Joint 48th IEEE Conference on Decision and Control and
28th Chinese Control Conference
Shanghai, P.R. China, December 16-18, 2009

Formation control of VTOL-UAVs

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***Abstract*— This paper considers the formation control problem of a group of Vertical Take-Off and Landing (VTOL) Unmanned Aerial Vehicles (UAV) in SE(3). The vehicles among the team are required to track a reference velocity signal and maintain a desired formation. For each vehicle in the formation, we propose a design methodology based on the separation of the translational and the rotational dynamics, using the desired orientation, in terms of the unit-quaternion, as an intermediate variable to achieve our position tracking objective. Global asymptotic stability result of the closed loop system is established using the Lyapunov method. The communication topology between formation team members is assumed to be fixed and bidirectional. Our control scheme can also be applied to the position control of a single VTOL-UAV and constitute, in its own right, an interesting contribution since global results are seldom achieved in the available literature. Finally, simulation results of a scenario of four VTOL-UAVs in a formation are provided to show the effectiveness of the proposed control scheme.**

I. INTRODUCTION

Cooperative control of mobile agents has received an extensive interest among the research community in the past years. This interest is motivated by the idea that through efficient coordination many inexpensive, simple vehicles can achieve better performance at lower cost than a single monolithic vehicle. Many interesting results have been reported in the literature, see for instance, [1]-[4]. These works mainly deal with simple dynamic models such as linear systems and single or double integrators. However, much work remains to be done when it comes to dealing with complex vehicle dynamics.

More recently, a growing interest in unmanned aerial vehicles (UAVs) has been shown due to their potential applications in areas such as high buildings and monuments investigation, search and rescue missions, and surveillance. An important class of UAVs are the Vertical Take-Off and Landing (VTOL) vehicles, which are suitable for broad range of applications requiring stationary flights. These vehicles are generally *under-actuated i.e.*, equipped with fewer actuators than degrees-of-freedom. Several control schemes have been proposed for the attitude stabilization of rigid bodies including VTOL vehicles, see for instance, [5], [6]. The attitude synchronization of rigid bodies in space has also been extensively dealt with in the recent years, see for instance [7], [8] and references therein. On the other hand, the position control of underactuated VTOL vehicles in SE(3) is more challenging than the attitude control problem since global asymptotic stability is difficult to achieve for this class of mechanical systems. Several attempts have been reported in the literature, such as the feedback linearization method in [9], the backstepping approach in [10], the sliding mode technique in [11], and several other propositions have been developed [12], [13], [14]. The authors in [10] and [15] propose a design strategy for the stabilization of hovering VTOLs, and a hierarchical controller composed of a high level position control and a low level attitude control is presented. The proposed idea consists in using the thrust and the vehicle's orientation as control variables to stabilize the vehicle's position, and then applying a classical backstepping procedure to determine torque-inputs capable of stabilizing the requested orientation. In [16], a similar control architecture is applied to solve the trajectory tracking problem, where the angular velocity is used as an intermediate variable instead of the orientation, and a high gain controller is used to determine the torque signals capable of tracking the requested angular velocity. The difficulty with the latter design is to prove the stability of the global cascaded system. More recently, the authors in [17] proposed a backstepping design for the trajectory tracking problem of a class of underactuated systems, including VTOL vehicles, where the states are guaranteed to converge to a ball near the origin.

In this paper, we consider a formation of a group of VTOL-UAVs, and propose a control scheme such that all vehicles track a reference velocity signal while maintaining a prescribed formation pattern. For this purpose, we exploit the cascaded nature of the system and first design a control input for the translational dynamics for each vehicle, from which we can extract the magnitude and direction of the necessary thrust input for each vehicle. The direction of the thrust will then define a time-varying desired attitude for each aerial vehicle to be tracked by the rotational dynamics with an appropriate design of the torque input. The novelty in our work, with respect to the existing literature, is the use of a singularity-free unit-quaternion for the orientation representation. Indeed, we make use of an extraction method for the desired direction of the vehicle's thrust, which provides always a realizable solution under the condition that the translational control input is upper bounded by a well defined quantity. It is clear that in a formation control design, each vehicle uses information from its neighbors' to maintain a prescribed formation, and it is not always possible to maintain an upper bound of the control if the number of neighbors is large. The proposed formation control law is guaranteed to be upper bounded by a prescribed quantity

