m iterative variables, while the ILC scheme 2 uses $p + m$ iterative variables. Although, the number of iterative variables is increased in the ILC scheme 2 with respect to the ILC scheme 1, the fact of using the tracking error from the previous iteration in the iterative law (5), offers more flexibility for the choice of the filter $K_2(s)$. In fact, since the tracking error at the previous iteration is available over the whole operation time-interval, the filter $K_2(s)$ can be non-causal. In this case, it is not possible to take $K_2 = 2K_1$ since K_1 must be causal. Therefore, the following condition[3] must be used instead of the one used in Proposition 2

$$\|(I + GK_1)^{-1}(I + G(K_1 - K_2))\|_\infty < 1.$$

Remark 7. As a particular case, if $G(s)$ is square and ESPR and if $K_2(s) = \alpha I$, with α being a positive scalar, then the convergence condition in Propositions 1 and 2 is satisfied. Therefore, before applying the ILC schemes, one can first try to design a feedback controller making the closed-loop system ESPR. In fact, in Sun et al. (1994) necessary and sufficient conditions, given in terms of solutions to algebraic Riccati equations, for the existence of a feedback controller making the closed-loop system ESPR are provided.

4. ILC Design in the time-domain

Let us consider the class of MIMO–LTI systems described by the following minimal state-space representation[4]:

$$\dot{x}_k(t) = Ax_k(t) + Bu_k(t),$$
$$y_k(t) = Cx_k(t), \tag{17}$$

where $x_k \in \mathbb{R}^n$ denotes the state vector, $u_k \in \mathbb{R}^p$ denotes the input vector and $y_k \in \mathbb{R}^p$ denotes the output vector, at the iteration k.

Let us consider the ILC scheme in Fig. 1, where $K_1(s)$ and $K_2(s)$ are real matrices, that is

$$u_k(t) = K_1 e_k(t) + \theta_k(t), \tag{18}$$

$$\theta_k(t) = \theta_{k-1}(t) + K_2 e_k(t), \tag{19}$$

with $\theta_{-1}(t) = 0$, $K_2 \in \mathbb{R}^{p\times p}$ and $K_1 \in \mathbb{R}^{p\times p}$.

Let the reference trajectory be given by $y_d(t) \in \mathscr{C}^1_{[0,T]}$, and assume that $rank(CB) = p$. Under these assumptions there exists a unique control input $u_d(t)$ such that $y_d(t)$ is the output of the following system:

$$\dot{x}_d(t) = Ax_d(t) + Bu_d(t), \quad x_d(0) = x_k(0),$$
$$y_d(t) = Cx_d(t), \tag{20}$$

where x_d is the desired state vector.

Using (17) and (20), in view of (18), one can obtain the following error model:

$$\dot{\tilde{x}}_k(t) = \bar{A}\tilde{x}_k(t) + B(u_d - \theta_k), \quad \tilde{x}_k(0) = 0,$$
$$e_k(t) = C\tilde{x}_k(t), \tag{21}$$

where $\tilde{x}_k = x_d - x_k$, $e_k = y_d - y_k$, $\bar{A} = A - BK_1C$.

[3] This condition can be obtained in the same way as we did to obtain (10) by using the ILC scheme (4)–(5) instead of (2)–(3).

[4] For the sake of presentation simplicity, we considered here the case where the direct transmission matrix is equal to zero, i.e., $D \equiv 0$, which is the most common case in practical applications.

Now, we are in position to state the following result:

Proposition 3. *Consider system* (17) *under the ILC scheme* (18)–(19), *over a finite time-interval* $[0, T]$. *Let* $K_2 = \Gamma_1\Gamma_2$, *where* $\Gamma_1 \in \mathbb{R}^{p\times p}$ *is a symmetric positive definite matrix. Let* $K_1 \in \mathbb{R}^{p\times p}$ *and* $\Gamma_2 \in \mathbb{R}^{p\times p}$ *be such that rank*$(\Gamma_2) = p$ *and the transfer function defined by the triple* $\{\bar{A}, B, \Gamma_2 C\}$ *is PR. Then* $\tilde{x}_k(t)$, *is bounded for all* $k \in \mathbb{Z}_+$ *and all* $t \in [0, T]$, *and* $\lim_{k\to\infty} \tilde{x}_k(t) = \lim_{k\to\infty} e_k(t) = 0$, $\forall t \in [0, T]$.

