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On the Coordinated Attitude Alignment of a Group of Spacecraft Without Velocity Measurements

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***Abstract*— We consider the coordinated attitude control problem for a group of spacecraft without velocity measurements. Our approach is based on the introduction of an auxiliary dynamical system for each spacecraft (playing the role of velocity observers in a certain sense) to generate the individual and relative damping terms in the absence of the actual angular velocities and relative angular velocities. Our main interest is to provide new design methodologies to minimize the order of the controllers as well as the information flow requirement between spacecraft in the team. We will address the following two problems: 1) Design a velocity-free attitude tracking and synchronization control scheme, that allows the team members to align their attitudes and track a time-varying reference trajectory (simultaneously). 2) Design a velocity-free synchronization control scheme, in the case where no reference attitude is specified, and all spacecraft are required to synchronize their attitudes to the same final time-varying attitude. Throughout this paper, the communication flow between spacecraft is assumed to be fixed and undirected. Simulation results are provided to show the effectiveness of the proposed control schemes.**

I. INTRODUCTION

In the past few years, the problem of controlling and maintaining the relative attitudes of formation flying spacecraft, or rigid bodies in general, have been the interest of many researchers. While several papers on the subject consider the full information case, where the angular velocities are available for feedback, [3]-[13], only few consider the velocity-free problem. In [14], the authors introduced a local passivity based control law for multi-spacecraft attitude alignment with zero desired angular velocity without velocity measurements, assuming a ring communication topology. Reference [15] extended the work of [14] to the case of a general undirected communication topology. In both papers, the authors consider the case where the final angular velocity is zero, and the extension of the obtained results to the trajectory tracking case is not obvious. Reference [16] presents an output tracking solution to a leader/follower spacecraft. The authors consider the dynamics of the relative attitude, and uniform practical stability is shown. As relative attitude is involved, this method can hardly be extended to a formation with large number of members.

In [1] and [2], based on the work of [17], we have considered the attitude alignment problem without velocity measurements and proposed solutions to the velocity-free synchronization with tracking, leader follower and the consensus seeking problems. The basic idea in these works consists of associating an auxiliary dynamic system to each spacecraft and to each pair of spacecraft with a communication link in order to recover and generate the necessary damping that would have been generated by the actual angular velocities and relative angular velocities. Although the proposed control schemes guarantee global asymptotic convergence of the system states, and do not increase the communication requirements as compared to the full information case, their implementation requires the use of a number of auxiliary dynamical systems, for each spacecraft, which increases with the number of its neighbors, hence augmenting considerably the order of the system as the number of spacecraft in the formation is large. The main contribution of this paper is to extend our previous results, and present a velocity-free control scheme that requires the implementation of a single dynamic auxiliary system for each spacecraft. First, we present a new design methodology for the input of the individual auxiliary systems, which is function of the states of neighboring spacecraft. With the proposed scheme, the order of the control system will not be affected by the number of neighbors in the formation as compared to [1] and [2]. Then, we propose a second design method of the input of the single auxiliary system for each spacecraft that reduces the information flow requirement between spacecraft.

In this paper, we propose solutions to solve two different problems. First, we consider the case where spacecraft are required to achieve simultaneous tracking and synchronization without velocity measurements and without any restriction on the graph topology. Second, the attitude alignment without reference trajectory is solved, where no reference trajectory is assigned to dictate the group's objective, and it is required that spacecraft align their attitudes with the same (not necessarily constant) final angular velocity, and only the spacecraft absolute attitudes are transmitted between communicating spacecraft.

II. SPACECRAFT DYNAMICS AND PROBLEM FORMULATION

Consider a group of n spacecraft modeled as rigid bodies. The equations of motion of the j^{th} spacecraft are

$$I_{f_j}\dot{\omega}_j = \tau_j - \mathbf{S}(\omega_j)I_{f_j}\omega_j, \qquad (1)$$

$$\dot{\mathbf{q}}_j = \frac{1}{2}\mathbf{q}_j \odot \bar{\omega}_j = \frac{1}{2}\begin{pmatrix} \eta_j\ I_3 + \mathbf{S}(q_j) \\ -q_j^T \end{pmatrix}\omega_j, \qquad (2)$$

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where $\bar{\omega}_j^T = (\omega_j^T\ ,\ 0)$, and $\omega_j \in \mathbb{R}^3$ denotes the angular velocity of the j^{th} spacecraft expressed in the body-fixed frame $\mathscr{F}_j$. $I_{f_j} \in \mathbb{R}^{3\times 3}$ is a constant symmetric positive definite inertia matrix of the j^{th} spacecraft with respect to $\mathscr{F}_j$. The vector τ_j is the external torque applied to the j^{th} spacecraft expressed in $\mathscr{F}_j$. The unit-quaternion $\mathbf{q}_j = (q_j^T, \eta_j)^T$ is composed of a real part $\eta_j \in \mathbb{R}$ and a vector part $q_j \in \mathbb{R}^3$, and represents the orientation of the j^{th} spacecraft frame $\mathscr{F}_j$ with respect to the inertial frame $\mathscr{F}_i$. The elements of a unit-quaternion $\mathbf{q}_j$ are subject to the constraint

