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Attitude stabilization of a four-rotor aerial robot

A. Tayebi and S. McGilvray

Abstract— In this paper, we propose a new quaternion-based feedback control scheme for exponential attitude stabilization of a four-rotor vertical take-off and landing (VTOL) aerial robot known as the quadrotor aircraft. The proposed controller is based upon the compensation of the Coriolis and gyroscopic torques and the use of a PD^2 feedback structure, where the proportional action is in terms of the quaternion vector and the two derivative actions are in terms of the airframe angular velocity and the quaternion velocity. We also show that the model-independent PD controller, where the proportional action is in terms of the quaternion vector and the derivative action is in terms of the airframe angular velocity, without compensation of the Coriolis and gyroscopic torques, provides asymptotic stability for our problem. Simulation results are also provided to show the effectiveness of the proposed controller.

I. Introduction

Unmanned vehicles are important when it comes to performing a desired task in a dangerous and/or unaccessible environment. Unmanned indoor and outdoor mobile robots have been successfully used for some decades. More recently, a growing interest for unmanned aerial vehicles has been shown among the research community. Being able to design an unmanned VTOL vehicle which is highly maneuverable and extremely stable is an important contribution to the field of aerial robotics since potential applications are tremendous (e.g., high buildings and monuments investigation, rescue missions, film making, etc). In practical applications, the position in space of the VTOL unmanned aerial vehicle is generally controlled by an operator through a remote control system using a visual feedback from an on-board camera, while the attitude is automatically stabilized via an on-board controller. The attitude controller is an important feature since it allows the vehicle to maintain a desired horizontal orientation and hence prevents the vehicle from flipping over and crashing when the pilot performs the desired maneuvers.

The attitude control problem of a rigid body has been investigated by several researchers and a wide class of controllers has been proposed (see, for instance, [14], [13], [1], [5], [8], and the list is not exhaustive). This is a particularly interesting problem in dynamics since the angular velocity of the body cannot be integrated to obtain the attitude of the body [5]. The paper [13] contains an interesting list of references and proposes several control algorithms guaranteeing asymptotic stability and, under certain initial conditions, local exponential stability is shown. As for the rest of the papers proposed in the literature, only asymptotic stability is guaranteed. Most of the existing attitude controllers in the literature are based upon the use of the quaternion representation to describe the attitude of the body. This particular representation allows to avoid the singularities inherent to the direction cosine matrix obtained from Euler rotations [10], [4], [6]. The existing controllers are generally based upon the use of a PD feedback structure and the compensation of the Coriolis torques, where the proportional action is in terms of the vector part of the quaternion and the derivative action is in terms of the angular velocity of the body.

In our paper, we consider the attitude stabilization problem of the quadrotor aircraft. The dynamical model describing the attitude of the quadrotor aircraft contains an additional gyroscopic term caused by the combination of the rotations of the airframe and the four rotors. In the case where the gyroscopic term is set to zero, this dynamical model reduces to the well known model used in the literature concerning the attitude control of a rigid body. Despite this additional gyroscopic term, we show that the classical model-independent PD controller can asymptotically stabilize the attitude of the quadrotor aircraft. Moreover, using a new Lyapunov function, we derive an exponentially stabilizing controller based upon the compensation of the Coriolis and gyroscopic torques and the use of a PD^2 feedback structure, where the proportional action is in terms of the quaternion vector and the two derivative actions are in terms of the airframe angular velocity and the quaternion velocity. As it will be confirmed later in our simulation results, the exponential stability property of our new controller, due mainly to the introduction of the derivative action in terms of the quaternion vector, offers a great advantage in practical applications in terms of transient performance and disturbance rejection. In our simulation results, we also confirmed the fact that the compensation of the Coriolis and gyroscopic torques provides better transient performance than the pure PD schemes.

