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Cascaded Iterative Learning Control for a Class of Uncertain Time-Varying Nonlinear Systems

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Abstract— We propose a simple recursive iterative learning control (ILC) scheme for a class of uncertain time-varying nonlinear systems. The proposed constructive backstepping-like procedure provides, in a systematic way, a control Lyapunov-like function leading to a simple recursive ILC scheme guaranteeing the existence of a finite time-interval on which a perfect tracking can be achieved when the number of iteration goes to infinity. Simulation results are also provided to show the effectiveness of the proposed ILC scheme.

I. Introduction

Iterative learning control is a technique that aims to generate, in an iterative manner, an adequate control input leading to a 'perfect' tracking over a finite time-interval for systems performing repetitive tasks (See, for instance, [2], [3], [5], [14], [15], [22]). Several ILC schemes for nonlinear systems have been proposed in the literature (see, for instance, the following non-exhaustive list of references and the references therein [1], [6], [7], [8], [10], [12], [17], [18], [19], [20], [21], [23], [24], [25]). They are generally based upon 1) the global Lipschitz condition assumption and the use of the λ-norm, or 2) the use of some structural assumptions as well as a partial knowledge of the system dynamics, restricting, thereby, the class of systems considered, or 3) the description of a given nonlinear system by a set of blended linear models and a validity function providing a time-varying weight (or probability) for each model according to the region of operation of the nonlinear system [19].

The design of ILC schemes is generally performed within two major frameworks: The contraction mapping framework and the Lyapunov framework. The contraction mapping approach, which was introduced in early 1980s for robot manipulators, is generally based on the use of the time-weighted norm (or λ-norm) and consists of adjusting the previous control input with an adequate correcting term depending, generally, on the current (or the previous) tracking error profile. In the mid 1990s, an adaptive ILC approach, based on the Lyapunov theory, has been introduced to overcome some of the limitations of the original approach (i.e., contraction mapping). This new design methodology, provided powerful tools to handle complex systems that were difficult to handle using the contraction mapping approach. In this new framework, the previous control input is adjusted indirectly through an adequate iterative adjustment of some parameters in the control law. As a natural consequence, this new framework has been very helpful in extending some of the results from standard nonlinear theory[1] to ILC design. The backstepping approach [13], in its simplest form, has been used to design an ILC scheme for a Ball and Beam system in [11]. It has also been applied in [9], [16], [21] for a class of uncertain nonlinear systems in strict-feedback form. However, it is well known that, despite its huge benefits, the backstepping approach, in general, leads to very complicated analytical expressions due to the successive derivatives of the virtual control inputs.

In the present paper, we propose a constructive backstepping-like design method leading to a simple ILC scheme for a quite large class of uncertain nonlinear systems. The analytical simplicity of the controller is achieved by avoiding direct cancelation of the successive derivatives of the virtual control inputs. In fact, those derivatives are considered as time-varying unknowns that can be handled by a set of recursive adaptive schemes. The proposed procedure provides, in a systematic way, a control Lyapunov-like function leading to a simple ILC scheme guaranteeing the existence of a finite time-interval on which a perfect tracking can be achieved when the number of iterations goes to infinity. Finally, we provide a numerical example to show the performance of our proposed controller.

II. Problem formulation

Let us consider the following nonlinear system

$$\begin{array}{rcl} \dot{x}_i(t) &=& x_{i+1}(t) + f_i(\bar{x}_i, t), \quad i = 1, \dots, n-1 \\ \dot{x}_n(t) &=& u + f_n(x, t) \\ y(t) &=& x_1(t), \end{array} \tag{1}$$

where $y \in \mathbb{R}$, $u \in \mathbb{R}$, $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ are, respectively, the system output, the control input and the state vector. The functions $f_i : \mathbb{R}^n \times [0, T] \mapsto \mathbb{R}$ are assumed to be well defined over $\mathbb{R}^n \times [0, T]$ and completely unknown. We assume that the system is in

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[1] By 'standard', we mean non-ILC techniques

triangular form, i.e., $\bar{x}_i = (x_1, \ldots, x_i)$, $i \in \{1, \ldots, n-1\}$. We also assume that the system under consideration is operated repeatedly over a finite time-interval $[0, T]$. Therefore, we introduce a subscript k to denote the operation or iteration number and rewrite system (2) as follows:

$$\begin{array}{rcl} \dot{x}_{i,k}(t) &=& x_{i+1,k}(t) + f_i(\bar{x}_{i,k}, t), \quad i = 1, \ldots, n-1 \\ \dot{x}_{n,k}(t) &=& u_k + f_n(x_k, t) \\ y_k(t) &=& x_{1,k}(t), \end{array} \tag{2}$$

where $x_k = (x_{1,k}, \ldots, x_{n,k})^T$ and $\bar{x}_{i,k} = (x_{1,k}, \ldots, x_{i,k})^T$. Our objective is to design an ILC scheme $u_k(t)$ guaranteeing the boundedness of the state variables, over the finite time-interval $[0, T]$, for all k, as well as the convergence of the system output $y_k(t)$ to the desired reference trajectory $y_d(t) \in \mathcal{C}^n_{[0,T]}$, for all $t \in [0, T]$, when k tends to infinity.