$$\eta_j^2 + q_j^T q_j = 1 \tag{3}$$

The quaternion multiplication "$\odot$" of two unit-quaternion $\mathbf{q}_j = (q_j^T, \eta_j)^T$ and $\mathbf{q}_k = (q_k^T, \eta_k)^T$ is distributive and associative but not commutative, and is defined as

$$\mathbf{q}_j \odot \mathbf{q}_k = \begin{pmatrix} \eta_j q_k + \eta_k q_j + \mathbf{S}(q_j) q_k \\ \eta_j \eta_k - q_j^T q_k \end{pmatrix} \tag{4}$$

where the matrix $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $S(\mathbf{x})V = \mathbf{x} \times V$ for any vector $V \in \mathbb{R}^3$, and is given by

$$S(\mathbf{x}) = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} \tag{5}$$

with $\mathbf{x} = (x_1, x_2, x_3)^T$. The unit-quaternion inverse is given by $\mathbf{q}_j^{-1} = (-q_j^T, \eta_j)^T$. The orthogonal rotation matrix related to the unit-quaternion $\mathbf{q}_j$, that brings the inertial frame into the body frame, is defined as $R(\mathbf{q}_j)$, and can be obtained through the Rodriguez formula as

$$R(\mathbf{q}_j) = (\eta_j^2 - q_j^T q_j) I_3 + 2 q_j q_j^T - 2\eta_j S(q_j) \tag{6}$$

Assume that the desired trajectory is represented by the unit-quaternion $\mathbf{q}_d = (q_d^T, \eta_d)^T$ that describes the orientation of the desired frame, denoted by $\mathscr{F}_d$, with respect to $\mathscr{F}_i$, and satisfies the unit-quaternion dynamics: $\dot{\mathbf{q}}_d = \frac{1}{2}\mathbf{q}_d \odot \bar{\omega}_d$, with $\bar{\omega}_d^T = (\omega_d^T, 0)$, and $\omega_d \in \mathbb{R}^3$ is the angular velocity of $\mathscr{F}_d$ expressed in $\mathscr{F}_d$, which is assumed to be bounded as well as its first and second time-derivatives. The discrepancy between the absolute attitude of the j^{th} spacecraft and the desired attitude defines the attitude tracking error for spacecraft j, namely $\tilde{\mathbf{q}}_j = (\tilde{q}_j^T, \tilde{\eta}_j)^T$, is given by $\tilde{\mathbf{q}}_j = \mathbf{q}_d^{-1} \odot\ \mathbf{q}_j$, which is governed by the unit-quaternion dynamics

$$\dot{\tilde{q}}_j = \frac{1}{2}(\tilde{\eta}_j\ I_3 + \mathbf{S}(\tilde{q}_j))\tilde{\omega}_j, \quad \dot{\tilde{\eta}}_j = -\ \frac{1}{2}\tilde{q}_j^T \tilde{\omega}_j, \tag{7}$$

$$\tilde{\omega}_j = \omega_j - R(\tilde{\mathbf{q}}_j)\ \omega_d, \tag{8}$$

where $\tilde{\omega}_j$ is the angular velocity error vector describing the relative angular velocity of $\mathscr{F}_j$ with respect to $\mathscr{F}_d$ expressed in $\mathscr{F}_j$. Matrix $R(\tilde{\mathbf{q}}_j)$ is the rotation matrix, related to $\tilde{\mathbf{q}}_j$, that brings $\mathscr{F}_d$ onto $\mathscr{F}_j$ and is given by $R(\tilde{\mathbf{q}}_j) = R(\mathbf{q}_j) R(\mathbf{q}_d)^T$, [19]. The angular velocity error dynamics for the j^{th} spacecraft can be written, using (8) and (1), as

$$\begin{aligned} I_{f_j}\dot{\tilde{\omega}}_j = \ & \tau_j - \mathbf{S}\left(\tilde{\omega}_j + R(\tilde{\mathbf{q}}_j)\omega_d\right) I_{f_j}\left(\tilde{\omega}_j + R(\tilde{\mathbf{q}}_j)\omega_d\right) \\ & + I_{f_j}\left(\mathbf{S}(\tilde{\omega}_j) R(\tilde{\mathbf{q}}_j)\omega_d - R(\tilde{\mathbf{q}}_j)\dot{\omega}_d\right). \end{aligned} \tag{9}$$

After some algebraic manipulations, and using the cross product properties and the fact that $I_{f_j} = I_{f_j}^T > 0$, one can show that

$$\tilde{\omega}_j^T I_{f_j} \dot{\tilde{\omega}}_j = \tilde{\omega}_j^T(\tau_j - \mathbf{F}(\omega_d, \dot{\omega}_d, \tilde{\mathbf{q}}_j)) \tag{10}$$

with $\mathbf{F}(\cdot) = I_{f_j} R(\tilde{\mathbf{q}}_j)\dot{\omega}_d + \mathbf{S}(R(\tilde{\mathbf{q}}_j)\omega_d) I_{f_j} R(\tilde{\mathbf{q}}_j)\omega_d$.