II. Mathematical model

The aerial robot under consideration consists of a rigid cross frame equipped with four rotors as shown in figure 1. The up-down motion is achieved by increas-

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The authors are with the Department of Electrical Engineering, Lakehead University, Thunder Bay, Ontario, P7B 5E1, Canada. {atayebi, smcgilvr}@lakeheadu.ca

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ing or decreasing the total thrust while maintaining an equal individual thrust. The forward/backward, left/right and the yaw motions are achieved through a differential control strategy of the thrust generated by each rotor. In order to avoid the yaw drift due to the reactive torques, the quadrotor aircraft is configured such that the set of rotors (right-left) rotates clockwise and the set of rotors (front-rear) rotates counter-clockwise. There is no change in the direction of rotation of the rotors (i.e., $\omega_i \geq 0$, $i \in \{1, 2, 3, 4\}$). If a yaw motion is desired, one has to reduce the thrust of one set of rotors and increase the thrust of the other set while maintaining the same total thrust to avoid an up-down motion. Hence, the Yaw motion is then realized in the direction of the induced reactive torque. On the other hand, forward and backward motion is achieved by pitching in the desired direction by increasing the front (rear) rotor thrust and decreasing the rear (front) rotor thrust to maintain the total thrust. Finally, a sideways motion is achieved by rolling in the desired direction by increasing the left (right) rotor thrust and decreasing the right (left) rotor thrust to maintain the total thrust.

Let $\mathcal{I} = \{e_x, e_y, e_z\}$ denote an inertial frame, and $\mathcal{A} = \{e_1, e_2, e_3\}$ denote a frame rigidly attached to the aircraft as shown in Figure 1. The dynamical model described in [2], with a slight modification of the gyroscopic torques expression, is given as follows:

$$\dot{p} = v, \tag{1}$$

$$\dot{v} = ge_z - \frac{1}{m}TRe_z, \tag{2}$$

$$\dot{R} = RS(\Omega), \tag{3}$$

$$I_f\dot{\Omega} = -\Omega \times I_f\Omega - G_a + \tau_a, \tag{4}$$

$$I_r\dot{\omega}_i = \tau_i - Q_i, \quad i \in \{1, 2, 3, 4\} \tag{5}$$

where m denotes the mass of the airframe, g denotes the acceleration due to gravity, $e_z = (0, 0, 1)^T$ denotes the unit vector in the frame $\mathcal{I}$, the vector $p = (x, y, z)^T$ denotes the position of the origin of the body-fixed frame $\mathcal{A}$ with respect to the inertial frame $\mathcal{I}$, the vector $v = (v_x, v_y, v_z)^T$ denotes the linear velocity of the origin of $\mathcal{A}$ expressed in $\mathcal{I}$, Ω denotes the angular velocity of the airframe expressed in the body-fixed frame $\mathcal{A}$. The orientation of the airframe is given by the orthogonal rotation matrix $R \in SO(3)$. $I_f \in \mathbb{R}^{3\times3}$ is a symmetric positive definite constant inertia matrix of the airframe with respect to the frame $\mathcal{A}$ whose origin is at the center of mass. The speed and the moment of inertia of the rotor i are denoted, respectively, by ω_i and I_r. The matrix $S(\Omega)$ is a skew-symmetric matrix such that $S(\Omega)V = \Omega \times V$ for any vector $V \in \mathbb{R}^3$, where $\times$ denotes the vector cross-product. In other words, for a given vector $\Omega = (\Omega_1, \Omega_2, \Omega_3)^T$,the skew-symmetric matrix $S(\Omega)$ is defined as follows:

$$S(\Omega) = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \tag{6}$$

The reactive torque generated, in free air, by the rotor i due to rotor drag is given by

$$Q_i = \kappa\omega_i^2, \tag{7}$$

and the total thrust generated by the four rotors is given by

$$T = \sum_{i=1}^{4} |f_i| = b\sum_{i=1}^{4} \omega_i^2, \tag{8}$$

where $f_i = -b\omega_i^2 e_3$ is the lift generated by the rotor i in free air (expressed in $\mathcal{A}$), and $\kappa > 0$ and $b > 0$ are two parameters depending on the density of air, the radius, the shape, the pitch angle of the blade and other factors (see [11], [9] for more details).
The vector G_a contains the gyroscopic torques, due to the combination of the rotation of the airframe and the four rotors, and is given by

$$G_a = \sum_{i=1}^{4} I_r(\Omega \times e_z)(-1)^{i+1}\omega_i. \tag{9}$$

The airframe torques generated by the rotors are given by $\tau_a = (\tau_a^1, \tau_a^2, \tau_a^3)^T$, with

$$\begin{array}{rcl} \tau_a^1 & = & db(\omega_2^2 - \omega_4^2) \\ \tau_a^2 & = & db(\omega_1^2 - \omega_3^2) \\ \tau_a^3 & = & \kappa(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{array} \tag{10}$$

where d represents the distance from the rotors to the center of mass of the quadrotor aircraft.