Proof. First, note that the condition *rank*$(\Gamma_2) = p$ is required so that *rank*$(K_2) = p$ which ensures that $e_k(t)$ equals zero once $\theta_k(t) = \theta_{k-1}(t)$. Now, the error model (21) can be written as follows:

$$\dot{\tilde{x}}_k(t) = \bar{A}\tilde{x}_k(t) + B\tilde{\theta}_k(t), \quad \tilde{x}_k(0) = 0,$$

$$e_k(t) = C\tilde{x}_k(t), \tag{22}$$

where $\tilde{\theta}_k(t) = u_d(t) - \theta_k(t)$.

Since the transfer function $\Gamma_2 C(sI - \bar{A})^{-1}B$ is PR, according to Kalman–Yakubovich–Popov lemma (Ioannou & Sun, 1996; Khalil, 2002), there exist $P = P^{\mathrm{T}} > 0$, and $Q = Q^{\mathrm{T}} \geqslant 0$ such that

$$\bar{A}^{\mathrm{T}}P + P\bar{A} = -Q,$$

$$PB = (\Gamma_2 C)^{\mathrm{T}}. \tag{23}$$

Now, let us consider the following Lyapunov-like functional candidate:

$$V_k(\tilde{x}_k, \tilde{\theta}_k) = \frac{1}{2}\tilde{x}_k^{\mathrm{T}}P\tilde{x}_k + \frac{1}{2}\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma_1^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau. \tag{24}$$

The difference of the Lyapunov-like functional (24), in view of (18) and (23), is given by

$$\begin{aligned}
\Delta V_k &= V_k - V_{k-1}\\
&= \frac{1}{2}\tilde{x}_k^{\mathrm{T}}P\tilde{x}_k - \frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}}P\tilde{x}_{k-1}\\
&\quad + \frac{1}{2}\int_0^t (\tilde{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_k - \tilde{\theta}_{k-1}^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_{k-1})\,\mathrm{d}\tau,\\
&= -\frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}}P\tilde{x}_{k-1} + \frac{1}{2}\tilde{x}_k(0)^{\mathrm{T}}P\tilde{x}_k(0)\\
&\quad + \frac{1}{2}\int_0^t \frac{\mathrm{d}}{\mathrm{d}\tau}(\tilde{x}_k^{\mathrm{T}}P\tilde{x}_k)\,\mathrm{d}\tau\\
&\quad - \frac{1}{2}\int_0^t (\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\bar{\theta}_k + 2\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_k)\,\mathrm{d}\tau\\
&= -\frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}}P\tilde{x}_{k-1} - \frac{1}{2}\int_0^t (\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\bar{\theta}_k + 2\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_k)\,\mathrm{d}\tau\\
&\quad + \frac{1}{2}\int_0^t (\tilde{x}_k^{\mathrm{T}}(\bar{A}^{\mathrm{T}}P + P\bar{A})\tilde{x}_k + 2\tilde{x}_k^{\mathrm{T}}PB\tilde{\theta}_k)\,\mathrm{d}\tau\\
&= -\frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}}P\tilde{x}_{k-1} - \frac{1}{2}\int_0^t (\tilde{x}_k^{\mathrm{T}}Q\tilde{x}_k - 2e_k^{\mathrm{T}}\Gamma_2^{\mathrm{T}}\tilde{\theta}_k)\,\mathrm{d}\tau\\
&\quad - \frac{1}{2}\int_0^t (\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\bar{\theta}_k + 2\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_k)\,\mathrm{d}\tau,
\end{aligned} \tag{25}$$

where $\bar{\theta}_k = \theta_k - \theta_{k-1}$. Now, using (19), Eq. (25) leads to

$$\begin{aligned}
\Delta V_k &= -\frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}}P\tilde{x}_{k-1} - \frac{1}{2}\int_0^t \bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\bar{\theta}_k\,\mathrm{d}\tau\\
&\quad - \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}}Q\tilde{x}_k\,\mathrm{d}\tau\\
&= -\frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}}P\tilde{x}_{k-1}\\
&\quad - \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}}(Q + C^{\mathrm{T}}\Gamma_2^{\mathrm{T}}\Gamma_1\Gamma_2 C)\tilde{x}_k\,\mathrm{d}\tau \leqslant 0.
\end{aligned} \tag{26}$$

Since $y_d(t) \in \mathscr{C}^1_{[0,T]}$, it is clear that $V_0(t)$ is bounded over the finite-time interval $[0, T]$. Therefore, from the fact that $V_k(t)$ is non-increasing, one can conclude that $\tilde{x}_k(t)$, $e_k(t)$ and $\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma_1^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau$ are bounded over $[0, T]$. Since $y_d(t) \in \mathscr{C}^1_{[0,T]}$, it is clear that $u_d(t)$ is bounded over any finite time-interval and hence, it is clear that $\int_0^t \theta_k^{\mathrm{T}}(\tau)\Gamma_1^{-1}\theta_k(\tau)\,\mathrm{d}\tau$ as well as the $\mathscr{L}_2$-norm of $u_k(t)$ are bounded for all $k \in \mathbb{Z}_+$ and all $t \in [0, T]$.