In the sequel, we say that the j^{th} and k^{th} spacecraft are neighbors, or connected by a communication link, if they have access to their relative information. In our case, two neighbors need to know their relative attitudes. The relative attitude between the j^{th} and k^{th} spacecraft, namely $\mathbf{q}_{jk} = (q_{jk}^T, \eta_{jk})^T$, is defined as

$$\mathbf{q}_{jk} = \mathbf{q}_k^{-1} \odot \mathbf{q}_j, \tag{11}$$

and is governed by the following dynamics

$$\dot{q}_{jk} = \frac{1}{2}(\eta_{jk}\ I_3 + \mathbf{S}(q_{jk}))\omega_{jk}, \quad \dot{\eta}_{jk} = -\ \frac{1}{2} q_{jk}^T \omega_{jk}, \tag{12}$$

with

$$\omega_{jk} = \omega_j - R(\mathbf{q}_{jk})\omega_k, \tag{13}$$

where $\mathbf{q}_{jk}$ represents the rotation from $\mathscr{F}_k$ to $\mathscr{F}_j$, $R(\mathbf{q}_{jk})$ is the rotation matrix related to $\mathbf{q}_{jk}$, and the vector ω_{jk} is the relative angular velocity of $\mathscr{F}_j$ with respect to $\mathscr{F}_k$ expressed in $\mathscr{F}_j$. Using (11), the following relations can be easily verified

$$R(\mathbf{q}_{kj})^T = R(\mathbf{q}_{jk}), \quad q_{kj} = -\ q_{jk} = -R(\mathbf{q}_{kj})\ q_{jk}. \tag{14}$$

In this paper, our main objective is to design coordinated attitude control laws without angular velocity measurements for each spacecraft to solve two problems. First, we design a velocity-free attitude tracking and synchronization scheme such as the following tasks are simultaneously achieved without velocity measurements:

- All relative attitudes and angular velocities between the team members converge to zero, *i.e.*, $\mathbf{q}_j \to \mathbf{q}_k$ and $\omega_j \to \omega_k$, for all $j, k \in \{1, ..., n\}$.
- Each spacecraft tracks the desired trajectory, *i.e.,* $\mathbf{q}_j(t) \to \mathbf{q}_d(t)$ and $\omega_j(t) \to \omega_d(t)$ as $t \to \infty$.

Second, we assume that no reference signal is available to any spacecraft, and we want to design a velocity-free synchronization scheme such that spacecraft align their attitudes, *i.e.,* $\mathbf{q}_j \to \mathbf{q}_k$ and $\omega_j \to \omega_k$, using only local information transmitted between neighbors among the group.

III. SIMULTANEOUS ATTITUDE TRACKING AND SYNCHRONIZATION

In this section, we consider the problem of the design of a simultaneous attitude tracking and synchronization scheme without velocity measurements, allowing a group of spacecraft to align their attitudes with a reference attitude (possibly time-varying), while maintaining the same relative attitude during formation maneuvers. In order to remove the angular velocity requirement in the control, we introduce the following auxiliary system to each spacecraft in the team

$$\dot{\mathbf{p}}_j = \frac{1}{2}\mathbf{p}_j \odot \bar{\beta}_j, \tag{15}$$

with $\bar{\beta}_j = (\beta_j^T \ , \ 0)^T$ and $\beta_j \in \mathbb{R}^3$ to be designed later. The mismatch between the auxiliary system output and the attitude tracking error for the j^{th} spacecraft is defined by the unit-quaternion $\tilde{\mathbf{p}}_j = (\tilde{p}_j^T, \tilde{\varepsilon}_j)^T$ given by

$$\tilde{\mathbf{p}}_j = \mathbf{p}_j^{-1} \odot \mathbf{q}_j, \tag{16}$$

satisfying the unit-quaternion dynamics

$$\dot{\tilde{p}}_j = \frac{1}{2}(\tilde{\varepsilon}_j \ I_3 + \mathbf{S}(\tilde{p}_j))\Omega_j, \quad \dot{\tilde{\varepsilon}}_j = -\ \frac{1}{2}\tilde{p}_j^T \Omega_j, \tag{17}$$

with

$$\Omega_j = \omega_j - R(\tilde{\mathbf{p}}_j)\beta_j, \tag{18}$$

where $R(\tilde{\mathbf{p}}_j)$ is the rotation matrix related to $\tilde{\mathbf{p}}_j$.