Finally, τ_i represents the torque produced by the rotor i.

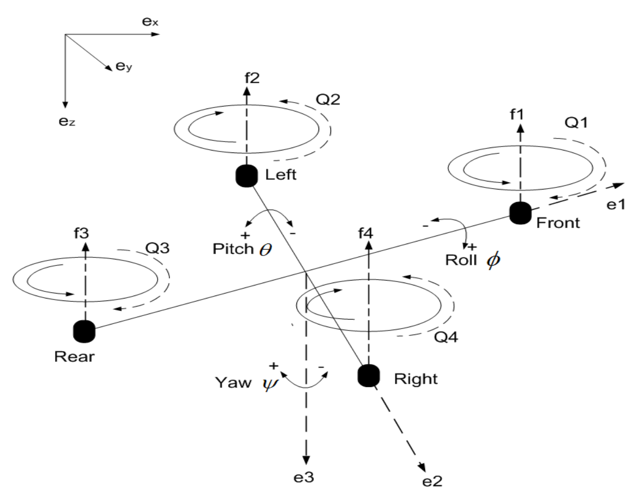

Fig. 1. Four-rotor aerial robot with forces and reactive torques

III. Attitude control design

In this section, we aim to design a feedback control scheme for the attitude stabilization of the quadrotor aircraft. To this end, we will make use of equations (3)-(5). Equations (1)-(2) describing the position and the linear velocity of the center of the body-attached frame are not used. Our approach consists of two parts. In the first part, we design the desired airframe torques τ_a for the attitude stabilization. In this part, we present two control schemes; the first one is model-dependent and guarantees exponential stability while the second one is model-independent but guarantees only asymptotic stability. In the second part, we design the rotor torques required to obtain the desired airframe torques designed in the first part.

In our approach, we make use of the quaternion to describe the quadrotor aircraft orientation. In fact, using the Euler angles [12], the rotation matrix R, describing the quadrotor aircraft orientation, is given by

$$R = \begin{pmatrix} c_\theta c_\psi & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\psi & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix} \quad (11)$$

where $c_x \overset{\triangle}{=} \cos x$, $s_x \overset{\triangle}{=} \sin x$, $x \in \{\phi, \theta, \psi\}$, with ϕ, θ and ψ denoting, respectively, the roll, the pitch and the yaw. One of the drawbacks related to the use of the direct-cosine matrix R is the inherent geometric singularity at $c_\theta = 0$. This drawback can be avoided by using the four-parameter description of the orientation called the quaternion representation [4], [6], [5], [7], [10], [13], which is based upon the fact that any rotation of a rigid body can by described by a single rotation about a fixed axis [12]. This globally nonsingular representation of the orientation, is given by vector $(q, q_0)^T$, with

$$q = \hat{k}\sin(\frac{\gamma}{2}), \quad q_0 = \cos(\frac{\gamma}{2}), \quad (12)$$

where γ is the equivalent rotation angle about the axis described by the unit vector $\hat{k} = (\hat{k}_1, \hat{k}_2, \hat{k}_3)$, subject to the constraint

$$q^T q + q_0^2 = 1. \quad (13)$$

The rotation matrix R is related to the quaternion through the Rodriguez formula [3], [12]

$$\begin{aligned} R &= I + 2q_0 S(q) + 2S(q)^2 \\ &= I + \sin\gamma S(\hat{k}) + (1-\cos\gamma) S(\hat{k})^2. \end{aligned} \quad (14)$$