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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978-1-4244-3872-3/09/$25.00 ©2009 IEEE

regardless of the number of neighbors of each vehicle. It is important to mention that the proposed formation control scheme is applicable to solve the trajectory tracking problem of VTOL-UAVs, when the number of vehicles in the team is one, and the same results apply. This constitutes a new contribution to the tracking problem of this type of UAVs.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this work, we consider n-VTOL aircraft modeled as rigid-bodies. The equations of motion of the j^{th} aircraft are described by

$$(\Sigma_j): \begin{cases} (\Sigma_{1_j}): \begin{cases} \dot{p}_j = v_j, \\ \dot{v}_j = g\hat{e}_3 - \frac{T_j}{m_j} R(\mathbf{q}_j)^T \hat{e}_3, \end{cases} \\ (\Sigma_{2_j}): \begin{cases} \dot{\mathbf{q}}_j = \frac{1}{2}\mathbf{q}_j \odot \bar{\omega}_j, \\ I_{f_j}\dot{\omega}_j = \tau_j - S(\omega_j) I_{f_j}\omega_j, \end{cases} \end{cases} \tag{1}$$

where p_j and v_j denote, respectively, the position and velocity of the center of gravity of the j^{th} vehicle with respect to the inertial frame, $\mathscr{F}_i \triangleq \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$, expressed in the inertial frame $\mathscr{F}_i$. m_j and g are the vehicle mass and the gravitational acceleration. The vector $\bar{\omega}_j = (\omega_j^T, 0)^T$, and ω_j denotes the angular velocity of the j^{th} aerial vehicle expressed in the j^{th} body-fixed frame $\mathscr{F}_j \triangleq \{\hat{e}_{1j}, \hat{e}_{2j}, \hat{e}_{3j}\}$. $I_{f_j} \in \mathbb{R}^{3\times 3}$ is the symmetric positive definite constant inertia matrix of the j^{th} vehicle with respect to $\mathscr{F}_j$. The scalar T_j and the vector τ_j represent respectively the magnitude of the thrust applied to the j^{th} vehicle in the direction of $\hat{e}_{3j}$, and the external torque applied to the system expressed in $\mathscr{F}_j$. The unit quaternion $\mathbf{q}_j = (q_j^T, \eta_j)^T$, composed of a vector component $q_j \in \mathbb{R}^3$ and a scalar component $\eta_j \in \mathbb{R}$, represents the orientation of the vehicle's body frame, $\mathscr{F}_j$, with respect to the inertial frame, $\mathscr{F}_i$, and are subject to the constraint

$$q_j^T q_j + \eta_j^2 = 1 \tag{2}$$

The rotation matrix related to the unit-quaternion $\mathbf{q}_j$, that brings the inertial frame into the body frame, can be obtained through the Rodriguez formula as

$$R(\mathbf{q}_j) = (\eta_j^2 - q_j^T q_j) I_3 + 2 q_j q_j^T - 2\eta_j S(q_j) \tag{3}$$

where I_3 is the 3-by-3 identity matrix and the matrix $S(\mathbf{x})$ is the skew-symmetric matrix such that $S(\mathbf{x})V = \mathbf{x}\times V$ for any vectors $\mathbf{x} \in \mathbb{R}^3$ and $V \in \mathbb{R}^3$. The quaternion multiplication between two unit quaternion, $\mathbf{q} = (q^T, \eta)$ and $\mathbf{p} = (p^T, \varepsilon)$, is defined by the following non-commutative operation; $\mathbf{q}\odot\mathbf{p} = \big(\eta p + \varepsilon q + S(q)p\ ,\ \eta\varepsilon - q^T p\big)$. The inverse or conjugate of a unit quaternion is defined by, $\mathbf{q}_j^{-1} = (-q_j^T, \eta_j)^T$, with the quaternion identity given by $(0,0,0,1)^T$, [18].

Our objective is to design a thrust and torque inputs for each VTOL vehicle in the formation to guarantee that all vehicles follow a reference velocity and maintain a prescribed formation, *i.e.*, maintain fixed relative distances between neighbors in the team, where the communication topology between vehicles is assumed to be fixed and undirected. More formally, our control objective is to guarantee;

$$v_j(t) \to v_d(t) \quad \text{and} \quad p_j - p_k \to \delta_{jk} \tag{4}$$

where the vector δ_{jk} defines the desired position offset between the j^{th} and k^{th} aircraft, and satisfies $\delta_{jk} = -\delta_{kj}$. Also, we assume that the reference velocity $v_d(t)$ is bounded and is available to all vehicles, and $\dot{v}_d(t)$, $\ddot{v}_d(t)$ and $v_d^{(3)}(t)$ are bounded. We define the velocity tracking error for each aircraft as

$$\tilde{v}_j = v_j - v_d \tag{5}$$

With this definition, velocity tracking is achieved if $\tilde{v}_j \to 0$.