Also, we consider a new unit-quaternion related to the output of the j^{th} and k^{th} auxiliary systems, defined as

$$\tilde{\mathbf{p}}_{jk} = \tilde{\mathbf{p}}_k^{-1} \odot \tilde{\mathbf{p}}_j \triangleq (\tilde{p}_{jk}^T, \tilde{\varepsilon}_{jk})^T, \tag{19}$$

where $\tilde{\mathbf{p}}_j = \mathbf{p}_j^{-1} \odot \mathbf{q}_j$ and $\tilde{\mathbf{p}}_k = \mathbf{p}_k^{-1} \odot \mathbf{q}_k$, satisfying the unit quaternion dynamics

$$\dot{\tilde{p}}_{jk} = \frac{1}{2}(\tilde{\varepsilon}_{jk} I + \mathbf{S}(\tilde{p}_{jk}))\Omega_{jk}, \quad \dot{\tilde{\varepsilon}}_{jk} = -\frac{1}{2}\tilde{p}_{jk}^T \Omega_{jk}, \tag{20}$$

with

$$\Omega_{jk} = \Omega_j - R(\tilde{\mathbf{p}}_{jk})\Omega_k, \tag{21}$$

where Ω_j is given in (18). The following properties can be easily shown

$$R(\tilde{\mathbf{p}}_{kj})^T = R(\tilde{\mathbf{p}}_{jk}), \tag{22}$$

$$\tilde{p}_{kj} = -\tilde{p}_{jk} = -R(\tilde{\mathbf{p}}_{kj})\tilde{p}_{jk}. \tag{23}$$

The main idea behind the introduction of the auxiliary systems is to use the vector parts of the unit quaternion $\tilde{\mathbf{p}}_j$ and $\tilde{\mathbf{p}}_{jk}$ in the control law instead of the real angular velocities and relative angular velocities to generate the necessary damping for the overall closed loop stability.

We consider the following velocity-free attitude tracking control law for the j^{th} spacecraft

$$\tau_j = \mathbf{F}(\omega_d, \dot{\omega}_d, \tilde{\mathbf{q}}_j) - \alpha_{1j}\tilde{q}_j - \alpha_{2j}\tilde{p}_j - \sum_{k=1}^{n}(k_{jk}^p \ q_{jk} + k_{jk}^d \ \tilde{p}_{jk}), \tag{24}$$

where α_{1j} and α_{2j} are strictly positive gains, n is the number of spacecraft in the formation and k_{jk}^p, k_{jk}^d are positive gains defined such that $k_{jj}^\star \triangleq 0$ and

$$\begin{cases} k_{jk}^\star = k_{kj}^\star > 0, & \text{if spacecraft } j \text{ and } k \text{ are connected} \\ k_{jk}^\star = k_{kj}^\star = 0, & \text{otherwise} \end{cases} \tag{25}$$

for $j,k \in \{1,...,n\}$, $\star \in \{p,d\}$. The magnitude of a nonzero k_{jk}^p and/or k_{jk}^d determines the strength of the connection between spacecraft. In addition, by restrictions (25), we are assuming that the communication flow between spacecraft is undirected.

Our result is stated in the following theorem.

Theorem 1: Consider the formation given in (1)-(2) under the control law (24), with (25), and let the inputs of the auxiliary systems (15) be

$$\beta_j = R(\tilde{\mathbf{p}}_j)^T \left(\Gamma_j \mathscr{X}_j + R(\tilde{\mathbf{q}}_j)\omega_d\right), \quad \Gamma_j = \Gamma_j^T > 0, \tag{26}$$

with

$$\mathscr{X}_j = \alpha_{2j}\tilde{p}_j + \sum_{k=1}^{n} k_{jk}^d \tilde{p}_{jk} \tag{27}$$

If the control gains satisfy

$$\alpha_{1j} > 2\sum_{k=1}^{n} k_{jk}^p \quad , \quad \alpha_{2j} > 2\sum_{k=1}^{n} k_{jk}^d \tag{28}$$

for $j \in \{1,...,n\}$, then all the signals are bounded and $q_j(t) \to q_k(t) \to q_d(t)$ and $\omega_j(t) \to \omega_k(t) \to \omega_d(t)$ asymptotically, $\forall j,k \in \{1,...,n\}$. Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$ and $\tilde{\varepsilon}_j(t) > 0$, for all $t \geq T$ and $j \in \{1,...,n\}$, then the same convergence results are obtained without condition (28).

Proof: Consider the following Lyapunov function candidate

$$V = \sum_{j=1}^{n}\left(\frac{1}{2}\tilde{\omega}_j^T I_{f_j}\tilde{\omega}_j + 2\alpha_{1j}(1-\tilde{\eta}_j) + 2\alpha_{2j}(1-\tilde{\varepsilon}_j)\right) + \sum_{j=1}^{n}\sum_{k=1}^{n}\left(k_{jk}^p(1-\eta_{jk}) + k_{jk}^d(1-\tilde{\varepsilon}_{jk})\right) \tag{29}$$

Note that: $2(1-\tilde{\eta}_j) = \tilde{q}_j^T\tilde{q}_j + (1-\tilde{\eta}_j)^2$, and this relation is valid for the elements of $\tilde{\mathbf{q}}_{jk}$, $\tilde{\mathbf{p}}_j$ and $\tilde{\mathbf{p}}_{jk}$. The time derivative of V evaluated along the closed loop dynamics of the j^{th} spacecraft, with (8), (18) and (24), is given by

$$\dot{V} = \sum_{j=1}^{n}\alpha_{2j}\tilde{p}_j^T(\tilde{\omega}_j + R(\tilde{\mathbf{q}}_j)\omega_d - R(\tilde{\mathbf{p}}_j)\beta_j) - \sum_{j=1}^{n}\alpha_{2j}\tilde{\omega}_j^T\tilde{p}_j - \sum_{j=1}^{n}\sum_{k=1}^{n}\left(\tilde{\omega}_j^T(k_{jk}^p q_{jk} + k_{jk}^d\tilde{p}_{jk}) - \frac{1}{2}k_{jk}^p q_{jk}^T\omega_{jk} - \frac{1}{2}k_{jk}^d\tilde{p}_{jk}^T\Omega_{jk}\right) \tag{30}$$