An algorithm for the quaternion extraction is presented in [7]. In fact, q and q_0 are obtained from R, [8], as follows:

$$S(q) = \frac{1}{2\sqrt{1+trR}}(R - R^T), \quad (15)$$

from which, one can obtain q. Finally q_0 is obtained from (13). Note that trR denotes the trace of the matrix R. Although the quaternion representation is nonsingular, it contains a sign ambiguity (i.e., (q, q_0) and $(-q, -q_0)$ lead to the same orientation) which can be resolved by choosing the following differential equations [3], [13]

$$\begin{aligned} \dot{q} &= \frac{1}{2}(S(q) + q_0 I_{3\times3})\Omega, \\ \dot{q}_0 &= -\frac{1}{2}q^T\Omega, \end{aligned} \quad (16)$$

where $I_{3\times3}$ is a 3×3 identity matrix.

A. Step1: Airframe torques design

In this part, we consider τ_a as a control input to be designed for the attitude stabilization of the quadrotor aircraft. Except for the gyroscopic term, which is new, the dynamical model (3)-(4) is similar to the well known model used in the literature concerning the attitude control of a rigid body. Our objective, is to stabilize the equilibrium point $(\phi, \theta, \psi, \Omega) = 0$, or $(R = I, \Omega = 0)$. This can be achieved by the stabilization of the two equilibrium points $(q = 0, q_0 = \pm1, \Omega = 0)$ for (16) and (4). Since $q_0 = 1$ corresponds to $\gamma = 0$ and $q_0 = -1$ corresponds to $\gamma = 2\pi$, it is clear that $q_0 = \pm1$ correspond to the same physical point. Hence the two equilibrium points $(q = 0, q_0 = \pm1, \Omega = 0)$ are in reality a unique physical equilibrium point corresponding to $(R = I, \Omega = 0)$. In the sequel, the Euler angles and the equivalent rotation angle γ are taken between $-\pi$ and π, which imply that $0 \le q_0 \le 1$.

Now, one can state the following result:

Theorem 1: consider (3) and (4) under the following control law

$$\tau_a = \Omega \times I_f\Omega + G_a + I_f\dot{\bar{\Omega}} - \Gamma_2\tilde{\Omega} - \alpha q, \quad (17)$$

where $\tilde{\Omega} = \Omega - \bar{\Omega}$, $\bar{\Omega} = -\Gamma_1 q$, and $\dot{\bar{\Omega}} = -\Gamma_1\dot{q} = -\frac{1}{2}\Gamma_1(q_0 I_{3\times3} + S(q))\Omega$, with Γ_1 and Γ_2 being 3×3 symmetric positive definite matrices and α is a positive parameter. Then, the equilibrium point $(R = I, \Omega = 0)$ is globally exponentially stable.

Proof: Let us use the following Lyapunov function candidate

$$\begin{aligned} V &= \alpha q^T q + \alpha(q_0 - 1)^2 + \frac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega} \\ &= 2\alpha(1 - q_0) + \frac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega} \end{aligned} \quad (18)$$

whose time derivative, in view of (4) and (16) is given by

$$\dot{V} = \alpha q^T(\tilde{\Omega} + \bar{\Omega}) + \tilde{\Omega}^T(-(\Omega \times I_f\Omega) - G_a + \tau_a - I_f\dot{\bar{\Omega}}). \quad (19)$$

Using (17), we obtain

$$\dot{V} = -\alpha q^T \Gamma_1 q - \tilde{\Omega}^T \Gamma_2 \tilde{\Omega}. \quad (20)$$

which implies that the state variables of (4) and (16) are bounded and $\lim_{t\to\infty} \tilde{\Omega}(t) = \lim_{t\to\infty} q(t) = 0$. This implies that $\lim_{t\to\infty} \Omega(t) = \lim_{t\to\infty} \bar{\Omega}(t) = 0$. Hence, from (13), one can conclude that $\lim_{t\to\infty} q_0(t) = \pm1$.