III. CONTROL DESIGN PROCEDURE

The main idea in our work is to exploit the cascaded nature of the system and first design an intermediary control input for the translational dynamics for each aircraft, from which we can extract the magnitude and direction of the necessary thrust input for each vehicle. The magnitude of the thrust $T_j(t)$ will be the input to the translational dynamics, and its direction will define a time-varying desired attitude for each aerial vehicle, namely $\mathbf{q}_{d_j}(t)$, to be tracked by the rotational dynamics with an appropriate design of the torque input for each subsystem (Σ_{2_j}). In this section, we will present an extraction procedure of the thrust magnitude and direction for each VTOL vehicle in the formation using the unit quaternion representation of the attitude. We can rewrite the second equation of subsystem (Σ_{1_j}) as

$$\dot{v}_j = F_j - \frac{T_j}{m_j}\Big(R(\mathbf{q}_j)^T - R(\mathbf{q}_{d_j})^T\Big)\hat{e}_3 \tag{6}$$

$$F_j = g\hat{e}_3 - \frac{T_j}{m_j} R(\mathbf{q}_{d_j})^T \hat{e}_3 \tag{7}$$

where the variable F_j is the intermediary control input to the translational dynamics. As it is clear that (7) can have multiple solutions for the thrust magnitude and direction, the following Lemma [19] gives one possible extraction that is free from singularities if the control input satisfies some conditions. Since this procedure applies for all VTOL vehicles in the formation, we will omit the subscript "j" in the following result for clarity of presentation.

Lemma 1: [19]
Consider equation (7) and let the vector $F \triangleq (\mu_1, \mu_2, \mu_3)^T$. It is always possible to extract the thrust magnitude and direction from (7) as

$$T = m\big((g-\mu_3)^2 + \mu_1^2 + \mu_2^2\big)^{1/2} \tag{8}$$

$$q_d = \frac{m}{2T\eta_d}(\mu_2, -\mu_1, 0)^T \quad , \quad \eta_d = \sqrt{\frac{1}{2} + \frac{m(g-\mu_3)}{2T}} \tag{9}$$

under the condition that the elements of F satisfy

$$(\mu_1, \mu_2, \mu_3) \neq (0, 0, x), \quad \text{for} \quad x \geq g \tag{10}$$

In addition, we can write the desired angular velocity of each aircraft in terms of the intermediary control, F, as

$$\omega_d = \Xi(F)\dot{F}, \tag{11}$$

$$\Xi(F) = \frac{1}{\gamma_1^2\gamma_2}\begin{pmatrix} -\mu_1\mu_2 & -\mu_2^2 + \gamma_1\gamma_2 & \mu_2\gamma_2 \\ \mu_1^2 - \gamma_1\gamma_2 & \mu_1\mu_2 & -\mu_1\gamma_2 \\ \mu_2\gamma_1 & -\mu_1\gamma_1 & 0 \end{pmatrix},$$

with $\gamma_1 = (T/m)$ and $\gamma_2 = \gamma_1 + (g - \mu_3)$.

Proof: Let $\mathbf{q}_d = (q_{d1}, q_{d2}, q_{d3}, \eta_d)^T$. Using (2) and (3), equation (7) is equivalent to

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} - \frac{T}{m} \begin{pmatrix} 2q_{d1}q_{d3} + 2\eta_d q_{d2} \\ 2q_{d2}q_{d3} - 2\eta_d q_{d1} \\ 1 - 2(q_{d1}^2 + q_{d2}^2) \end{pmatrix} \tag{12}$$

from which, it is clear that there are multiple solutions for the desired attitude $\mathbf{q}_d$, and hence for the thrust magnitude. One possible solution can be obtained by fixing one of the above variables. We consider that $q_{d3} = 0$, and take the sum of the squares of the two first equations to obtain

$$\mu_1^2 + \mu_2^2 = 4\frac{T^2}{m^2}\eta_d^2(1 - \eta_d^2) \tag{13}$$

from which one solution for η_d can be obtained as; $\eta_d^2 = \frac{m}{2T}\left(\frac{T}{m} + \sqrt{\frac{T^2}{m^2} - \mu_1^2 - \mu_2^2}\right)$, under the conditions that $\mu_1^2 + \mu_2^2 \leq \frac{T^2}{m^2}$, and $T \neq 0$. Also, from the last equation of (12) and (2), we can write the expression of the thrust given in (8), which is always positive. Note that with this expression of the thrust, the condition; $\mu_1^2 + \mu_2^2 \leq \frac{T^2}{m^2}$ is always verified. Hence, using (8) and exploiting the first two equations of (12), we obtain the elements of the desired attitude vector $\mathbf{q}_d$ given in (9), under the conditions that $\eta_d \neq 0$ and $T \neq 0$. We can see that $T = 0$ only if $(\mu_1, \mu_2, \mu_3) = (0, 0, g)$, and $\eta_d = 0$ if $T = -(g - \mu_3)$, which is only possible if; $\mu_3 \geq g$ and $\mu_1^2 + \mu_2^2 = 0$. Finally, we can conclude that if the elements of the control F satisfy condition (10), the solution (8)-(9) always exist, and is singularity-free.