Using the fact that spacecraft are required to align their attitudes to the same desired angular velocity, the following equations can be derived easily:

$$\mathbf{q}_{jk} = \tilde{\mathbf{q}}_k^{-1} \odot \tilde{\mathbf{q}}_j, \quad \omega_{jk} = \tilde{\omega}_j - R(\mathbf{q}_{jk})\tilde{\omega}_k, \tag{31}$$

Exploiting equations (14), (25) and (31), we can show that

$$\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p q_{jk}^T\omega_{jk} = \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p\tilde{\omega}_j^T q_{jk} \tag{32}$$

Similarly, using the expression of Ω_{jk}, given in (21), with (8), (18), (22), (23), and (25) we get

$$\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\tilde{p}_{jk}^T\Omega_{jk} = \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\tilde{p}_{jk}^T\Omega_j = \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\tilde{p}_{jk}^T\left(\tilde{\omega}_j + R(\tilde{\mathbf{q}}_j)\omega_d - R(\tilde{\mathbf{p}}_j)\beta_j\right) \tag{33}$$

Then, from equations (30)-(33) and using the fact that $q^T R(\mathbf{q}) = q^T$ for any quaternion $\mathbf{q} = (q^T, \eta)^T$, we obtain

$$\dot{V} = -\sum_{j=1}^{n}\mathscr{X}_j^T\left(R(\tilde{\mathbf{p}}_j)\beta_j - R(\tilde{\mathbf{q}}_j)\omega_d\right) \tag{34}$$

Then choosing the auxiliary systems' inputs as given in (26), we obtain

$$\dot{V} = -\sum_{j=1}^{n}\mathscr{X}_j^T\Gamma_j\mathscr{X}_j \tag{35}$$

and we conclude that $V(t) \leq V(0)$, and $\tilde{\mathbf{q}}_j$, $\tilde{\mathbf{p}}_j$, $\tilde{\omega}_j$, $\mathbf{q}_{jk}$ and $\tilde{\mathbf{p}}_{jk}$ are bounded. In addition, we can verify that $\dot{\tilde{p}}_j$ and $\dot{\tilde{p}}_{jk}$ are bounded, and so is $\ddot{V}$. Hence, invoking Barbalat's lemma, [20], we can conclude that $\mathscr{X}_j \to 0$ as $t \to \infty$, for $j \in \{1,...,n\}$.

Using a similar procedure as in [1], the set of equations: $\mathscr{X}_j = 0$ for $j \in \{1,...,n\}$ can be rewritten in a matrix form, using the Kronecker product $\otimes$, as

$$(G(t) \otimes I_3)\tilde{P} = 0 \tag{36}$$

where $\tilde{P} \in \mathbb{R}^{3n}$ is the column vector composed of all the vectors $\tilde{p}_j$, for $j \in \{1,...,n\}$, and the matrix $G(t) = [G_{jk}(t)] \in \mathbb{R}^{n\times n}$ is given by: $G_{jj}(t) = \alpha_{2j} + \sum_{k=1}^{n} k_{jk}^{d}\tilde{\varepsilon}_k$, and $G_{jk}(t) = -k_{jk}^{d}\tilde{\varepsilon}_j$, and we can verify that if the second condition in (28) is satisfied, then $G(t)$ will be strictly diagonally dominant, and consequently, $\tilde{p}_j = 0$ for $j \in \{1,...,n\}$ is the only solution to (36). Hence, we can conclude that $\tilde{p}_j \to 0$ and $\tilde{\varepsilon}_j \to \pm 1$, and $R(\tilde{\mathbf{p}}_j) \to I_3$. In addition, we can see that $\beta_j \to R(\tilde{\mathbf{p}}_j)^T R(\tilde{\mathbf{q}}_j)\omega_d$ and $\Omega_j \to \tilde{\omega}_j$.

Now, since $\dot{\omega}_d$ is bounded, one can show that $\ddot{\tilde{\mathbf{p}}}_j$ is bounded, and since we have already shown that $\tilde{\mathbf{p}}_j \to (0,0,0,\pm 1)^T$, by Barballat's lemma we have $\dot{\tilde{\mathbf{p}}}_j \to 0$, and from equations (17) we can conclude that $\Omega_j \to 0$, and consequently, $\tilde{\omega}_j \to 0$. Furthermore, one can easily verify that $\ddot{\tilde{\omega}}_j$ is bounded since $\ddot{\omega}_d$ is bounded, and so, invoking barballat's lemma, we conclude that $\dot{\tilde{\omega}}_j \to 0$.