Finally, to show the convergence of $e_k(t)$ to zero when k tends to infinity, let us rewrite V_k as follows:

$$V_k = V_0 + \sum_{j=1}^{j=k} \Delta V_j \leqslant V_0 - \frac{1}{2}\sum_{j=1}^{j=k} \tilde{x}_{j-1}^{\mathrm{T}}P\tilde{x}_{j-1},$$

which leads to

$$\sum_{j=1}^{j=k} \tilde{x}_{j-1}^{\mathrm{T}}(t)P\tilde{x}_{j-1}(t) \leqslant 2(V_0(t) - V_k(t)) \leqslant 2V_0(t). \tag{27}$$

Since $V_0(t)$ and $\tilde{x}_k(t)$ are bounded for all $k \in \mathbb{Z}_+$ and $t \in [0, T]$, one can conclude that $\lim_{k\to\infty}\tilde{x}_k(t) = 0$ and consequently $\lim_{k\to\infty} e_k(t) = 0$, $\forall t \in [0, T]$. □

Now, let us consider the ILC scheme in Fig. 2, where $K_1(s)$ and $K_2(s)$ are real matrices, that is

$$u_k(t) = K_1 e_k(t) + \theta_k(t), \tag{28}$$

$$\theta_k(t) = \theta_{k-1}(t) + K_2 e_{k-1}(t), \tag{29}$$

with $\theta_{-1}(t) = 0$, $e_{-1}(t) = 0$, $K_2 \in \mathbb{R}^{p\times p}$ and $K_1 \in \mathbb{R}^{p\times p}$. Our result concerning the ILC scheme 2 can be stated as follows:

Proposition 4. *Consider system* (17) *under the ILC scheme* (28)–(29), *over a finite time-interval* $[0, T]$. *Let* $K_2 = \Gamma_1\Gamma_2$. *If there exist* K_1, $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} \geqslant 0$, $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ *and* $\Gamma_2 \in \mathbb{R}^{p\times p}$ *such that rank*$(\Gamma_2) = p$ *and*

$$\bar{A}^{\mathrm{T}}P + P\bar{A} + PB\Gamma_1 B^{\mathrm{T}}P = -Q,$$

$$PB = (\Gamma_2 C)^{\mathrm{T}}. \tag{30}$$

Then $\tilde{x}_k(t)$ *is bounded for all* $k \in \mathbb{Z}_+$ *and all* $t \in [0, T]$, *and* $\lim_{k\to\infty} \tilde{x}_k(t) = \lim_{k\to\infty} e_k(t) = 0$, $\forall t \in [0, T]$.

Proof. Let us consider the following Lyapunov-like functional candidate:

$$V_k(\tilde{x}_{k-1}, \tilde{\theta}_k) = \frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}} P \tilde{x}_{k-1} + \frac{1}{2}\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma_1^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau, \quad (31)$$

whose difference, in view of (28) and (30), is given by

$$\begin{aligned}\Delta V_k &= \frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}} P \tilde{x}_{k-1} - \frac{1}{2}\tilde{x}_{k-2}^{\mathrm{T}} P \tilde{x}_{k-2} \\ &\quad + \frac{1}{2}\int_0^t (\tilde{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_k - \tilde{\theta}_{k-1}^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_{k-1})\,\mathrm{d}\tau, \\ &= -\frac{1}{2}\tilde{x}_{k-2}^{\mathrm{T}} P \tilde{x}_{k-2} \\ &\quad + \frac{1}{2}\int_0^t (\tilde{x}_{k-1}^{\mathrm{T}}(\bar{A}^{\mathrm{T}}P + P\bar{A})\tilde{x}_{k-1} + 2e_{k-1}^{\mathrm{T}}\Gamma_2^{\mathrm{T}}\tilde{\theta}_{k-1})\,\mathrm{d}\tau \\ &\quad - \frac{1}{2}\int_0^t (\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\bar{\theta}_k + 2\bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1} \\ &\quad \times (\tilde{\theta}_k - \tilde{\theta}_{k-1} + \tilde{\theta}_{k-1})\,\mathrm{d}\tau. \end{aligned} \quad (32)$$