Now, let us show the exponential stability. From the fact that $|q_0| \leq 1$ and (13), we have

$$\|q\|^2 = 1 - q_0^2 \geq 1 - q_0. \tag{21}$$

Consequently, V can be bounded from above as follows:

$$V \leq max\{2\alpha, \frac{\lambda_{max}(I_f)}{2}\}(\|q\|^2 + \|\tilde{\Omega}\|^2). \tag{22}$$

On the other hand $\dot{V}$ can be bounded from above as follows:

$$\dot{V} \leq -min\{\alpha\lambda_{min}(\Gamma_1), \lambda_{min}(\Gamma_2)\}(\|q\|^2 + \|\tilde{\Omega}\|^2). \tag{23}$$

Hence, from (22) and (23), one can conclude that

$$\dot{V} \leq -\beta V, \tag{24}$$

where $\beta = \dfrac{min\{\alpha\lambda_{min}(\Gamma_1), \lambda_{min}(\Gamma_2)\}}{max\{2\alpha, 0.5\lambda_{max}(I_f)\}}$, with $\lambda_{min}(*)$ and $\lambda_{max}(*)$ denote, respectively, the minimum and maximum eigenvalue of $(*)$.

□

Remark 1: It is worth noting that the Lyapunov function used in the proof of Theorem 1 can be obtained using the backstepping approach. In fact, the first step consists to consider Ω as a virtual control input in (16) and use a first Lyapunov function $V_1 = 2\alpha(1 - q_0)$ to design the virtual control input $\bar{\Omega}$. The second and last step consists to augment V_1 by the quadratic term in $\tilde{\Omega}$ to obtain the final Lyapunov function V leading the control input τ_a.

Note that the control law (17) requires the compensation of the Coriolis and gyroscopic torques involving the airframe inertia I_f and the rotor inertia I_r. Now, we will show that the classical PD feedback control without compensation of the Coriolis and gyroscopic torques (i.e., model-independent control) can asymptotically stabilize the attitude of the quadrotor aircraft.

Theorem 2: Consider (3) and (4) under the following control law

$$\tau_a = -\Gamma_4\Omega - \alpha q, \tag{25}$$

where Γ_4 is a 3×3 symmetric positive definite matrix and α is a positive parameter. Then, the equilibrium point $(R = I, \Omega = 0)$ is globally asymptotically stable.

Proof: Let us consider the following Lyapunov function candidate

$$\begin{aligned} V &= \alpha q^T q + \alpha(q_0 - 1)^2 + \frac{1}{2}\Omega^T I_f \Omega \\ &= 2\alpha(1 - q_0) + \frac{1}{2}\Omega^T I_f \Omega, \end{aligned} \tag{26}$$

whose time derivative, in view of (4) and (16), is given by

$$\dot{V} = \alpha q^T \Omega + \Omega^T(-(\Omega \times I_f \Omega) - G_a + \tau_a). \tag{27}$$

Taking the control input as in (25), and using the fact that $\Omega^T(\Omega \times I_f\Omega) = 0$ and $\Omega^T G_a = \Omega^T(\Omega \times e_z)\sum_{i=1}^{4} I_r(-1)^{i+1}\omega_i = 0$, we obtain

$$\dot{V} = -\Omega^T \Gamma_4 \Omega. \tag{28}$$

From (26) and (28), and the fact that q and q_0 satisfy (13), one can conclude that Ω, q and q_0 are bounded. Using La Salle's invariance theorem, one can easily show that the equilibrium point $(q = 0, q_0 = \pm 1, \Omega = 0)$ is asymptotically stable. In fact, on can conclude that $\lim_{t\to\infty} \Omega(t) = 0$, which implies that $\lim_{t\to\infty} \dot{\Omega}(t) = 0$. Hence, from (4) and (25), one has $\lim_{t\to\infty} \tau_a(t) = -\alpha q = 0$. Finally, from (13), one can conclude that $\lim_{t\to\infty} q_0(t) = \pm 1$.

□

Remark 2: It is worth noting that the control law (17) can be written in the following form

$$\tau_a = \Omega \times I_f \Omega + G_a - (\alpha I_{3\times 3} + \Gamma_2\Gamma_1)q - \Gamma_2\Omega - I_f\Gamma_1\dot{q}, \tag{29}$$

which is basically a PD^2 feedback with Coriolis and gyroscopic torques compensation. The two derivative actions are related to the angular velocity (Ω) and the "quaternion velocity" ($\dot{q}$). Note that $\dot{q}$ is obtained explicity from (16).