Using the above concluding results, the closed loop dynamics (9), with (24), reduces to

$$\alpha_{1j}\tilde{q}_j + \sum_{k=1}^{n} k_{jk}^{p}\, q_{jk} = 0, \quad \text{for } j \in \{1,...,n\} \tag{37}$$

which, following the same steps as in [1], can be rewritten in matrix form as

$$(M(t) \otimes I_3)\tilde{Q} = 0 \tag{38}$$

where $\tilde{Q} \in \mathbb{R}^{3n}$ is the column vector composed of all the vectors $\tilde{q}_j$, for $j \in \{1,...,n\}$, and the matrix $M(t) = [m_{jk}(t)] \in \mathbb{R}^{n\times n}$ is given by: $m_{jj}(t) = \alpha_{1j} + \sum_{k=1}^{n} k_{jk}^{p}\tilde{\eta}_k$, and $m_{jk}(t) = -k_{jk}^{p}\tilde{\eta}_j$, where $M(t)$ is strictly diagonally dominant if the first condition in (28) is satisfied, [1], and we can conclude that $\tilde{q}_j = 0$ for $j \in \{1,...,n\}$ is the only solution to (38).

Finally, we can conclude that $\tilde{q}_j \to 0$ and $\tilde{\eta}_j \to \pm 1$, or equivalently $q_j \to q_k \to q_d$. Moreover, since $\tilde{\omega}_j \to 0$, $R(\tilde{\mathbf{q}}_j) \to I_3$ and $R(\mathbf{q}_{jk}) \to I_3$, we conclude that $\omega_j \to \omega_k \to \omega_d(t)$, $\forall j,k \in \{1,...,n\}$.

Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$ and $\tilde{\varepsilon}_j(t) > 0$, for all $t \geq T$ and $j \in \{1,...,n\}$, Matrices $G(t)$ and $M(t)$ are strictly diagonally dominant without condition (28), and the same convergence results hold. ■

In order to implement the proposed control scheme, neighboring spacecraft must communicate their absolute attitudes $\mathbf{q}_j$ and the output of their individual auxiliary systems $\tilde{\mathbf{p}}_j$. This does not increase the communication requirements as compared to the full information case, where both attitudes and angular velocities are communicated.

In our previous work [1], we have obtained similar results with the same communication flow requirement, where several auxiliary systems were introduced for each spacecraft and for each pair of communicating spacecraft, making the order of the controller of each spacecraft proportional to the number of its neighbors. The main contribution in this part is that the same convergence results are obtained with the introduction of a single dynamical auxiliary system for each spacecraft, reducing considerably the order of the controller as compared to the results in [1].

Remark 1: It is important to note that the control scheme presented in (24) consist of pure unit-quaternion feedback terms, and terms depending on the desired angular velocity, its derivative and the inertia matrix. Consequently, the control effort is bounded (regardless of the angular velocities) as follows: $\|\tau_j\| \leq \|I_{f_j}\|(\vartheta + \rho^2) + \alpha_{1j} + \alpha_{2j} + \sum_{k=1}^{n}(k_{jk}^{p} + k_{jk}^{d})$, with ϑ and ρ are the upper bounds of $\dot{\omega}_d(t)$ and $\omega_d(t)$ respectively.

IV. ATTITUDE ALIGNMENT WITHOUT REFERENCE TRAJECTORY

In this section, we consider the case where it is required to synchronize a group of spacecraft to reach an agreement on the final attitude without velocity measurements. We assume that no desired reference trajectory is assigned, and spacecraft are required to converge to the same (not necessarily constant) angular velocity while maintaining the same attitudes during formation maneuvers, i.e., $q_j \to q_k$ and $\omega_j \to \omega_k$. We assume that the communication between spacecraft is fixed and bidirectional and the spacecraft angular velocities are not available.

Using the definition of Ω_j in (18), and exploiting the cross product properties, we can easily verify that

$$\Omega_j^T I_{f_j}\dot{\Omega}_j = \Omega_j^T\left(\tau_j - \mathbf{H}(\beta_j, \dot{\beta}_j, \tilde{\mathbf{p}}_j)\right) \tag{39}$$

with

$$\mathbf{H}(\cdot) = I_{f_j}R(\tilde{\mathbf{p}}_j)\dot{\beta}_j + \mathbf{S}(R(\tilde{\mathbf{p}}_j)\beta_j)I_{f_j}R(\tilde{\mathbf{p}}_j)\beta_j \tag{40}$$

Then, we consider the following control action

$$\tau_j = \mathbf{H}(\beta_j, \dot{\beta}_j, \tilde{\mathbf{p}}_j) - \sum_{k=1}^{n} k_{jk}^{p} q_{jk}, \tag{41}$$

with the formation-keeping gains defined as in (25).

In the sequel analysis, we describe the communication flow between spacecraft using undirected graphs. Then, the information flow between spacecraft can be described by the undirected graph $\mathscr{G}_1 = (\mathscr{N}, \mathscr{E}, \mathscr{K}_p)$. $\mathscr{N} = \{1,...,n\}$ is the set of nodes or vertices, describing the set of spacecraft in the formation, $\mathscr{E}$ is the set of unordered pairs of nodes, called edges. An edge (j,k) indicates that spacecraft j and k are neighbors and can obtain information from one another. $\mathscr{K}_p$ is the set of weights, k_{jk}^{p}, associated to the links in the graph. For more details on graph properties, the reader is referred to [18].