From the attitude kinematics, the desired angular velocity is defined as, [18],

$$\omega_d = 2\begin{pmatrix} \eta_d I_3 + S(q_d) \\ -q_d^T \end{pmatrix}^T \dot{\mathbf{q}}_d, \tag{14}$$

Hence, taking the time derivative of (9) and with simple computation, we can write the expression of ω_d in terms of the elements of the intermediary control input as given in (11) for each aircraft. ■

IV. FORMATION CONTROL

A. *Intermediary control design:*

We first consider the translational dynamics. In view of system (Σ_{1j}) in (1) and equations (5)-(7), the velocity tracking error dynamics can be written as

$$\dot{\tilde{v}}_j = F_j - \dot{v}_d - \frac{T_j}{m_j} f(\mathbf{q}_j, \mathbf{q}_{d_j}) \tag{15}$$

with $f(\mathbf{q}_j, \mathbf{q}_{d_j}) = \left(R(\mathbf{q}_j)^T - R(\mathbf{q}_{d_j})^T\right)\hat{e}_3$. It can be seen that (15) describes the dynamics of a linear system with a nonlinear perturbation described by the term; $\frac{T_j}{m_j} f(\mathbf{q}_j, \mathbf{q}_{d_j})$. It is worth noticing that at this stage of control, this nonlinear perturbation term is completely unknown, and since $f(\mathbf{q}_j, \mathbf{q}_{d_j})$ contains orthogonal rotation matrices, we know that it is bounded if T_j is bounded.

In order to design an intermediary control F_j for each aircraft that achieves our control objectives, we have to take into consideration some important requirements. First, it is important to note that for the extraction condition (10) to be satisfied, the upper bound of the control input F_j must be determined *a priori*. Since our objective is to design a control action for each aircraft that guarantees a prescribed formation of a group of VTOL UAVs, we will require the implementation of a term containing some information, mainly the position, of neighboring aircraft. It is clear that if this term can be guaranteed to be bounded, using some saturation functions for example, its upper bound will grow with the number of neighboring vehicles. The challenge is then to guarantee that the force control input F_j for each aircraft can be upper bounded *a priori* regardless of the number of neighboring aircraft.

The second requirement that we should consider can be seen from the expression of ω_{d_j} in (11), which suggests that $\dot{\omega}_{d_j}$ is function of $\ddot{F}_j$. Hence, in order to implement a trajectory tracking attitude controller, that necessarily requires the knowledge of $\dot{\omega}_{d_j}$, we need to implement in each aircraft the second derivatives of each signal used in the control F_j. Therefore, if relative positions are implemented in the intermediary control law, we will need the knowledge of neighboring aircraft accelerations which are not available for feedback. Finally, the last requirement that we should satisfy is to design a bounded control law that guarantees global stability results.

In order to achieve our control objective and overcome the above mentioned problems, we introduce the following new variables for the j^{th} vehicle

$$\xi_j = p_j - \theta_j \quad , \quad z_j = \tilde{v}_j - \dot{\theta}_j := \dot{\xi}_j - v_d \tag{16}$$

for $j = 1, ..., n$, where θ_j is a design variable to be determined later. Note that the new variable θ_j is introduced to modify (during the transient) the desired trajectory to simplify the control design. Once the tracking is achieved, the variable θ_j and its time derivative are forced to converge to zero, ensuring hence the tracking of the original desired trajectories.

The translational error dynamics can then be written as

$$\dot{z}_j = F_j - \frac{T_j}{m_j} f(\mathbf{q}_j, \mathbf{q}_{d_j}) - \ddot{\theta}_j - \dot{v}_d \tag{17}$$

It is clear that if one is able to design a bounded control F_j for each vehicle in the formation such that the signals z_j, $(\xi_j - \xi_k - \delta_{jk})$, θ_j and $\dot{\theta}_j$ converge to zero, we will guarantee that $p_j - p_k \to \delta_{jk}$ and $\tilde{v}_j \to 0$ for all $j, k \in \{1, ..., n\}$, if every aircraft can communicate with at least one other aircraft in the group, *i.e,* the communication graph is connected.

We propose the following control input for each VTOL vehicle

$$F_j = \dot{v}_d - \Phi(\theta_j) \tag{18}$$

$$\ddot{\theta}_j = -\Phi(\theta_j) + k_j^z z_j + \sum_{k=1}^{n} k_{jk}^p (\xi_{jk} - \delta_{jk}) \tag{19}$$

with $\Phi(\theta_j) = \left(k_{\theta_{1j}} \tanh(\theta_j) + k_{\theta_{2j}} \tanh(\dot{\theta}_j)\right)$, $\xi_{jk} = \xi_j - \xi_k$, k_j^z, $k_{\theta_{1j}}$ and $k_{\theta_{2j}}$ are positive scalar gains. The gains k_{jk}^p are the formation-keeping gains defined such that $k^p_{jj} \triangleq 0$ and

$$\begin{cases} k^p_{jk} = k^p_{kj} > 0, & \text{if aircrafts } j \text{ and } k \text{ are connected} \\ k^p_{jk} = k^p_{kj} = 0, & \text{otherwise} \end{cases} \tag{20}$$

for $j,k \in \{1,...,n\}$. We say that two aircraft are "connected" if they can communicate with each other and share their states information. The magnitude of a nonzero k^p_{jk} determines the strength of the connection between members of the formation. In addition, by restrictions (20), we are assuming that the communication flow between aircraft is undirected.