The control law (25) is a classical PD feedback, where the derivative action is related to the angular velocity (Ω). It is similar to the control laws proposed in [5], [8], [14] for the attitude control of a rigid body. The main advantage of the control law (25) with respect to the control law (17) is the fact that the model parameters are not required and the control law is much simpler to implement. The main advantage of the control law (17) with respect to the control law (25) is the exponential convergence property mainly due to the compensation of the Coriolis and gyroscopic terms and the use of the vector-quaternion time-derivative $\dot{q}$.

B. Step2: Rotor torques design

In reality, the control inputs in (1)-(5) are the four rotor torques τ_i, $i \in \{1, 2, 3, 4\}$. To design the rotor torques, one has to find the desired speed of each rotor $\omega_{d,i}$, $i \in \{1, 2, 3, 4\}$, corresponding to the desired airframe torques $\tau_a = (\tau_a^1, \tau_a^2, \tau_a^3)^T$ obtained from (17) or (25). To this end, we must specify the desired total thrust T. The desired speed for the four motors can be obtained from (8) and (10). That is $\varpi_d = M^{-1}\mu$, with $\varpi_d = (\omega_{d,1}^2, \omega_{d,2}^2, \omega_{d,3}^2, \omega_{d,4}^2)^T$, $\mu = (\tau_a^1, \tau_a^2, \tau_a^3, T)^T$ and

$$M = \begin{pmatrix} 0 & db & 0 & -db \\ db & 0 & -db & 0 \\ \kappa & -\kappa & \kappa & -\kappa \\ b & b & b & b \end{pmatrix} \tag{30}$$

Note that M is nonsingular as long as $db\kappa \neq 0$. Now, having $\omega_{d,i}$, $i \in \{1, 2, 3, 4\}$, one can design τ_i as follows:

$$\tau_i = Q_i + I_r\dot{\omega}_{d,i} - k_i\tilde{\omega}_i, \tag{31}$$

where k_i, $i \in \{1,2,3,4\}$, are four positive parameters, and $\tilde{\omega}_i = \omega_i - \omega_{d,i}$. In fact, applying (31) to (5) leads to

$$\dot{\tilde{\omega}}_i = -\frac{k_i}{I_r}\tilde{\omega}_i, \tag{32}$$

which shows the exponential convergence of ω_i to $\omega_{d,i}$ and hence the convergence of the airframe torques to the desired values leading to the attitude stabilization of the quadrotor aircraft.

Intuitively, for better transient performance, one has to ensure that the convergence of ω_i to $\omega_{d,i}$ is faster than the convergence of the attitude to zero. This could be realized by taking $\frac{k_i}{I_r} > \beta$, where β is defined in (24).

Remark 3: In the control law (31), the derivative of the desired rotors speed is required. It is possible to derive the analytical expression for $\dot{\omega}_{d,i}$ as follows:

$$\dot{\omega}_d = \frac{1}{2}A_\omega M^{-1}\dot{\mu}, \tag{33}$$

where, $\dot{\omega}_d = (\dot{\omega}_{d,1}, \dot{\omega}_{d,2}, \dot{\omega}_{d,3}, \dot{\omega}_{d,4})^T$, $A_\omega = diag(\frac{1}{\omega_{d,1}}, \frac{1}{\omega_{d,2}}, \frac{1}{\omega_{d,3}}, \frac{1}{\omega_{d,4}})$ and $\dot{\mu} = [\dot{\tau}_a^T, \dot{T}]^T$. If we consider that the desired total thrust is a strictly positive constant and $\omega_{d,i} \neq 0$, one can obtain $\dot{\omega}_{d,i}$ from (33), using $\dot{T} = 0$. The derivative of the desired airframe torque can be obtained explicitly from (17) or (25). For the control law (25), one can obtain $\dot{\tau}_a$ using (4) and (16) as follows

$$\begin{array}{rcl} \dot{\tau}_a & = & -\Gamma_3 I_f^{-1}(-\Omega \times I_f\Omega - G_a + \tau_a) \\ & - & \dfrac{\alpha}{2}(S(q) + q_0 I_{3\times 3})\Omega. \end{array} \tag{34}$$

However, to simplify the control implementation and to avoid the singularity occuring for $\omega_{d,i} = 0$, one can obtain $\dot{\omega}_{d,i}$ from $\omega_{d,i}$ using a filtered derivative also called the “dirty derivative” (i.e., $\dot{\omega}_{d,i} = \frac{s}{1+T_f s}[\omega_{d,i}]$, with $T_f = \frac{1}{2\pi f_c}$, where f_c is the cut-off frequency).