III. Constructive ILC Design

We propose the following recursive ILC scheme:

$$\begin{array}{rcl} \alpha_{i,k}(t) &=& k_i z_{i,k}(t) + s_{i-1,k}(t) + \mathbf{sat}\left(\frac{z_{i,k}(t)}{\phi_i(t)}\right)\theta_{i,k}(t) \\ \theta_{i,k}(t) &=& \theta_{i,k-1}(t) + \bar{k}_i |s_{i,k}(t)|, \end{array} \tag{3}$$

for $i = 1, \ldots, n$, with $s_{0,k}(t) = 0$, and

$$u_k(t) = \left\{ \begin{array}{l} \alpha_{n,k}(t) \text{ if } \sup\limits_{i \in \{1,..,n\}, t \in [0,T]} |s_{i,k-1}(t)| > \sigma \text{ or } k = 0 \\ u_{k-1}(t) \text{ if } \sup\limits_{i \in \{1,..,n\}, t \in [0,T]} |s_{i,k-1}(t)| \leq \sigma \text{ and } k \geq 1 \end{array} \right. \tag{4}$$

where $\theta_{i,-1}(t) = 0$, for $i = 1, \ldots, n$, $z_{1,k} = y_d - x_{1,k}$, $z_{i,k} = \alpha_{i-1,k} - x_{i,k}$, for $i = 2, \ldots, n$. The signals $s_{i,k}(t)$ are generated by

$$s_{i,k}(t) = z_{i,k}(t) - \phi_i(t)\mathbf{sat}(z_{i,k}(t)/\phi_i(t)), \tag{5}$$

with $\phi_i(t) = \epsilon_i e^{-k_i t}$, $\epsilon_i \geq |z_{i,k}(0)|$, $k_i > 0$ and $\bar{k}_i > 0$. The function **sat** is a saturation defined as follows:

$$\mathbf{sat}(z_{i,k}(t)/\phi_i(t)) = \left\{ \begin{array}{ccl} 1 & \text{if} & z_{i,k}(t) > \phi_i(t) \\ z_{i,k}(t)/\phi_i(t) & \text{if} & |z_{i,k}(t)| \leq \phi_i(t) \\ -1 & \text{if} & z_{i,k}(t) < -\phi_i(t) \end{array} \right. \tag{6}$$

where $\phi_i(t)$ is the width of the boundary layer which is time-varying but iteration-invariant. It is worth noting that $s_{i,k}(0) = 0$ and $s_{i,k}(t)\mathbf{sat}(z_{i,k}(t)/\phi_i(t)) = |s_{i,k}(t)|$ for all $i \in \{1, \ldots, n\}$ and $k \in \mathbb{Z}_+$. The saturation function has been introduced for two reasons: 1) to smooth out the virtual control inputs $\alpha_{i,k}$ by avoiding the use of the sign function, which helps in bounding the unknown terms including the derivatives of the virtual control inputs at each step of the backstepping procedure; 2) to relax the reinitialization assumption [7], which is a commonly used in most existing ILC schemes.

The control law is derived as follows:

We assume that $\sup\limits_{i \in \{1,..,n\}, t \in [0,T]} |s_{i,k-1}(t)| > \sigma$ and we apply the following steps:

Step 1: Let us consider the first equation of (2), i.e., for $i = 1$, where $x_{2,k}$ is viewed as a virtual control input. Denoting the tracking error by $z_{1,k} = y_d - x_{1,k}$, we have

$$\dot{z}_{1,k} = -x_{2,k} - f_1(\bar{x}_{1,k}, t) + \dot{y}_d. \tag{7}$$

Let us consider the following Lyapunov-like functional

$$V_{1,k} = \frac{1}{2}s^2_{1,k} + \frac{1}{2\bar{k}_1}\int_0^t \tilde{\theta}^2_{1,k} d\tau, \tag{8}$$

where $\tilde{\theta}_{1,k}(t) = \theta^*_1(t) - \theta_{1,k}(t)$, and $\theta^*_1(t) \in \mathcal{C}^0_{[0,T_1]}$ is such that there exists[2] a finite time $T_1 > 0$ such that

$$\begin{array}{rcl} |-f_1(\bar{x}_{1,k}, t) + \dot{y}_d(t)| &\leq& |-f_1(\bar{x}_{1,k}, t) + \dot{y}_d(t)| + \phi_2(t) \\ &\leq& \theta^*_1(t), \quad \forall t \in [0, T_1], \ \forall k \in \mathbb{Z}_+ \end{array} \tag{9}$$

From (8), we have

$$\begin{array}{rcl} \Delta V_{1,k} &=& V_{1,k} - V_{1,k-1} \\ &=& \frac{1}{2}s^2_{1,k}(0) + \int_0^t \frac{d}{d\tau}\left(\frac{1}{2}s^2_{1,k}\right) d\tau - \frac{1}{2}s^2_{1,k-1} \\ &-& \frac{1}{2\bar{k}_1}\int_0^t \bar{\theta}^2_{1,k} d\tau - \frac{1}{\bar{k}_1}\int_0^t \tilde{\theta}_{1,k}\bar{\theta}_{1,k} d\tau \\ &=& -\frac{1}{2}s^2_{1,k-1} - \frac{1}{2\bar{k}_1}\int_0^t \bar{\theta}^2_{1,k} d\tau \\ &+& \int_0^t \frac{d}{d\tau}\left(\frac{1}{2}s^2_{1,k}\right) d\tau - \frac{1}{\bar{k}_1}\int_0^t \tilde{\theta}_{1,k}\bar{\theta}_{1,k} d\tau \end{array} \tag{10}$$

where $s_{1,k}(0) = 0$ and $\bar{\theta}_{1,k} = \theta_{1,k} - \theta_{1,k-1}$ have been used. Now, let us evaluate $\frac{d}{dt}\left(\frac{1}{2}s^2_{1,k}\right)$.