It is worth noticing from the proposed control (18) that F_j is guaranteed to be bounded as

$$\|F_j\| \leq \|\ddot{p}_d\| + \sqrt{3}(k_{\theta_{1j}} + k_{\theta_{2j}}) \tag{21}$$

regardless of the number of neighbors of vehicle j. In addition, an upper bound of the extracted value of the thrust T_j, in (8), can be determined *a priori* and is given as

$$T_j \leq m\left(g + \|\ddot{p}_d\| + \sqrt{3}(k_{\theta_{1j}} + k_{\theta_{2j}})\right) := \Lambda_j \tag{22}$$

with Λ_j a positive constant.

B. Attitude control design:

In this section, we consider the rotational dynamics and design a torque input for each aircraft in order to track the desired orientation, $\mathbf{q}_{d_j}(t)$, extracted according to (9) from F_j given in (18). We define the attitude tracking error for each vehicle, namely $\tilde{\mathbf{q}}_j = (\tilde{q}_j^T, \tilde{\eta}_j)^T$, and is given by; $\tilde{\mathbf{q}}_j = \mathbf{q}_{d_j}^{-1} \odot \mathbf{q}_j$, and is governed by the unit-quaternion dynamics

$$\dot{\tilde{q}}_j = \frac{1}{2}(\tilde{\eta}_j\ I_3 + S(\tilde{q}_j))\tilde{\omega}_j, \quad \dot{\tilde{\eta}}_j = -\ \frac{1}{2}\tilde{q}_j^T\tilde{\omega}_j, \tag{23}$$

$$\tilde{\omega}_j = \omega_j - R(\tilde{\mathbf{q}}_j)\ \omega_{d_j}, \tag{24}$$

where $\tilde{\omega}_j$ is the angular velocity error vector. $R(\tilde{\mathbf{q}}_j)$ is the rotation matrix, related to $\tilde{\mathbf{q}}_j$, and is given by $R(\tilde{\mathbf{q}}_j) = R(\mathbf{q}_j)R(\mathbf{q}_{d_j})^T$, [18]. The vector ω_{d_j} is the (desired) angular velocity and is given in (11) for each aircraft. The time derivative of the desired angular velocity can be obtained as; $\dot{\omega}_{d_j} = \dot{\Xi}(F_j)\dot{F}_j + \Xi(F_j)\ddot{F}_j$, for each aircraft, where $\dot{\Xi}(F_j)$ can be easily derived from $\dot{F}_j$. From (18), and after simple computations, we can easily verify that in order to evaluate ω_{d_j} and $\dot{\omega}_{d_j}$ for each aircraft, we will need only available signals and aircraft must transmit only the variables ξ_j and z_j. In addition, ω_{d_j} and $\dot{\omega}_{d_j}$ are guaranteed to be bounded if the signals $\dot{z}_j$, θ_j, $\dot{\theta}_j$ and $\ddot{\theta}_j$ are bounded for $j \in \{1,...,n\}$.

In order to design an attitude tracking control law, we introduce the following new variable for each vehicle as

$$\Omega_j = \tilde{\omega}_j - \beta_j \tag{25}$$

with β_j a design variable to be determined later. Exploiting the rotational dynamics (Σ_{2_j}) in (1), we can easily show that

$$I_{f_j}\dot{\Omega}_j = \tau_j - \mathbf{H}_j(\omega_j, \dot{\beta}_j, \dot{\omega}_{d_j}, \omega_{d_j}, \tilde{\mathbf{q}}_j) \tag{26}$$

$$\mathbf{H}_j(\cdot) = S(\omega_j)I_{f_j}\omega_j - I_{f_j}S(\tilde{\omega}_j)R(\tilde{\mathbf{q}}_j)\omega_{d_j} + I_{f_j}R(\tilde{\mathbf{q}}_j)\dot{\omega}_{d_j} + I_{f_j}\dot{\beta}_j$$

We propose the following torque control for each vehicle

$$\tau_j = \mathbf{H}_j(\omega_j, \dot{\beta}_j, \dot{\omega}_{d_j}, \omega_{d_j}, \tilde{\mathbf{q}}_j) - \alpha_j\tilde{q}_j - k_j^{\Omega}\Omega_j \tag{27}$$

$$\beta_j = -k_j^{\beta}\tilde{q}_j \tag{28}$$

with α_j, k_j^{Ω} and k_j^{β} are positive scalar gains, and

$$\dot{\beta}_j = \frac{-k_j^{\beta}}{2}(\tilde{\eta}_j I_3 + S(\tilde{q}))\tilde{\omega}_j \tag{29}$$