Now, using (29), Eq. (32) leads to

$$\Delta V_k = -\frac{1}{2}\tilde{x}_{k-2}^{\mathrm{T}} P \tilde{x}_{k-2} - \frac{1}{2}\int_0^t \tilde{x}_{k-1}^{\mathrm{T}} Q \tilde{x}_{k-1}\,\mathrm{d}\tau \leqslant 0. \quad (33)$$

The remaining of the proof is omitted since it follows the lines of the proof of Proposition 3. □

Remark 8. As mentioned in Remark 3, the control laws (18)–(19) and (28)–(29) can be rewritten as follows:

$$u_k = u_{k-1} + F_1 e_k + F_2 e_{k-1}, \quad (34)$$

with $F_1 = K_1 + K_2$, $F_2 = -K_1$ for the control law (18)–(19) and $F_1 = K_1$, $F_2 = K_2 - K_1$ for the control law (28)–(29). Many authors in the literature, considered the ILC scheme (34), but, to the best of our knowledge, there is no reference in the literature stating that it is straightforward to obtain F_1 and F_2 for systems with relative degree one, and discussing the possibility of implementing the control scheme (34) as (18)–(19). In fact, the learning gains F_1 and F_2, for the ILC scheme (34), could be designed using Proposition 3 and $F_1 = K_1 + K_2$, $F_2 = -K_1$, for PR systems (or systems that can be made PR via an adequate choice of Γ_2 and K_1). As for the implementation, it is preferable to implement (18)–(19) instead of (34). Indeed, by implementing (34), we have to save u_k and e_k at each sampling time, while by implementing (18)–(19), we have just to save θ_k at each sampling time.

Remark 9. It is worth noting that if the system is PR, then the result of Proposition 3 can be simplified by taking $\Gamma_2 = I$ and $K_1 = 0$. Note that in the case where $\Gamma_2 = I$ and $K_1 = 0$, the authors in Kuc and Lee (1996) show the convergence of the tracking error to zero for SPR systems, while in our Proposition 3, the PR condition is enough.

Remark 10. It is worth pointing out that if the transfer function defined by the triple$\{A, B, \Gamma_2 C\}$ is PR, then K_1 can be taken equal to zero. Otherwise, K_1 and Γ_2 have to be designed to make the transfer function defined by the triple $\{A - BK_1C, B, \Gamma_2 C\}$ PR. In fact, for a given system described by the triple $\{A, B, C\}$, a necessary and sufficient condition, formulated as an LMI problem, for the existence of a gain K such that the transfer function defined by the triple $\{A - BKC, B, C\}$ is SPR is given in Huang, Ioannou, Maroulas, and Safonov (1999). Moreover, if the condition is satisfied and a certain matrix is obtained using LMIs, an explicit solution for K is provided. To use this result in our case, one should take $K_1 = K\Gamma_2$ and design K and Γ_2 according to Huang et al. (1999).

Remark 11. Note that, in contrast to the result in Proposition 3, the PR condition is not sufficient to guarantee the convergence for the ILC of Proposition 4. In fact, the convergence condition of Proposition 4 can be satisfied by choosing $\Gamma_1 > 0$ with sufficiently small eigenvalues, and K_1 and Γ_2 such that $\Gamma_2 C(sI - \bar{A})^{-1}B$ is SPR. Note that the result in Proposition 4, with $K_1 = 0$, has been addressed in Arimoto and Naniwa (2000, 2001) with a different proof.

Remark 12. From a theoretical point of view it is clear that there is no advantage of applying the ILC scheme 2 over the ILC scheme 1. Nevertheless, in practical applications perhaps, the ILC scheme 2, involving the tracking error from the previous iteration, might give the designer the ability to manipulate the tracking error profile saved in the memory before actually applying it to the system (i.e., for instance, filtering the data off-line to reduce the measurement noise).