$$\begin{array}{l} \frac{d}{dt}\left(\frac{1}{2}s^2_{1,k}\right) = s_{1,k}\dot{s}_{1,k} \\ = \left\{ \begin{array}{ccl} s_{1,k}(\dot{z}_{1,k} - \dot{\phi}_1) & \text{if} & z_{1,k}(t) > \phi_1(t) \\ 0 & \text{if} & |z_{1,k}(t)| \leq \phi_1(t) \\ s_{1,k}(\dot{z}_{1,k} + \dot{\phi}_1) & \text{if} & z_{1,k}(t) < -\phi_1(t) \end{array} \right. \\ = s_{1,k}(\dot{z}_{1,k} - \mathbf{sgn}(s_{1,k})\dot{\phi}_1) \\ = s_{1,k}(-x_{2,k} - f_1(\bar{x}_{1,k}, t) + \dot{y}_d - \mathbf{sgn}(s_{1,k})\dot{\phi}_1) \\ \leq s_{1,k}(-x_{2,k} + \mathbf{sgn}(s_{1,k})\theta^*_1 - \mathbf{sgn}(s_{1,k})\dot{\phi}_1). \end{array} \tag{11}$$

Using (11) in (10) and substituting $x_{2,k}$ by the virtual control $\alpha_{1,k}$ given in (3), we obtain

$$\begin{array}{rcl} \Delta V_{1,k} &\leq& -\frac{1}{2}s^2_{1,k-1} - \frac{1}{2\bar{k}_1}\int_0^t \bar{\theta}^2_{1,k} d\tau \\ &-& \frac{1}{\bar{k}_1}\int_0^t \tilde{\theta}_{1,k}\bar{\theta}_{1,k} d\tau \\ &+& \int_0^t s_{1,k}\left(-k_1 z_{1,k} - \mathbf{sat}(z_{1,k}(t)/\phi_1(t))\theta_{1,k}\right) d\tau \\ &+& \int_0^t s_{1,k}(\mathbf{sgn}(s_{1,k})\theta^*_1 - \mathbf{sgn}(s_{1,k})\dot{\phi}_1) d\tau. \end{array} \tag{12}$$

[2] The existence of such a finite time, in this step and all the following steps, will be discussed latter on.

Using the fact that

$$\begin{aligned} & s_{1,k}\left(-k_1 z_{1,k} - \mathbf{sgn}(s_{1,k})\dot{\phi}_1\right) \\ &= s_{1,k}\left(-k_1 s_{1,k} - k_1\phi_1 \mathbf{sat}(z_{1,k}(t)/\phi_1(t)) - \mathbf{sgn}(s_{1,k})\dot{\phi}_1\right) \\ &= -k_1 s_{1,k}^2 - |s_{1,k}|(k_1\phi_1 + \dot{\phi}_1) \\ &= -k_1 s_{1,k}^2, \end{aligned} \tag{13}$$

and using $\bar{\theta}_{1,k}$ given in (3), inequality (12) becomes

$$\begin{aligned} \Delta V_{1,k} &\le -\frac{1}{2}s_{1,k-1}^2 - \frac{1}{2\bar{k}_1}\int_0^t \bar{\theta}_{1,k}^2 d\tau \\ &- \frac{1}{\bar{k}_1}\int_0^t \tilde{\theta}_{1,k}\bar{k}_1|s_{1,k}|d\tau \\ &+ \int_0^t \tilde{\theta}_{1,k}|s_{1,k}|d\tau - \int_0^t k_1 s_{1,k}^2 d\tau \\ &= -\frac{1}{2}s_{1,k-1}^2 - \frac{1}{2\bar{k}_1}\int_0^t \bar{\theta}_{1,k}^2 d\tau - \int_0^t k_1 s_{1,k}^2 d\tau. \end{aligned} \tag{14}$$

Step 2: Now, let us introduce a new variable $z_{2,k} = \alpha_{1,k} - x_{2,k}$ leading to

$$\dot{z}_{2,k} = -x_{3,k} - f_2(\bar{x}_{2,k}, t) + \dot{\alpha}_{1,k}. \tag{15}$$

Let us consider the following Lyapunov-like functional

$$V_{2,k} = V_{1,k} + \frac{1}{2}s_{2,k}^2 + \frac{1}{2\bar{k}_2}\int_0^t \tilde{\theta}_{2,k}^2 d\tau, \tag{16}$$

where $\tilde{\theta}_{2,k}(t) = \theta_2^*(t) - \theta_{2,k}(t)$, and $\theta_2^*(t) \in \mathcal{C}^0_{[0,T_2]}$ is such that there exists a finite $T_2 > 0$ such that

$$\begin{aligned} & |-f_2(\bar{x}_{2,k}, t) + \dot{\alpha}_{1,k}(t)| \le |-f_2(\bar{x}_{2,k}, t) + \dot{\alpha}_{1,k}(t)| + \phi_3(t) \\ & \le \theta_2^*(t), \ \forall t \in [0, T_2], \ \forall k \in \mathbb{Z}_+ \end{aligned} \tag{17}$$