IV. Simulation results

To explore the performance of each controller, two simulations have been performed with results for each plotted separately as aircraft angles, and angular velocity seen in Figures 2 - 5 below. Simulation 1 uses control law (17) with the following gains: $\Gamma_1 = diag(100, 100, 100)$, $\Gamma_2 = diag(0.025, 0.025, 0.025)$ and $\alpha = 0.4$. Simulation 2 uses control law (25) with the following gains: $\Gamma_4 = diag(0.025, 0.025, 0.025)$ and $\alpha = 0.4$. The desired thrust, gains k_i and the initial roll, pitch and yaw angles for both simulation 1 and 2 are given as $T = 15N$, $k_i = 0.001$, for all $i \in \{1,2,3,4\}$, $\phi = -25°$, $\theta = 30°$ and $\psi = -10°$ respectively.
The quadrotor aircraft model parameters for both simulations are given in the table below:
It can be clearly seen from Figure 2 that exponential convergence of the aircraft angles has been achieved. Figure 3 however, shows the convergence of the attitude angles with a large overshoot and a longer settling time. It should be noted that better performance using control law (25) has been observed by increasing the controller gains namely α and Γ_4 but at some cost to the control effort and subsequently, the angular velocity of the aircraft.
Figures 4 and 5 show the angular velocity of the quadrotor aircraft for simulations 1 and 2 respectively. Similar maximum angular velocities are observed during both simulations indicating a fair comparison of control laws (17) and (25).

Parameter	Value	Units
g	9.81	m/s^2
m	0.468	kg
d	0.225	m
I_r	3.357×10^{-5}	$kg \cdot m^2$
I_{f_ϕ}	4.856×10^{-3}	$kg \cdot m^2$
I_{f_θ}	4.856×10^{-3}	$kg \cdot m^2$
I_{f_ψ}	8.801×10^{-3}	$kg \cdot m^2$
b	2.980×10^{-6}	
κ	1.140×10^{-7}	

TABLE I
Quadrotor aircraft model parameters for simulation

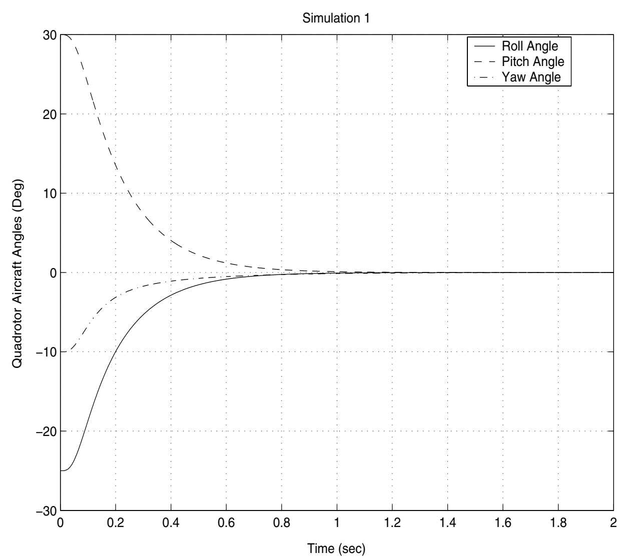

Fig. 2. Aircraft Angles, Simulation 1

V. Conclusion

In this paper, we showed that it is possible to achieve global exponential stability for the attitude stabilization problem of the quadrotor aircraft. This result is based upon the compensation of the Coriolis and gyroscopic torques and the use of a PD^2 feedback structure, where the proportional action is in terms of the quaternion vector and the two derivative actions are in terms of the airframe angular velocity and the