Remark 13. Under the same feedback gain K_1, the ILC scheme (18)–(19) exhibits more robustness with respect to reinitialization errors[5] than the ILC scheme (28)–(29). This is due to the fact that Γ_1 (and hence K_2) can be chosen arbitrarily large in the control scheme (18)–(19), while Γ_1 is generally restricted to be sufficiently small for the control law (28)–(29) according to Remark 11. In fact, if we assume that $\|\tilde{x}_k(0)\| \leqslant \sigma$ for all k, and following the steps of the proof of Proposition 3, we obtain

$$\begin{aligned}\Delta V_k &= \frac{1}{2}\tilde{x}_k(0)^{\mathrm{T}} P \tilde{x}_k(0) - \frac{1}{2}\tilde{x}_{k-1}^{\mathrm{T}} P \tilde{x}_{k-1} \\ &\quad - \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}}(Q + C^{\mathrm{T}}\Gamma_2^{\mathrm{T}}\Gamma_1\Gamma_2 C)\tilde{x}_k\,\mathrm{d}\tau \\ &\leqslant \frac{1}{2}\lambda_{\max}(P)\sigma^2 - \frac{1}{2}\lambda_{\min}(P)\|\tilde{x}_{k-1}(t)\|^2 \\ &\quad - \frac{1}{2}\lambda_{\min}(Q + C^{\mathrm{T}}\Gamma_2^{\mathrm{T}}\Gamma_1\Gamma_2 C)\int_0^t \|\tilde{x}_k(\tau)\|^2\,\mathrm{d}\tau, \end{aligned} \quad (35)$$

[5] A detailed discussion on initial conditions in ILC can be found in Xu and Yan (2005).

where $\lambda_{\min}(\star)$ (resp. $\lambda_{\max}(\star)$) denotes the minimum (resp. maximum) eigenvalue of $(\star)$. Therefore, it is possible to overcome the effect of the positive term $\frac{1}{2}\lambda_{\max}(P)\sigma^2$, over $(0, T]$, by choosing Γ_1 sufficiently large.

On the other hand, under the ILC scheme (28)–(29), if we assume that $\|\tilde{x}_{k-1}(0)\| \leqslant \sigma$ for all k, and following the steps of the proof of Proposition 4, we obtain

$$\begin{aligned}\Delta V_k = {} & \frac{1}{2}\tilde{x}_{k-1}(0)^{\mathrm{T}} P \tilde{x}_{k-1}(0) \\ & - \frac{1}{2}\tilde{x}_{k-2}^{\mathrm{T}} P \tilde{x}_{k-2} - \frac{1}{2}\int_0^t \tilde{x}_{k-1}^{\mathrm{T}} Q \tilde{x}_{k-1}\,\mathrm{d}\tau \\ \leqslant {} & \frac{1}{2}\lambda_{\max}(P)\sigma^2 - \frac{1}{2}\lambda_{\min}(P)\|\tilde{x}_{k-2}(t)\|^2 \\ & - \frac{1}{2}\lambda_{\min}(Q)\int_0^t \|\tilde{x}_{k-1}(\tau)\|^2\,\mathrm{d}\tau. \end{aligned} \tag{36}$$

Therefore, to overcome the effect of the positive term $\frac{1}{2}\lambda_{\max}(P)\sigma^2$ one should rely on the feedback gain K_1 leading (if possible) to a sufficiently small value for $\lambda_{\max}(P)/\lambda_{\min}(Q)$.

The same observation can be made regarding the robustness of the two ILC schemes with respect to bounded disturbances since, similarly to the reinitialization error case, the disturbances will generate an additional term in Eqs. (35) and (36).

Remark 14. Concerning the convergence rates achieved by the two ILC schemes under consideration, one can argue that, for the same feedback gain K_1, the ILC scheme (18)–(19) will potentially lead to higher convergence rates (by increasing Γ_1) than the ILC scheme (28)–(29) for which the convergence generally restricts Γ_1 to be sufficiently small. In fact, from (26), one can see that $\Delta V_k \overset{\triangle}{=} V_k - V_{k-1}$ can be made arbitrarily large (negative) by increasing Γ_1.