The difference of (16), in view of (15), is given by

$$\begin{aligned} & \Delta V_{2,k} = V_{2,k} - V_{2,k-1} \\ &= -\frac{1}{2}\sum_{i=1}^2 s_{i,k-1}^2 - \sum_{i=1}^2 \frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau \\ &- \sum_{i=1}^2 \frac{1}{\bar{k}_i}\int_0^t \tilde{\theta}_{i,k}\bar{\theta}_{i,k} d\tau \\ &+ \int_0^t s_{1,k}\left(-x_{2,k} - f_1(\bar{x}_{1,k}, t) + \dot{y}_d - \mathbf{sgn}(s_{1,k})\dot{\phi}_1\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-x_{3,k} - f_2(\bar{x}_{2,k}, t) + \dot{\alpha}_{1,k} - \mathbf{sgn}(s_{2,k})\dot{\phi}_2\right) d\tau \end{aligned} \tag{18}$$

Substituting $x_{3,k}$ by the virtual control $\alpha_{2,k}$ given in (3) and $x_{2,k}$ by $\alpha_{1,k} - z_{2,k}$, we obtain

$$\begin{aligned} & \Delta V_{2,k} \le -\frac{1}{2}\sum_{i=1}^2 s_{i,k-1}^2 - \sum_{i=1}^2 \frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau \\ &- \sum_{i=1}^2 \frac{1}{\bar{k}_i}\int_0^t \tilde{\theta}_{i,k}\bar{\theta}_{i,k} d\tau \\ &+ \int_0^t s_{1,k}\left(-k_1 z_{1,k} - \mathbf{sgn}(s_{1,k})\dot{\phi}_1\right) d\tau \\ &+ \int_0^t s_{1,k}\left(z_{2,k} - \mathbf{sat}(z_{1,k}/\phi_1)\theta_{1,k} - f_1(\bar{x}_{1,k}, t) + \dot{y}_d\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-k_2 z_{2,k} - \mathbf{sgn}(s_{2,k})\dot{\phi}_2\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-s_{1,k} - \mathbf{sat}(z_{2,k}/\phi_2)\theta_{2,k} - f_2(\bar{x}_{2,k}, t) + \dot{\alpha}_{1,k}\right) d\tau. \end{aligned} \tag{19}$$

Since $s_{1,k}z_{2,k} - s_{2,k}s_{1,k} = s_{1,k}(s_{2,k} + \phi_2\mathbf{sat}(z_{2,k}/\phi_2)) - s_{2,k}s_{1,k} = s_{1,k}\phi_2\mathbf{sat}(z_{2,k}/\phi_2)$, equation (19) becomes

$$\begin{aligned} & \Delta V_{2,k} \le -\frac{1}{2}\sum_{i=1}^2 s_{i,k-1}^2 - \sum_{i=1}^2 \frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau \\ &- \sum_{i=1}^2 \frac{1}{\bar{k}_i}\int_0^t \tilde{\theta}_{i,k}\bar{\theta}_{i,k} d\tau \\ &+ \int_0^t s_{1,k}\left(-k_1 z_{1,k} - \mathbf{sgn}(s_{1,k})\dot{\phi}_1\right) d\tau \\ &+ \int_0^t s_{1,k}\left(-\mathbf{sat}(z_{1,k}/\phi_1)\theta_{1,k})\right) d\tau \\ &+ \int_0^t s_{1,k}\left(-f_1(\bar{x}_{1,k}, t) + \dot{y}_d + \phi_2\mathbf{sat}(z_{2,k}/\phi_2)\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-k_2 z_{2,k} - \mathbf{sgn}(s_{2,k})\dot{\phi}_2\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-\mathbf{sat}(z_{2,k}/\phi_2)\theta_{2,k} - f_2(\bar{x}_{2,k}, t) + \dot{\alpha}_{1,k}\right) d\tau \\ &\le -\frac{1}{2}\sum_{i=1}^2 s_{i,k-1}^2 - \sum_{i=1}^2 \frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau \\ &- \sum_{i=1}^2 \frac{1}{\bar{k}_i}\int_0^t \tilde{\theta}_{i,k}\bar{\theta}_{i,k} d\tau \\ &+ \int_0^t s_{1,k}\left(-k_1 z_{1,k} - \mathbf{sgn}(s_{1,k})\dot{\phi}_1\right) d\tau \\ &+ \int_0^t s_{1,k}\left(-\mathbf{sat}(z_{1,k}/\phi_1)\theta_{1,k} + \mathbf{sgn}(s_{1,k})\theta_1^*\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-k_2 z_{2,k} - \mathbf{sgn}(s_{2,k})\dot{\phi}_2\right) d\tau \\ &+ \int_0^t s_{2,k}\left(-\mathbf{sat}(z_{2,k}/\phi_2)\theta_{2,k} + \mathbf{sgn}(s_{2,k})\theta_2^*\right) d\tau \end{aligned} \tag{20}$$

Following the same argument given in step1 and using $\bar{\theta}_{i,k}$, $i = 1, 2$, from (3), we obtain

$$\Delta V_{2,k} \le -\frac{1}{2}\sum_{i=1}^2 s_{i,k-1}^2 - \sum_{i=1}^2 \frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau - \sum_{i=1}^2 \int_0^t k_i s_{i,k}^2 d\tau. \tag{21}$$

Step $(n-1)$: In this step we introduce a new variable $z_{n-1,k} = \alpha_{n-2,k} - x_{n-1,k}$ leading to

$$\dot{z}_{n-1,k} = -x_{n,k} - f_{n-1}(\bar{x}_{n-1,k}, t) + \dot{\alpha}_{n-2,k}. \quad (22)$$