C. Stability Analysis:

Now, we can state our result in the following theorem

Theorem 1: Consider the VTOL-UAVs formation modeled as in (1), where the closed loop system is given by (17) and (26), with the control inputs (18)-(19) and (27)-(28). If the control gains satisfy

$$k_{\theta_{1j}} + k_{\theta_{2j}} < (\sqrt{3})^{-1}(g - \|\ddot{p}_d(t)\|), \tag{30}$$

$$k_j^z > \sigma_j, \quad \text{and} \quad \alpha_j k_j^{\beta}(m_j^2\sigma_j) > 2\Lambda_j^2, \tag{31}$$

for some constant $\sigma_j > 0$ and for $j \in \{1,...,n\}$, and Λ_j is given in (22), then all signals are bounded and $\tilde{v}_j \to 0$, $p_j - p_k \to \delta_{jk}$, $\tilde{q}_j \to 0$ and $\tilde{\omega}_j \to 0$ asymptotically for $j,k \in \{1,...,n\}$.

Proof: First, it is straightforward to check that if (30) is satisfied, then $\|F_j\| < g$ and condition (10) is always satisfied, and hence it is always possible to extract the magnitude of the thrust and its direction from (8) and (9) respectively for each VTOL vehicle in the team.

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\sum_{j=1}^{n}\left(z_j^T z_j + \frac{1}{2}\sum_{k=1}^{n}k_{jk}^p(\xi_{jk} - \delta_{jk})^T(\xi_{jk} - \delta_{jk})\right) + \sum_{j=1}^{n}\left(\frac{1}{2}\Omega_j^T I_{f_j}\Omega_j + 2\alpha_j(1 - \tilde{\eta}_j)\right) \tag{32}$$

The time derivative of V evaluated along the closed loop dynamics (17) and (26) with (23), and the control inputs (18)-(19) and (27) with (25), is given by

$$\dot{V} = \sum_{j=1}^{n} z_j^T\left(-k_j^z z_j - \frac{T_j}{m_j}f(\mathbf{q}_j, \mathbf{q}_{d_j}) - \sum_{k=1}^{n}k_{jk}^p(\xi_{jk} - \delta_{jk})\right) + \sum_{j=1}^{n}\alpha_j\tilde{q}_j^T\beta_j - \sum_{j=1}^{n}k_j^{\Omega}\Omega_j^T\Omega_j + \frac{1}{2}\sum_{k=1}^{n}k_{jk}^p\dot{\xi}_{jk}^T(\xi_{jk} - \delta_{jk})$$

Then, using (16) and restrictions (20), and the fact that $\delta_{jk} = -\delta_{kj}$, we can easily show that

$$\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p(z_j - z_k)^T(\xi_{jk} - \delta_{jk}) = \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p z_j^T(\xi_{jk} - \delta_{jk})$$

which, with (28) and the relation $\dot{\xi}_{jk} = z_j - z_k$, yields

$$\dot{V} = \sum_{j=1}^{n}\left(-k_j^z z_j^T z_j - \frac{T_j}{m_j}z_j^T f(\mathbf{q}_j, \mathbf{q}_{d_j}) - \alpha_j k_j^{\beta}\tilde{q}_j^T\tilde{q}_j - k_j^{\Omega}\Omega_j^T\Omega_j\right)$$

Exploiting equations (2) and (3), we can easily show that

$$\|f(\mathbf{q}_j, \mathbf{q}_{d_j})\| \leq \|I_3 - R(\tilde{\mathbf{q}}_j)\|_F = 2\sqrt{2}\|\tilde{q}_j\| \tag{33}$$

where $\|\cdot\|$ denotes the Euclidean norm and $\|\cdot\|_F$ is the Frobenius norm[1], and we have used the fact that the rotational matrix is orthogonal.

Then, the time derivative of V can be upper bounded using (33) as

$$\begin{aligned}\dot{V} \leq\ & \sum_{j=1}^{n}\left(-k_j^z\|z_j\|^2 + \frac{2\sqrt{2}T_j}{m_j}\|z_j\|\|\tilde{\mathbf{q}}_j\| - \alpha_j k_j^\beta\|\tilde{q}_j\|^2\right) \\ & -\sum_{j=1}^{n} k_j^\Omega\|\Omega_j\|^2 \\ \leq\ & \sum_{j=1}^{n}\left(-(k_j^z-\sigma_j)\|z_j\|^2 - (\alpha_j k_j^\beta - \frac{2T_j^2}{m_j^2\sigma_j})\|\tilde{q}_j\|^2\right) \\ & -\sum_{j=1}^{n} k_j^\Omega\|\Omega_j\|^2\end{aligned}$$

where we have used the fact that for any real numbers a and b, we have $2ab \leq a^2/\sigma + \sigma b^2$, for some $\sigma > 0$. Therefore, $\dot{V}$ is negative semi-definite if (31) is satisfied. Hence, we can conclude that z_j, ξ_{jk}, $\tilde{q}_j$, Ω_j are bounded for $j,k \in \{1,...,n\}$. Consequently, $\dot{\theta}_j$ and $\dot{z}_j$ are bounded for $j \in \{1,...,n\}$, from (17) and (18)-(19).