Before we state our result in this section, we give a result from [2] in the following Lemma.

Lemma 1: Consider the relative attitude tracking error vectors q_{jk}. If the communication graph is a tree[1], then the only solution to the set of equations

$$\sum_{k=1}^{n} k_{jk}^{p} q_{jk} = 0, \quad \text{for} \quad j \in \{1,...,n\} \tag{42}$$

is $q_{jk} = 0$ for $j,k \in \{1,...,n\}$, where k_{jk}^{p} are defined as in (25). Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$, (or $\tilde{\eta}_j(t) < 0$), for all $t \geq T$ and $j \in \{1,...,n\}$, then $q_{jk} = 0$ for $j,k \in \{1,...,n\}$ is the only solution to (42) for any connected undirected graph.

Proof: See [2] ■

Theorem 2: Consider the formation given in (1)-(2) under the control law (41), with restrictions (25), and let the inputs of the auxiliary systems (15) be defined as

$$\dot{\beta}_j = -\Gamma_j \beta_j - R(\tilde{\mathbf{p}}_j)^T \sum_{k=1}^{n} k_{jk}^{p} q_{jk}, \quad \Gamma_j = \Gamma_j^T > 0 \tag{43}$$

If the information flow graph is a tree, then all the signals are bounded and $q_j \to q_k$ and $\omega_j \to \omega_k$ asymptotically, for all $j,k = 1,...,n$. Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$, (or $\tilde{\eta}_j(t) < 0$), for all $t \geq T$ and $j \in \{1,...,n\}$, then the above result holds for any connected undirected graph.

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2} \sum_{j=1}^{n} (\Omega_j^T I_{f_j} \Omega_j + \beta_j^T \beta_j) + \sum_{j=1}^{n} \sum_{k=1}^{n} k_{jk}^{p} (1 - \eta_{jk}) \tag{44}$$

The time-derivative of V in (44) evaluated along the systems dynamics (1) with (41) is given by

$$\dot{V} = \sum_{j=1}^{n} \sum_{k=1}^{n} \left(-k_{jk}^{p} \Omega_j^T q_{jk} + \frac{1}{2} k_{jk}^{p} q_{jk}^T \omega_{jk} \right) + \sum_{j=1}^{n} \beta_j^T \dot{\beta}_j \tag{45}$$

Using the relation,

$$\frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} k_{jk}^{p} q_{jk}^T \omega_{jk} = \sum_{j=1}^{n} \sum_{k=1}^{n} k_{jk}^{p} q_{jk}^T (\Omega_j + R(\tilde{\mathbf{p}}_j) \beta_j) \tag{46}$$

We obtain

$$\dot{V} = \sum_{j=1}^{n} \beta_j^T \dot{\beta}_j + \sum_{j=1}^{n} \sum_{k=1}^{n} k_{jk}^{p} q_{jk}^T R(\tilde{\mathbf{p}}_j) \beta_j = -\sum_{j=1}^{n} \beta_j^T \Gamma_j \beta_j \tag{47}$$

where we have used (43) to obtain the final result. We can see that $\dot{V}$ is negative semi-definite, and hence Ω_j, q_{jk} and β_j are bounded, and consequently ω_j is bounded. Invoking LaSalle's theorem, [20], we can show that $\beta_j \to 0$ and $\dot{\beta}_j \to 0$, and hence

$$\sum_{k=1}^{n} k_{jk}^{p} q_{jk} = 0, \quad \text{for} \quad j \in \{1,...,n\} \tag{48}$$

Then using the result in Lemma 1, we conclude that $q_j \to q_k$ and $\omega_j \to \omega_k$ for all $j,k \in \{1,...,n\}$, and this ends the proof. ■

Remark 2: In [2], we have proposed a solution to the above problem based on the introduction of several additional auxiliary systems for each spacecraft. The proposed scheme in this part considerably improves our previous results in that it requires a single auxiliary system for each spacecraft in the team, and reduces considerably the communication requirements between spacecraft since only spacraft absolute attitudes are transmitted between neighbors. However, it is important to mention that in the above control scheme, the inertia matrix is used in the control law to solve the problem, which was not a requirement in [2].

[1] an undirected graph is a tree if there is a path between any two distinct nodes on the graph, and it contains no cycles, [18]

V. SIMULATION RESULTS

TABLE I

SIMULATION PARAMETERS

$\mathbf{q}_1^T(0) = (0,0,1,0)$, $\mathbf{q}_2^T(0) = (1,0,0,0)$, $\mathbf{q}_3^T(0) = (0,1,0,0)$,
$\mathbf{q}_4^T(0) = (0,0,sin(-\pi/4),\cos(-\pi/4))$, $\omega_1^T(0) = (-0.5,0.5,-0.45)$,
$\omega_2^T(0) = (0.5,-0.3,0.1)$, $\omega_3^T(0) = (0.1,0.6,-0.1)$,
$\omega_4^T(0) = (0.4,0.4,-0.5)$, $\Gamma_j = 0.06I_3$, $\mathbf{p}_j^T(0) = (1,0,0,0)$
Theorem1: $\alpha_{1j} = 70,\ \alpha_{2j} = 90,\ k_{jk}^{p} = 5,\ \ k_{jk}^{d} = 5,\ for\ j,k \in \mathscr{E}_1$.
Theorem2: $\beta_j(0) = (0.1,0.1,0.1)$, $k_{jk}^{p} = 25,\ for\ j,k \in \mathscr{E}_2$