Now, let us consider the following Lyapunov-like functional

$$V_{n-1,k} = V_{n-2,k} + \frac{1}{2}s_{n-1,k}^2 + \frac{1}{2\bar{k}_{n-1}}\int_0^t \tilde{\theta}_{n-1,k}^2 d\tau, \quad (23)$$

with $\tilde{\theta}_{n-1,k}(t) = \theta_{n-1}^*(t) - \theta_{n-1,k}(t)$, where $\theta_{n-1}^*(t) \in \mathcal{C}^0_{[0,T_{n-1}]}$ is such that there exists a finite $T_{n-1} > 0$ such that

$$\begin{aligned} &|-f_{n-1}(\bar{x}_{n-1,k}, t) + \dot{\alpha}_{n-2,k}(t)| \leq \\ &|-f_{n-1}(\bar{x}_{n-1,k}, t) + \dot{\alpha}_{n-2,k}(t)| + \phi_n(t) \leq \theta_{n-1}^*(t), \\ &\forall t \in [0, T_{n-1}],\ \forall k \in \mathbb{Z}_+ \end{aligned} \quad (24)$$

Substituting $x_{n,k}$ by the virtual control $\alpha_{n-1,k}$ given in (3), one can show that

$$\begin{aligned} \Delta V_{n-1,k} \leq & -\frac{1}{2}\sum_{i=1}^{n-1} s_{i,k-1}^2 - \sum_{i=1}^{n-1}\frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau \\ & - \sum_{i=1}^{n-1}\int_0^t k_i s_{i,k}^2 d\tau. \end{aligned} \quad (25)$$

Step n: In this last step, we introduce a new variable $z_{n,k} = \alpha_{n-1,k} - x_{n,k}$ leading to

$$\dot{z}_{n,k} = -u_k - f_n(x_k, t) + \dot{\alpha}_{n-1,k}. \quad (26)$$

Let us consider the following Lyapunov-like functional

$$V_{n,k} = V_{n-1,k} + \frac{1}{2}s_{n,k}^2 + \frac{1}{2\bar{k}_n}\int_0^t \tilde{\theta}_{n,k}^2 d\tau, \quad (27)$$

with $\tilde{\theta}_{n,k}(t) = \theta_n^*(t) - \theta_{n,k}(t)$, where $\theta_n^*(t) \in \mathcal{C}^0_{[0,T_n]}$ is such that there exists a finite $T_n > 0$ such that

$$|-f_n(x_k, t) + \dot{\alpha}_{n-1,k}(t)| \leq \theta_n^*(t), \quad \forall t \in [0, T_n],\ \forall k \in \mathbb{Z}_+ \quad (28)$$

Substituting u_k by the control law given in (4), we obtain

$$\begin{aligned} \Delta V_{n,k} \leq & -\frac{1}{2}\sum_{i=1}^{n} s_{i,k-1}^2 - \sum_{i=1}^{n}\frac{1}{2\bar{k}_i}\int_0^t \bar{\theta}_{i,k}^2 d\tau \\ & - \sum_{i=1}^{n}\int_0^t k_i s_{i,k}^2 d\tau. \end{aligned} \quad (29)$$

Since $\Delta V_{n,k} \leq 0$, it is clear that $V_{n,k}(t)$ is non-increasing with respect to k and, hence, bounded if $V_{n,0}(t)$ is bounded. The boundedness of $V_{n,0}(t)$ can be shown as follows: For $k = 0$, we have

$$\dot{V}_{n,0} \leq -\sum_{i=1}^{n} k_i s_{i,0}^2 + \sum_{i=1}^{n}\left(\frac{1}{2\bar{k}_i}\tilde{\theta}_{i,0}^2 + |s_{i,0}|\tilde{\theta}_{i,0}\right). \quad (30)$$

Since $\theta_{i,-1} = 0$, we have $\theta_{i,0} = \bar{k}_i|s_{i,0}|$. Therefore, (30) leads to

$$\begin{aligned} \dot{V}_{n,0} \leq & -\sum_{i=1}^{n} k_i s_{i,0}^2 + \sum_{i=1}^{n}\frac{1}{2\bar{k}_i}(\tilde{\theta}_{i,0}^2 + 2\tilde{\theta}_{i,0}\theta_{i,0}) \\ = & -\sum_{i=1}^{n} k_i s_{i,0}^2 - \sum_{i=1}^{n}\frac{1}{2\bar{k}_i}\tilde{\theta}_{i,0}^2 + \sum_{i=1}^{n}\frac{1}{\bar{k}_i}\tilde{\theta}_{i,0}\theta_i^* \\ \leq & -\sum_{i=1}^{n} k_i s_{i,0}^2 - \sum_{i=1}^{n}\frac{1}{2\bar{k}_i}\tilde{\theta}_{i,0}^2 + \sum_{i=1}^{n}\frac{\kappa_i}{\bar{k}_i}\tilde{\theta}_{i,0}^2 \\ & + \sum_{i=1}^{n}\frac{1}{4\kappa_i\bar{k}_i}\theta_{i,0}^{*2} \\ = & -\sum_{i=1}^{n} k_i s_{i,0}^2 - \sum_{i=1}^{n}\rho_i\tilde{\theta}_{i,0}^2 + \sum_{i=1}^{n}\frac{1}{4\kappa_i\bar{k}_i}\theta_i^{*2} \\ \leq & \sum_{i=1}^{n}\frac{1}{4\kappa_i\bar{k}_i}\theta_i^{*2}, \end{aligned} \quad (31)$$