Since $\tilde{q}_j$ is bounded, we can see that β_j, in (28), is bounded, and hence $\tilde{\omega}_j$ is bounded from (25). Hence, we can conclude that $\dot{\tilde{q}}_j$ and $\dot{\beta}_j$ are bounded from (23) and (29) respectively. In addition, from equation (26) with (27), we can conclude that $\dot{\Omega}_j$ is bounded and so is $\dot{\tilde{\omega}}_j$. As a result, $\ddot{V}$ is bounded. Hence, invoking Barbalat's lemma, [20], we can conclude that $z_j \to 0$, $\Omega_j \to 0$ and $\tilde{q}_j \to 0$, and therefore, $\tilde{\omega}_j \to 0$, $\tilde{\eta}_j \to \pm 1$ and $R(\tilde{\mathbf{q}}_j) \to I_3$, for $j \in \{1,...,n\}$.

From equation (17) with (18)-(19), and exploiting the above boundedness and convergence results, and invoking the extended Barbalat's lemma (see for example lemma 3 in [16]), we can conclude that $\dot{z}_j \to 0$. Hence, the closed loop translational dynamics, (17) with (18)-(19), reduce to

$$\sum_{k=1}^{n} k_{jk}^p(\xi_j - \xi_k - \delta_{jk}) = 0, \text{ for } \ j = 1,...n \tag{34}$$

which is equivalent to; $\sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}^p(\xi_j - \delta_j)^T(\xi_j - \xi_k - \delta_{jk}) = 0$, where the constant vector δ_j can be regarded as the desired position of the j^{th} aircraft with respect to the center of the formation. It is then clear that $\delta_{jk} = \delta_j - \delta_k$. Then using (20), equation (34) is equivalent to; $\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} k_{jk}^p(\xi_j - \xi_k - \delta_{jk})^T(\xi_j - \xi_k - \delta_{jk}) = 0$, from which we can conclude that $\xi_j - \xi_k \to \delta_{jk}$.

In order to complete the proof, we have to investigate the boundedness and asymptotic convergence to zero of θ_j and $\dot{\theta}_j$. It can be seen that (19) can be rewritten as

$$\ddot{\theta}_j = -k_{\theta_{1j}}\tanh(\theta_j) - k_{\theta_{2j}}\tanh(\dot{\theta}_j) + u_j \tag{35}$$

for $j = 1,...,n$ with $u_j = k_j^z z_j + \sum_{k=1}^{n} k_{jk}^p(\xi_{jk} - \delta_{jk})$. Using the above results, one can easily verify that u_j is bounded and tends to zero as t goes to infinity. One can show that the following Lyapunov function candidate

$$W_j = \frac{1}{2}\dot{\theta}_j^T\dot{\theta}_j + k_{\theta_{1j}}\mathbf{1}_3^T\log(\cosh(\theta_j)),$$

[1]The Frobenius norm of a square matrix $\mathbf{M}$ is given by: $\|\mathbf{M}\|_F = \sqrt{\mathrm{tr}(\mathbf{M}^T\mathbf{M})}$

TABLE I
SIMULATION PARAMETERS

$p_1(0) = (7,0,1)$, $p_2(0) = (6,0,-2)$, $p_3(0) = (13.5,-2,0.5)$,
$p_4(0) = (15,-0.5,1)$, $g = 9.8$, $v_j(0) = (0,0,0)$, $\mathbf{q}_j(0) = (0,0,0,1)$,
$\omega_j(0) = \theta_j(0) = \dot{\theta}_j(0) = (0,0,0)$, $k_j^z = 3$, $k_{\theta_1 j} = 1.8$, $k_{\theta_2 j} = 1.8$,
$k_j^\beta = 20$, $\alpha_j = 20$, $k_j^\Omega = 50$, for $j = 1,2,3,4$,
$k_{jk}^p = 2.5$, for $(j,k) \in \{(1,2),(2,3),(2,4)\}$,
$\delta_1 = (1,1,0)$, $\delta_2 = (-1,1,0)$, $\delta_3 = (-1,-1,0)$, $\delta_4 = (1,-1,0)$.

where $\mathbf{1}_3 = \mathrm{col}[1,1,1]$ and $\log(\cdot)$, $\cosh(\cdot)$ are defined element-wise for a vector, in view of (35), has a time-derivative given by

$$\dot{W}_j = -\dot{\theta}_j^T(k_{\theta_{2j}}\tanh(\dot{\theta}_j) - u_j). \tag{36}$$