Using SIMULINK, we consider a scenario where four spacecraft are required to align their attitudes under a bidirectional communication flow graph satisfying the conditions in Theorems 1 and 2. The spacecraft are modeled as rigid bodies whose inertia matrices are taken as $I_{f_j} = diag(20,20,30)$. The simulation parameters are illustrated in table I, with $\mathscr{E}_1 = \{(1,2),(1,3),(1,4),(2,3)\}$, and $\mathscr{E}_2 = \{(1,2),(2,3),(1,4)\}$.

The obtained results are illustrated in Figures (1)-(4). Figure 1 shows the components of the unit quaternion, $\mathbf{q}_j^i$, $i = 1,...4$, representing the attitude of the four spacecraft in the formation (we use the superscript (i) to denote the i^{th} component of a vector, and "$j = d$" stands for the desired trajectory), where we consider the desired reference trajectory defined by $\omega^d(t) = 0.1sin(0.1\pi t)(1,1,1)^T$ and $\bar{q}^d(0) = (0,0,0,1)^T$. Note that all spacecraft converge to the same desired attitude. In Figure 2 we illustrate the elements of spacecraft angular velocity vectors, from which the convergence to zero is clear. Figures 3 and 4 illustrate the obtained results in the synchronization problem without reference trajectory, where we can see that spacecraft reach an agreement and converge to the same final time varying attitude and angular velocity.

VI. CONCLUSION

We considered the quaternion-based attitude synchronization problem of a group of spacecraft without velocity measurements, under an undirected communication graph. Instrumental in our approach, the introduction of the so-called "auxiliary systems" playing the role of velocity observers allowing to generate the necessary damping in the absence of the actual spacecraft angular velocities and relative angular velocities. We improved our previous results in [1]-[2], and showed that only one auxiliary system for each spacecraft is capable to generate the necessary damping that would have been generated when the angular velocities and relative angular velocities are available for feedback, and the same

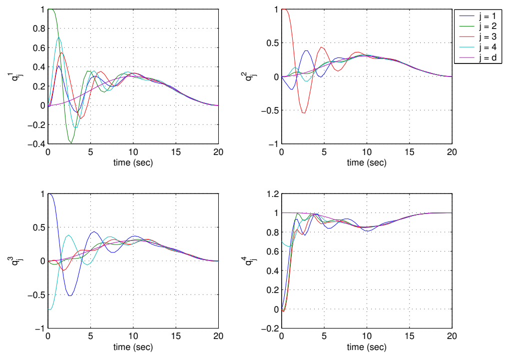

Fig. 1. Spacecraft attitudes in case of Theorem 1

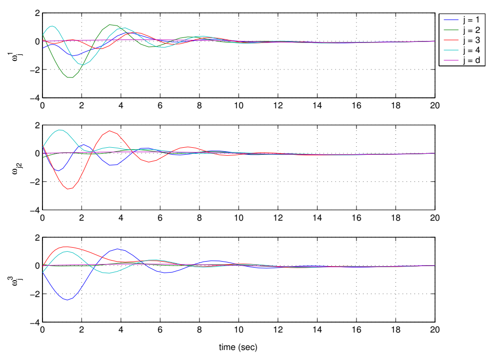

Fig. 2. The three elements of spacecraft angular velocities in case of Theorem 1

stability results are obtained. Simulation results have shown the effectiveness of the proposed control schemes.

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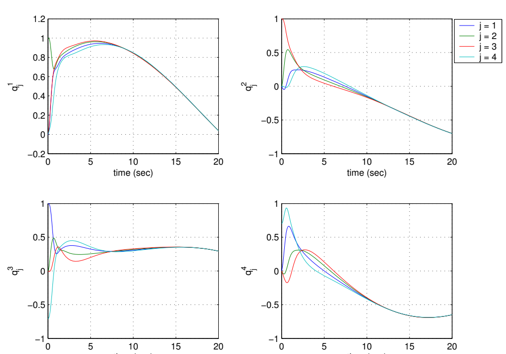

Fig. 3. Spacecraft attitudes in case of Theorem 2

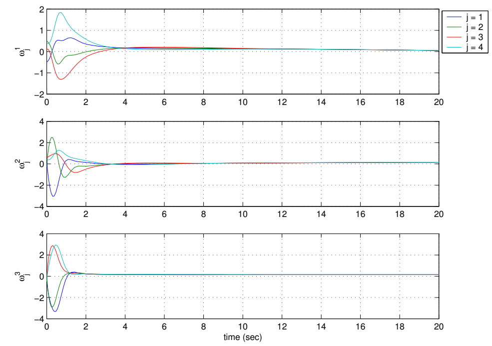

Fig. 4. The three elements of spacecraft angular velocities in case of Theorem 2

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