Now, one can show that the positive realness condition of Proposition 3 can be traded against a partial knowledge[6] of the first Markov parameter (or high-frequency gain matrix) *CB* as stated below

Proposition 5. *Consider system* (17), *under the ILC scheme* (18)–(19), *over a finite time-interval* $[0, T]$. *Let* $K_2 = \Gamma_1(CB)^{\mathrm{T}}P$, *where* $\Gamma_1 \in \mathbb{R}^{p\times p}$ *is a symmetric positive definite matrix. Let* $K_1 \in \mathbb{R}^{p\times p}$ *and* $P \in \mathbb{R}^{p\times p}$ *such that* $P = P^{\mathrm{T}} > 0$ *and* $\bar{A}^{\mathrm{T}}P_1 + P_1\bar{A} - P_1B\Gamma_1B^{\mathrm{T}}P_1 = -Q_1$, *with* $P_1 = C^{\mathrm{T}}PC$, $Q_1 = Q_1^{\mathrm{T}} \geqslant 0$. *Then* $\tilde{x}_k(t)$ *is bounded for all* $k \in \mathbb{Z}_+$ *and all* $t \in [0, T]$, *and* $\lim_{k\to\infty} e_k(t) = 0$, $\forall t \in [0, T]$.

Proof. Here, we consider the following Lyapunov-like functional candidate:

$$V_k(e_k, \tilde{\theta}_k) = \frac{1}{2}e_k^{\mathrm{T}} P e_k + \frac{1}{2}\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma_1^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau, \tag{37}$$

which leads to

$$\begin{aligned}\Delta V_k = {} & -\frac{1}{2}e_{k-1}^{\mathrm{T}} P e_{k-1} \\ & - \frac{1}{2}\int_0^t \bar{\theta}_k^{\mathrm{T}}\Gamma_1^{-1}\bar{\theta}_k\,\mathrm{d}\tau + \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}}(\bar{A}^{\mathrm{T}}P_1 + P_1\bar{A})\tilde{x}_k\,\mathrm{d}\tau \\ = {} & -\frac{1}{2}e_{k-1}^{\mathrm{T}} P e_{k-1} - \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}}(P_1B\Gamma_1B^{\mathrm{T}}P_1)\tilde{x}_k\,\mathrm{d}\tau \\ & + \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}}(\bar{A}^{\mathrm{T}}P_1 + P_1\bar{A})\tilde{x}_k\,\mathrm{d}\tau \\ = {} & -\frac{1}{2}e_{k-1}^{\mathrm{T}} P e_{k-1} - \frac{1}{2}\int_0^t \tilde{x}_k^{\mathrm{T}} Q_1\tilde{x}_k\,\mathrm{d}\tau \leqslant 0. \end{aligned} \tag{38}$$

The remaining of the proof follows the same lines of the proof of Proposition 3 and hence omitted. Note that $rank(CB) = p$ is required so that $e_k(t)$ equals zero once $\theta_k(t) = \theta_{k-1}(t)$. Note also that, this proof shows just the boundedness of $e_k(t)$. However, the boundedness of $\tilde{x}_k(t)$ is guaranteed over any finite time-interval, since there is no finite escape-time for system (22). □

Now, using the ILC scheme (28)–(29), one has the following result:

Proposition 6. *Consider system* (17), *under the ILC scheme* (28)–(29), *over a finite time-interval* $[0, T]$. *Let* $K_2 = \Gamma_1(CB)^{\mathrm{T}}P$. *Let* $K_1 \in \mathbb{R}^{p\times p}$, $P \in \mathbb{R}^{p\times p}$, $\Gamma_1 \in \mathbb{R}^{p\times p}$ *such that* $P = P^{\mathrm{T}} > 0$, $\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$ *and* $\bar{A}^{\mathrm{T}}P_1 + P_1\bar{A} + P_1B\Gamma_1B^{\mathrm{T}}P_1 = -Q_1$, *with* $P_1 = C^{\mathrm{T}}PC$, $Q_1 = Q_1^{\mathrm{T}} \geqslant 0$. *Then* $\tilde{x}_k(t)$ *is bounded for all* $k \in \mathbb{Z}_+$ *and all* $t \in [0, T]$, *and* $\lim_{k\to\infty} e_k(t) = 0$, $\forall t \in [0, T]$.

Proof. The proof is established by considering the following Lyapunov-like functional candidate:

$$V_k(e_{k-1}, \tilde{\theta}_k) = \frac{1}{2}e_{k-1}^{\mathrm{T}} P e_{k-1} + \frac{1}{2}\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma_1^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau, \tag{39}$$

and following the same lines of the proofs of Propositions 4 and 5. □

Remark 15. If there exists a non-singular matrix Λ such that $CB\Lambda = \Lambda^{\mathrm{T}}(CB)^{\mathrm{T}} > 0$, then it is possible to take $K_2 = \Lambda P$ in Propositions 5 and 6 and hence avoid the use of *CB* in the control laws (Ioannou & Sun, 1996). The proof of this claim follows directly from the proofs of Propositions 5 and 6 when Γ_1^{-1} is substituted by $(CB)^{\mathrm{T}}\Lambda^{-1} = (\Lambda^{-1})^{\mathrm{T}}(CB\Lambda)^{\mathrm{T}}\Lambda^{-1}$ in the Lyapunov-like functional.