where $\rho_i = \frac{1}{2\bar{k}_i}(1 - 2\kappa_i)$, with $0 < \kappa_i < \frac{1}{2}$. Since $\theta_i^*(t)$ is continuous over $[0, T_i]$, it is clear that $V_{n,0}(t)$ is bounded over any finite time-interval $[0, T]$, where $T \leq T^\star = \min\{T_1, \ldots, T_n\}$. Therefore, $V_{n,k}(t)$ is bounded for all $t \in [0, T]$ and all $k \in \mathbb{Z}_+$. Consequently, all the state variables as well as $\int_0^t \tilde{\theta}_{i,k}^2 d\tau,\ i \in \{1, \ldots, n\}$, are bounded for all $t \in [0, T]$ and all $k \in \mathbb{Z}_+$. Finally, to show the convergence of $s_{i,k}(t), i \in \{1, \ldots, n\}$, let us rewrite $V_{n,k}$ as follows:

$$V_{n,k} = V_{n,0} + \sum_{j=1}^{k}\Delta V_{n,j} \leq V_{n,0} - \frac{1}{2}\sum_{j=1}^{k}\sum_{i=1}^{n} s_{i,j-1}^2, \quad (32)$$

which leads to

$$\sum_{j=1}^{k}\sum_{i=1}^{n} s_{i,j-1}^2 \leq 2(V_{n,0}(t) - V_{n,k}(t)) \leq 2V_{n,0}(t). \quad (33)$$

Since $V_{n,0}(t)$ is bounded for all $k \in \mathbb{Z}_+$ and $t \in [0, T]$, from (33) and (4), one can conclude that there exists a finite iteration number $k^\star$ such that $\sup_{t\in[0,T]} |s_{i,k}(t)| \leq \sigma$, $\forall k \geq k^\star, \forall i \in \{0, \ldots, n\}$. Consequently, $|z_{i,k}(t)| \leq \phi_i(t) + \sigma$, $\forall k \geq k^\star$, $\forall i \in \{0, \ldots, n\}$, $\forall t \in [0, T]$.

Finally, it is important to notice that, at each step i, we assumed that there exists a function $\theta_i^*(t) \in \mathcal{C}^0_{[0,T_i]}$ bounding the unknown terms $f_i(\bar{x}_{i,k}, t)$ as well as the terms that we don't want to cancel out, i.e., $\dot{\alpha}_i$ and the successive derivatives of the reference trajectory, over the finite time-interval $[0, T_i]$. Now, we will show the existence of $\theta_i^*(t) \in \mathcal{C}^0_{[0,T_i]}$ for any finite iteration number, if the control gains k_i are sufficiently large.

Let us consider the Lyapunov function V_n without the terms involving the parametric errors, i.e.,

$$W_{n,k} = \frac{1}{2}\sum_{i=1}^{n} s_{i,k}^2, \quad (34)$$

whose time-derivative, in view of (2), (3) and (4) and according to the previous developments, can be shown to be

$$\begin{aligned}
\dot{W}_{n,k} &= -\sum_{i=1}^{n} k_i s_{i,k}^2 + \sum_{i=1}^{n} s_{i,k}(-f_i(\bar{x}_{i,k},t) + \dot{\alpha}_{i-1,k}) \\
&- \sum_{i=1}^{n} s_{i,k}\mathbf{sat}(z_{i,k}/\phi_i)\theta_{i,k} \\
&+ \sum_{i=2}^{n} s_{i-1,k}\phi_i \mathbf{sat}(z_{i,k}/\phi_i) \\
&\le -\sum_{i=1}^{n} k_i s_{i,k}^2 + \sum_{i=1}^{n} s_{i,k}(-f_i(\bar{x}_{i,k},t) + \dot{\alpha}_{i-1,k}) \\
&- \sum_{i=1}^{n} |s_{i,k}|\theta_{i,k} + \sum_{i=2}^{n} \epsilon_i |s_{i-1,k}| \\
&\le -\sum_{i=1}^{n} k_i s_{i,k}^2 + \sum_{i=1}^{n} s_{i,k}(-f_i(\bar{x}_{i,k},t) + \dot{\alpha}_{i-1,k}) \\
&- \sum_{i=1}^{n} \bar{k}_i s_{i,k}^2 - \sum_{i=1}^{n}(\bar{k}_i \sum_{j=0}^{k-1} |s_{i,k}| s_{i,j}) + \sum_{i=2}^{n} \epsilon_i |s_{i-1,k}|
\end{aligned} \tag{35}$$

where $\dot{\alpha}_{0,k} \equiv \dot{y}_d$. Now, using Young's inequality, we have

$$\begin{aligned}
&\sum_{i=1}^{n} s_{i,k}(-f_i(\bar{x}_{i,k},t) + \dot{\alpha}_{i-1,k}) \le \beta \sum_{i=1}^{n} s_{i,k}^2 \\
&+ \tfrac{1}{4\beta} \sum_{i=1}^{n} (-f_i(\bar{x}_{i,k},t) + \dot{\alpha}_{i-1,k})^2,
\end{aligned} \tag{36}$$

and

$$\sum_{i=1}^{n}(\bar{k}_i \sum_{j=0}^{k-1} |s_{i,k}| s_{i,j}) \le \sum_{i=1}^{n} \beta \bar{k}_i s_{i,k}^2 + \sum_{i=1}^{n} \frac{\bar{k}_i}{4\beta} \left(\sum_{j=0}^{k-1} s_{i,j} \right)^2, \tag{37}$$

for any $\beta > 0$.