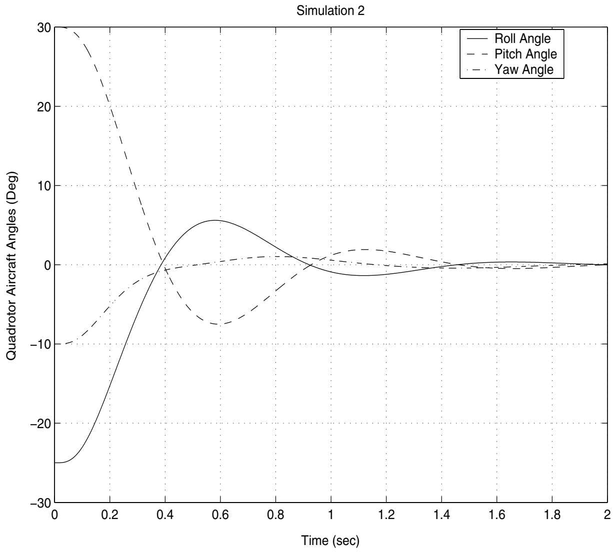

Fig. 3. Aircraft Angles, Simulation 2

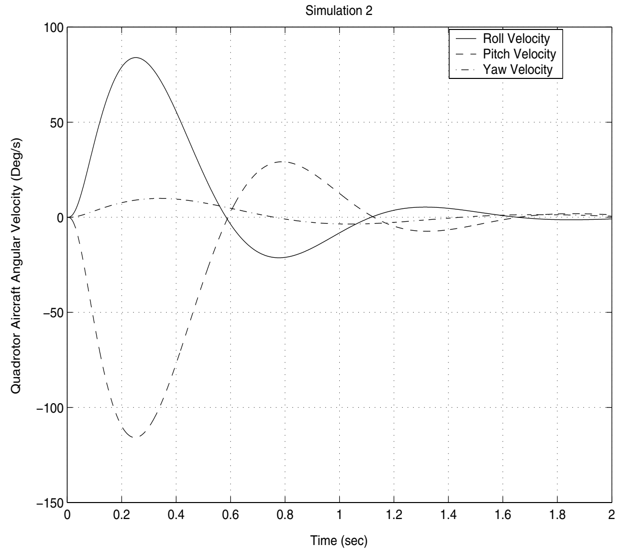

Fig. 5. Aircraft Angular Velocity, Simulation 2

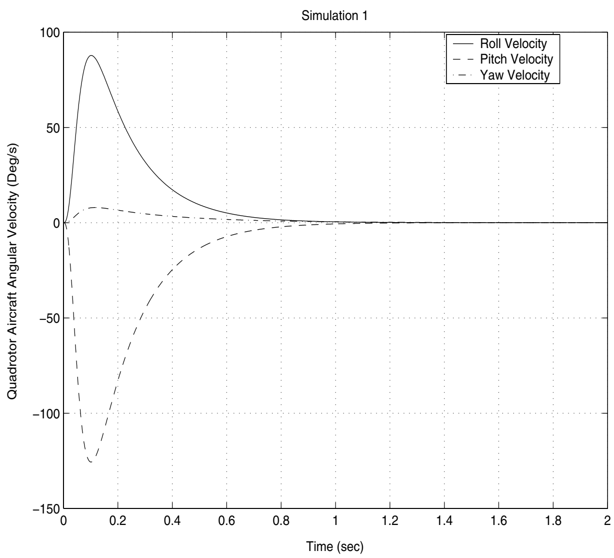

Fig. 4. Aircraft Angular Velocity, Simulation 1

quaternion velocity. We also showed that the model-independent PD controller, where the proportional action is in terms of the quaternion vector and the derivative action is in terms of the airframe angular velocity, without compensation of the Coriolis and gyroscopic torques, provides asymptotic stability for our problem.

Simulation results have been presented to show the effectiveness of the proposed controllers. It is worth noting that the proposed controllers have been successfully implemented on a small-scale quadrotor aircraft. The experimental results are not presented here because of space limitation. A video clip of the experiment can be seen at http://flash.lakeheadu.ca/~ tayebi/attitude_stabilization.wmv.

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