Due to the fact that $u_j(t)$ is bounded and converges to zero, one can show that θ_j and $\dot{\theta}_j$ are bounded and converge asymptotically to zero, under the condition that system (35) does not have a finite escape time. From (36), we can see that: $\dot{W}_j \leq \|\dot{\theta}_j\|\|u_j\|$. Using the relation $\|\dot{\theta}_j\|^2 \leq 2W_j$, we will obtain: $\dot{W}_j \leq A\sqrt{W_j}$, with $\sqrt{2}\|u_j\| \leq A$, which can be rewritten as: $\frac{dW_j}{\sqrt{W_j}} \leq Adt$. Suppose that there exists a finite time t_f such that $\lim_{t\to t_f} W_j(t) = +\infty$. Then, integrating the last inequality along the interval $[t_1, t_f]$ yields to: $(2\sqrt{W_j(t_f)} - 2\sqrt{W_j(t_1)}) = +\infty \leq A(t_f - t_1)$. We can see that this leads to a contradiction, since the left hand side of the inequality is infinite, whereas the right hand side is assumed to be finite. Hence, we can conclude that system (35) cannot have a finite escape time.

Finally, one can conclude that $\theta_j \to 0$ and $\dot{\theta}_j \to 0$ for all $j \in \{1,...,n\}$. As a result, we conclude that $p_j - p_k \to \delta_{jk}$ and $\tilde{v}_j \to 0$ for all $j,k \in \{1,...,n\}$. ■

Remark 1: It is important to mention that the proposed control scheme is also applied to solve the position trajectory tracking problem a single VTOL-UAV, with $k_{jk}^p = 0$, and global asymptotic stability is guaranteed, which is a new contribution to the trajectory tracking problem of this type of UAVs.

V. SIMULATION RESULTS

Using SIMULINK, we consider a scenario of four VTOL vehicles that are required to maintain a planar square formation while tracking a common desired velocity. The vehicles are modeled as rigid bodies whose inertia matrices are taken as $I_{f_j} = diag(0.065, 0.065, 0.02)$ and of mass $m_j = 5$ Kg. The simulation parameters are illustrated in table I, where the controller gains are selected such that conditions (30)-(31) are satisfied. The reference velocity is given by $v_d(t) = ((-2.5/\pi)\cos(t/(2\pi)), (1.25/\pi)\cos(t/(2\pi)), -0.5)$. The vectors δ_{jk} are computed according to the variables δ_j in the above table such that the desired formation pattern is a square, with $\delta_{jk} = \delta_j - \delta_k$. The obtained results are shown in Figs.1-2. Fig. 1 illustrates the three components of the velocity tracking errors for the four vehicles respectively, where the global convergence to zero is guaranteed after few seconds. Fig. 2 shows the position of the VTOL aircraft at instants of time, where, starting from an arbitrary initial

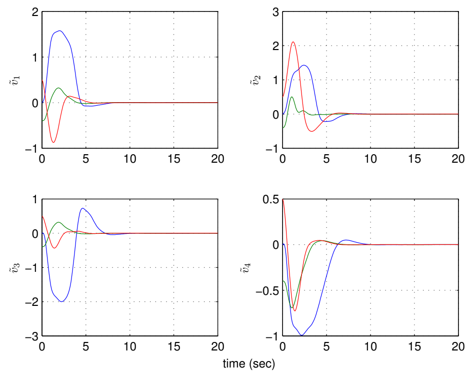

Fig. 1. Velocity tracking error of the four aircraft

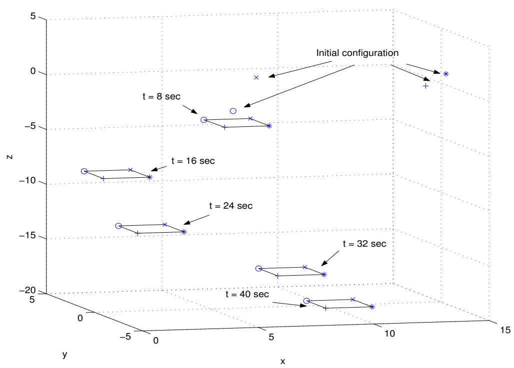

Fig. 2. VTOL aircraft formation

condition, we can see that the four aircraft converge to the desired square formation during the first 8 *sec*.

VI. Conclusion

We addressed the formation control problem of a class of under-actuated systems, the VTOL UAVs. Exploiting the cascaded nature of the system, a separate translational and rotational control design was presented, and global asymptotic stability of the closed-loop system was shown. The design procedure is based on the unit-quaternion extraction method of the desired orientation, which is guaranteed to be singularity-free, under the condition that the translational control input is upper bounded by a predefined quantity. This has led us to design a bounded control for each vehicle whose upper bound can be selected independently from the number of neighbors in the formation. Although in this work we present a formation control scheme, the proposed control law can be applied to a single VTOL-UAV leading to global asymptotic stability, which constitutes, in our opinion, an interesting contribution since global results in SE(3) are difficult to achieve for this class of systems. Still several interesting issues regarding the formation control problem, such as the collision avoidance, directed communication topology and the presence of time delays, need to be investigated. These topics will be the subject of our future work.

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