Remark 16. Note that for SISO systems, it is possible to take $K_2 = P\,\mathrm{sgn}(CB)$, where P is a positive parameter, in Propositions 5 and 6. This result is obtained by setting $\Gamma_1 = \mathrm{sgn}(CB)(CB)^{-1}$. Therefore, only the sign of the high-frequency gain is needed for systems with relative degree one satisfying the conditions of Propositions 5 and 6.

Remark 17. It is worth noting that, regardless of the stability of $\bar{A}$, one can see from (38), that it is possible to guarantee the

[6] For square MIMO systems we assume the knowledge of a matrix Λ such that $CB\Lambda = \Lambda^{\mathrm{T}}(CB)^{\mathrm{T}} > 0$, while for SISO systems we assume the knowledge of the sign of *CB*.

convergence of the ILC scheme of Proposition 5 by picking $P=P^{\mathrm{T}}>0$ and $\Gamma_1=\Gamma_1^{\mathrm{T}}>0$ such that the minimum eigenvalue of Γ_1 is sufficiently large. As for the ILC scheme of Proposition 6, the convergence condition could be satisfied by picking K_1 such that $\bar{A}$ is Hurwitz, $P=P^{\mathrm{T}}>0$ and $\Gamma_1=\Gamma_1^{\mathrm{T}}>0$ such that the maximum eigenvalue of Γ_1 is sufficiently small.

5. Conclusion

Two ILC schemes for MIMO–LTI systems have been discussed in this paper. Some remarks on the convergence, implementation, robustness with respect to reinitialization errors and disturbances, as well as positive realness issues of both ILC schemes have been provided. In particular, it is shown that, upon an appropriate choice of the filters $K_1(s)$ and $K_2(s)$, the convergence condition in the frequency domain for both schemes is the same, i.e., $GK_2+(GK_2)^*>0$. From this condition, it is clear that the design of convergent ILC schemes for MIMO–LTI systems is straightforward and does not require the knowledge of the system parameters, if the system is ESPR. Furthermore, we derived some sufficient convergence conditions, in the time-domain, for both schemes with constant ILC filters for a class of systems not necessarily SPR.

References

Amman, N., Owens, D. H., Rogers, E., & Wahl, A. (1996). An $\mathscr{H}_\infty$ approach to linear iterative learning control design. *International Journal of Adaptive Control and Signal Processing*, *10*, 767–781.

Arimoto, S. (1996). *Control theory of non-linear mechanical systems*. Oxford, UK: Oxford Science Publications.

Arimoto, S., Kawamura, S., & Miyazaki, F. (1984). Bettering operation of robots by learning. *Journal of Robotic Systems*, *1*, 123–140.

Arimoto, S., & Naniwa, T. (2000, 2001). Equivalence relations between learnability, output-dissipativity and strict positive realness. *International Journal of Control*, *73*(10), 824–831. (See also the Corrections to this paper in *74*(14), 1481–1482.)

Bien, Z., & Xu, J.-X. (1998). *Iterative learning control: Analysis, design, integration and applications*. Dordrecht: Kluwer Academic Publishers.

Chen, Y. & Wen, C. (1999). Iterative learning control: Convergence, robustness and applications. *Lecture notes in control and information sciences*. Berlin: Springer.

Chen, Y., Wen, C., & Sun, M. (1997). A robust high-order P-type iterative learning controller using current iteration tracking error. *International Journal of Control*, *68*(2), 331–342.

Chien, C.-J., & Liu, J. S. (1996). A P-type iterative learning controller for robust output tracking of non-linear time-varying systems. *International Journal of Control*, *64*, 319–334.

Chien, C.-J., & Yao, C.-Y. (2004). An output-based adaptive iterative learning controller for high relative degree uncertain linear systems. *Automatica*, *40*(1), 145–153.

French, M., Munde, G., Rogers, E., & Owens, D. H. (1999). Recent developments in adaptive iterative learning control. In *Proceedings of the IEEE conference on decision and control* (pp. 264–269) Phoenix, Arizona.

French, M., & Rogers, E. (2000). Nonlinear iterative learning by an adaptive Lyapunov technique. *International Journal of Control*, *73*(10), 840–850.