Now, using (36) and (37), inequality (35) leads to

$$\begin{aligned}
\dot{W}_{n,k} &\le -\sum_{i=1}^{n} (k_i - \bar{k}_i - \beta - \beta \bar{k}_i) s_{i,k}^2 \\
&+ \sum_{i=1}^{n} \frac{\bar{k}_i}{\beta} \left(\sum_{j=1}^{k-1} s_{i,j} \right)^2 + \sum_{i=2}^{n} \epsilon_i |s_{i-1,k}| \\
&+ \tfrac{1}{4\beta} \sum_{i=1}^{n} (-f_i(\bar{x}_{i,k},t) + \dot{\alpha}_{i-1,k})^2
\end{aligned} \tag{38}$$

which means that there exists a finite time interval $T^\star > 0$ over which (38) can be made negative (as long as $\sup_{i \in \{1,..,n\}, t \in [0,T]} |s_{i,k}(t)| > \sigma$) by picking β and k_i sufficiently large, with $k_i > \bar{k}_i + \beta + \beta \bar{k}_i$, for any finite iteration number k. Consequently, if k_i is sufficiently large, there exists a finite time interval $[0, T_i]$ over which $s_{i,k}(t)$ is bounded, which in turns guarantees the existence of $\theta_i^*(t) \in \mathcal{C}^0_{[0,T_i]}$ for any finite iteration number k.

Finally, one can summarize the previous development in the following theorem:

Theorem 1: Consider system (2), under the ILC scheme (3)-(4). For any $\sigma > 0$, there exist $k^\star$, $T^\star > 0$ and a sufficiently large $k_i > 0$, such that for all $0 \le T \le T^\star$, the following hold

(i) The state variables of the closed-loop system are bounded for all $t \in [0, T]$ and all k,

(ii) $|z_{i,k}(t)| \le \phi_i(t) + \sigma$, $\forall k \ge k^\star$, $\forall i \in \{1, \ldots, n\}$, $\forall t \in [0, T]$.

IV. Simulation results

We consider the following nonlinear system

$$\begin{aligned}
\dot{x}_{1,k} &= x_{2,k} + x_{1,k}^2 + t^2 \sin(x_{1,k}) \\
\dot{x}_{2,k} &= u_k + \cos(x_{2,k}) + x_{1,k}^2 + x_{2,k}^2 \\
y_k &= x_{1,k}.
\end{aligned} \tag{39}$$

The reference trajectory is taken as $y_d(t) = 1 - e^{-10t}$ over the finite time-interval $[0, 1]$ seconds. The initial conditions are taken equal to zero. Applying the control law (3)-(4) with $k_1 = k_2 = 10$, $\bar{k}_1 = \bar{k}_2 = 1$, $\epsilon_1 = \epsilon_2 = 5 \times 10^{-3}$ and $\sigma = 5 \times 10^{-2}$, we obtain the results shown in Fig. 1 and Fig. 2. In fact, Fig. 1 shows the evolution of the supremum norm of the tracking error function of the iteration number and Fig. 2 shows the time-evolution of the reference trajectory and the actual output at the first iteration ($k = 0$), the tenth iteration($k = 9$) and the 60th iteration ($k = 59$).

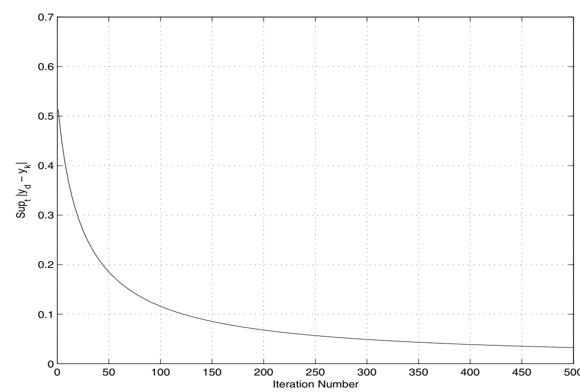

Fig. 1. Sup-norm of the tracking error versus the number of iterations

V. Conclusion

We proposed a simple recursive ILC scheme, composed of a cascade of n scalar adaptive iterative learning controllers, for a class of uncertain time-varying nonlinear systems. The only requirement on the n-dimensional nonlinear system is the explicit appearance of the state variable x_{i+1} in the $\dot{x}_i$-equation for all $i \in \{1, \ldots, n-1\}$. The nonlinear functions $f_i(\bar{x}_i, t)$, are assumed to be completely unknown. The proposed backstepping-like procedure provides, in a systematic way, a control Lyapunov-like function leading to a simple ILC scheme guaranteeing the existence of a finite time-interval on which the boundedness of the state variables of the closed-loop system as well as the convergence of the

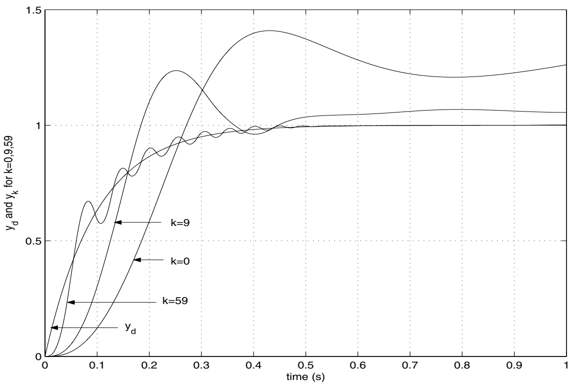

Fig. 2. Reference trajectory and actual output for $k = 0$, $k = 9$ and $k = 59$

tracking error to zero are guaranteed. The simplicity of the proposed controller is achieved by avoiding direct compensation of the successive derivatives (required in standard backstepping design) and considering them as time-varying unknowns to be handled along with the unknown functions $f_i(\bar{x}_i, t)$ through a cascade of n scalar adaptive ILC laws.

References

[1] H-S. Ahn, C-H. Choi and K-B. Kim, "Iterative learning control for a class of nonlinear systems,"Automatica, Vol.29, No. 6, pp. 1575-1578, 1993.

[2] S. Arimoto, Control theory of non-linear mechanical systems, Oxford Science Publications, Oxford, UK, 1996.

[3] Z. Bien and J-X. Xu, Iterative learning control: Analysis, Design, Integration and Applications, Kluwer Academic Publishers, 1998.

[4] W-J. Cao and J-X. Xu, "On functional approximation of the equivalent control using learning variable structure control," IEEE Transactions on Automatic Control, Vol. 47, No. 5, pp. 824-830, 2002.

[5] Y. Chen and C. Wen, Iterative learning control: Convergence, Robustness and Applications, Lecture notes in control and information sciences, Springer Verlag, 1999.

[6] Y. Chen, Z. Gong and C. Wen, "Analysis of a high-order iterative learning control algorithm for uncertain nonlinear systems with state delays,"Automatica, Vol. 34, No. 3, pp. 345-353, 1998.

[7] C-J. Chien and C-Y. Yao, "Iterative learning of model reference adaptive controller for uncertain nonlinear systems with only output measurement," Automatica, Vol. 40, No. 5, pp. 855 864, 2004.

[8] M. French and E. Rogers, "Nonlinear iterative learning by an adaptive Lyapunov technique," International Journal of Control, Vol. 73, No. 10, pp. 840-850, 2000.

[9] C. Ham, Z. Qu and J. Kaloust, "Nonlinear learning control for a class of nonlinear systems,"Automatica, Vol. 37, pp. 419-428, 2001.

[10] J. Hauser, "Learning control for a class of nonlinear systems," In Proc. of the 26th IEEE Conference on Decision and Control, pp. 859-860, 1987.

[11] H-K. Kim, D-H. Lee, T-Y. Kuc and T-C. Yi, "A backstepping design of adaptive robust learning controller for fast trajectory tracking of ball-beam dynamic systems," In Proc. of IEEE Int. Conf. on SMC, Vol. 3, pp. 2311-2314, 1996.

[12] T-Y. Kuc, J. S. Lee and K. Nam, "An iterative learning control theory for a class of nonlinear dynamic systems,"Automatica, Vol.28, No. 6, pp. 1215-1221, 1992.

[13] M. Krstic, I. Kanellakopoulos and P. Kokotovic, Nonlinear and adaptive control design, Wiley-Interscience Publication, New York, 1995.

[14] K.L. Moore, Iterative learning control for deterministic systems, Advances in Industrial Control, Springer Verlag, 1993.

[15] K.L. Moore, "Iterative Learning Control: An Expository Overview," Applied and Computational Controls, Signal Processing, and Circuits, Vol. 1, pp. 151-214, 1999.

[16] Z. Qu and J. Xu, "Asymptotic learning control for a class of cascaded nonlinear uncertain systems," IEEE Trans. Automat. Contr., Vol. 47, No. 8, pp. 1369-1376, 2002.

[17] T. Sugie and T. Ono, "An iterative learning control law for dynamical systems," Automatica, Vol. 27, No. 4, pp. 729-732, 1991.

[18] M. Sun and D. Wang, "Iterative learning control with initial rectifying action," Automatica, Vol. 38, No. 7, pp. 1177-1182, 2002.

[19] A. Tayebi and M.B. Zaremba, "Iterative learning control for nonlinear systems described by a blended multiple model representation," International Journal of Control, Vol. 75, No. 16/17, pp. 1376-1384, 2002.

[20] A. Tayebi and J.X. Xu, "Observer-based iterative learning control for a class of nonlinear systems," IEEE Transactions on Circuits and Systems—I: Fundamental Theory and Applications, Vol. 50, No. 3, pp. 452-455, 2003.

[21] Y-P. Tian and X. Yu, "Robust learning control for a class of nonlinear systems with periodic and aperiodic uncertainties," Automatica, Vol. 39, No. 11, pp. 1957-1966, 2003.

[22] J-X. Xu and Y. Tan, Linear and nonlinear iterative learning control, Lecture notes in control and information sciences, Springer Verlag, 2003.

[23] J-X. Xu and Y. Tan, "Adaptive robust iterative learning control with dead zone scheme," Automatica, Vol. 36, No. 1, pp. 91-99, 2000.

[24] J-X. Xu and Y. Tan, "A sub optimal learning control scheme for non-linear systems with time-varying uncertainties," Optimal Control, Applications and Methods, Vol. 22, pp. 111-126, 2001.

[25] J-X. Xu and Y. Tan, "A composite energy function based learning control approach for nonlinear systems with time-varying parametric uncertainties," IEEE Transactions on Automatic Control, Vol. 47, No. 11, pp. 1940-1945, 2002.