Huang, C. H., Ioannou, P. A., Maroulas, J., & Safonov, M. G. (1999). Design of strictly positive real systems using constant output feedback. *IEEE Transactions on Automatic Control*, *44*(3), 569–573.

Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control*. Englewood Cliffs, NJ: Prentice-Hall.

Khalil, H. K. (2002). *Nonlinear systems*. 3rd ed., Englewood-Cliffs, NJ: Prentice-Hall.

Kuc, T.-Y., & Lee, J. S. (1996). Learning strictly positive real linear systems with uncertain parameters and unknown input disturbances. *Automatica*, *32*(5), 791–792.

Kuc, T.-Y., Lee, J. S., & Nam, K. (1992). An iterative learning control theory for a class of nonlinear dynamic systems. *Automatica*, *28*(6), 1215–1221.

Moore, K. L. (1993). Iterative learning control for deterministic systems. *Advances in industrial control*. Berlin: Springer.

Moore, K. L. (1999). Iterative learning control: An expository overview. *Applied and Computational Controls, Signal Processing, and Circuits*, *1*, 151–214.

Norrlöf, M., & Gunnarsson, S. (2005). A note on causal and CITE iterative learning control algorithms. *Automatica*, *41*(2), 345–350.

Owens, D. H., Li, L. M., & Banks, S. P. (2004). Multi-periodic repetitive control system: A Lyapunov stability analysis for MIMO systems. *International Journal of Control*, *77*(5), 504–515.

Phan, M. Q., Longman, R. W., & Moore, K. L. (2000). Unified formulation of linear iterative learning control. In *Proceedings of IAAS/AIAA Space Flight Mechanics Meeting*, Clearwater, FL.

Saab, S. S. (1994). On the P-type learning control. *IEEE Transactions on Automatic Control*, *39*(11), 2298–2302.

Sun, M., & Wang, D. (2001). Sampled-data iterative learning control for nonlinear systems with arbitrary relative degree. *Automatica*, *37*(2), 283–289.

Sun, W., Khargonekar, P. P., & Shim, D. (1994). Solution to the positive real control problem for linear time-invariant systems. *IEEE Transactions on Automatic Control*, *39*(10), 2034–2046.

Tayebi, A. (2006a). Model reference adaptive iterative learning control for linear systems. *International Journal of Adaptive Control and Signal Processing*, *20*(9), 475–489.

Tayebi, A. (2006b). On ILC design for MIMO–LTI Systems. In *Proceedings of American control conference* (pp. 940–945), Minneapolis, Minnesota USA.

Tayebi, A., & Zaremba, M. B. (2003). Robust iterative learning control design is straightforward for uncertain LTI systems satisfying the robust performance condition. *IEEE Transactions on Automatic Control*, *48*(1), 101–106.

Xu, J.-X., Lee, T. H., & Zhang, H.-W. (2004). Analysis and comparison of two practical iterative learning control schemes. In *Proceedings of IEEE conference on control Application* (pp. 382–387), Taipei, Taiwan.

Xu, J.-X., & Tan, Y. (2003). *Linear and nonlinear iterative learning control. Lecture notes in control and information sciences*. Berlin: Springer.

Xu, J.-X., Wang, X.-W., & Heng, L. T. (1995). Analysis of continuous iterative learning control systems using current cycle feedback. In *Proceedings of American control conference* (pp. 4221–4225).

Xu, J.-X., & Yan, R. (2005). On initial conditions in iterative learning control. *IEEE Transactions on Automatic Control*, *50*(9), 1349–1354.

Abdelhamid Tayebi received his B.Sc. in Electrical Engineering from Ecole Nationale Polytechnique d'Alger, Algeria in 1992, his M.Sc. (DEA) in robotics from Université Pierre & Marie Curie, Paris, France in 1993, and his Ph.D. in Robotics and Automatic Control from Université d'Amiens, France in December 1997. He joined the Department of Electrical Engineering at Lakehead University in December 1999 where he is presently an Associate Professor. He is a Senior Member of IEEE and serves as an Associate Editor for IEEE Transactions on Systems, Man, and Cybernetics—Part B; Control Engineering Practice and IEEE CSS Conference Editorial Board. He is the founder and Director of the Automatic Control Laboratory at Lakehead University. His research interests are mainly related to linear and non-linear control theory including adaptive control, robust control and iterative learning control, with applications to mobile robots, robot manipulators and vertical take-off and landing aerial robots.