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Unit quaternion observer based attitude stabilization of a rigid spacecraft without velocity measurement

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Abstract— In this paper, we propose an alternative solution to the attitude stabilization problem without velocity measurement. Our approach consists of using a unit quaternion observer and a linear feedback control law in terms of the vector parts of the actual unit quaternion and the estimation-error quaternion. The closed loop system leads to a passive mapping between the observer input and the vector part of the estimation-error quaternion, which in turns allows to choose the observer input as a simple feedback in terms of the vector part of the estimation-error quaternion. The resulting control scheme, without velocity measurement and without the use of a lead filter, guarantees global asymptotic stability. Simulation results are provided to show the effectiveness of the proposed controller.

I. Introduction

The attitude control problem of a spacecraft, or a rigid body in space in general, has been extensively studied during the past four decades. This is a particularly interesting problem in dynamics since the angular velocity of the body cannot be integrated to obtain the attitude of the body [6]. From a practical point of view, the design of efficient and low-cost attitude controllers is an important issue which is of great interest for aerospace industry for instance. The attitude stabilization of a rigid body in space, using the unit quaternion an the angular velocity in the feedback control law, has been investigated by many researchers and a wide class of controllers has been proposed (see, for instance, [6], [12], [16], [17], and the list is not exhaustive). In [12], some quaternion based feedback controllers for the attitude stabilization have been proposed and tested experimentally on a quadrotor aircraft.

The attitude control of a rigid body in space with full states measurements (i.e., quaternion and angular velocity), being relatively well understood, the research has been directed towards other performance and implementation-cost optimization issues, by removing the requirement of the velocity measurement. The passivity property, was the main idea behind the design of the attitude controllers, without velocity measurement, in [4], [7], [14]. In fact, in [4], the authors used the passivity-based adaptive control approach for robotic manipulators to derive their adaptive attitude control scheme without velocity measurement. In [7], a quite similar passivity argument has been used to develop a velocity-measurement-free attitude stabilization controller using a lead filter. The results in [14] complements the work of [4], [7] by using the Rodrigues Parameters instead of the quaternion [11]. The second approach that has also been used to avoid the velocity measurement is based on the use of nonlinear observers. In fact, in [10], a nonlinear velocity observer, using just the torque and orientation measurements, has been proposed based on the analogy to second-order linear systems, where a separation principle-like property was conjectured. In, [13], based on the work of [10] and [15], an estimation algorithm for the constant gyro bias has been proposed. This algorithm, using the orientation and gyroscopic measurements, has been combined with the attitude control scheme proposed in [4]. In [8], a constant gyro bias estimator has been proposed directly in terms of the rotation matrix instead of unit quaternion, but the control design has not been investigated. In [2], two attitude tracking controllers without velocity measurement have been proposed. The first one is a locally exponentially stabilizing controller-observer scheme that requires the knowledge of the inertia matrix. The second scheme, generalizes the lead filter based regulation scheme of [7], to the attitude tracking control. Another alternative to the work of [2] has been proposed in [1] based on the results of [14] using the Rodrigues parameters instead of the unit quaternion.

In the present paper, we provide an alternative solution to the global attitude stabilization problem without velocity measurement. Our main idea is the introduction of a unit quaternion observer having the same structure as the actual unit quaternion-based attitude model. Under a linear feedback involving the vector parts of the actual quaternion and the estimation-error quaternion, we show that the map between the observer input and the vector part of the estimation-error quaternion is passive. Therefore, the observer input can be designed as a simple linear feedback in terms of the vector part of the estimation-error quaternion leading to global asymptotic attitude stabilization. Simulation results are also provided to support the theoretical developments.

II. Dynamical model and problem statement

The dynamical model of a spacecraft or a rigid body in space is given by

$$I_f \dot{\Omega} = -\Omega \times I_f \Omega + \tau, \tag{1}$$

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$$\dot{R} = RS(\Omega), \tag{2}$$

where Ω denotes the angular velocity of the body expressed in the body-fixed frame $\mathcal{A}$. The orientation of the rigid body is given by the orthogonal rotation matrix $R \in SO(3)$. $I_f \in \mathbb{R}^{3\times3}$ is a symmetric positive definite constant inertia matrix of the body with respect to the frame $\mathcal{A}$ whose origin is at the center of mass. The vector τ is the torque applied the rigid body, considered as the input vector. The matrix $S(\Omega)$ is a skew-symmetric matrix such that $S(\Omega)V = \Omega \times V$ for any vector $V \in \mathbb{R}^3$, where $\times$ denotes the vector cross-product.
Our objective is to design a feedback controller, without velocity measurement, for the stabilization of the equilibrium point $(R = I, \Omega = 0)$.

III. Unit quaternion

The orientation of a rigid body with respect to the inertial frame can be described by a four-parameters representation, namely unit quaternion [9]. A quaternion $Q = (q_0, q)$ is composed of a scalar component $q_0 \in \mathbb{R}$ and a vector $q \in \mathbb{R}^3$. The set of quaternion $\mathbb{Q}$ is a four-dimensional vector space over the reals, which forms a group with the quaternion multiplication denoted by "$\star$". The quaternion multiplication is distributive and associative but not commutative [9]. The multiplication of two quaternion $Q = (q_0, q)$ and $\bar{Q} = (\bar{q}_0, \bar{q})$ is defined as [9], [11]

$$Q \star \bar{Q} = (q_0\bar{q}_0 - q^T\bar{q}\ ,\ q_0\bar{q} + \bar{q}_0 q + q \times \bar{q}), \tag{3}$$

and has the quaternion $(1, \mathbf{0})$ as the identity element. Note that, for a given quaternion $Q = (q_0, q)$, we have $Q \star Q^{-1} = Q^{-1} \star Q = (1, \mathbf{0})$, where $Q^{-1} = \frac{(q_0, -q)}{\|Q\|}$.
The set of unit quaternion $\mathbb{Q}_u$ is a subset of $\mathbb{Q}$ such that

$$\mathbb{Q}_u = \{Q = (q_0, q) \in \mathbb{R} \times \mathbb{R}^3 \mid q_0^2 + q^T q = 1\}. \tag{4}$$

Note that in the case where $Q = (q_0, q) \in \mathbb{Q}_u$, the unit quaternion inverse is given by $Q^{-1} = (q_0, -q)$.

A rotation matrix R by an angle γ about the axis described by the unit vector $\hat{k} \in \mathbb{R}^3$, can be described by a unit quaternion $Q = (q_0, q) \in \mathbb{Q}_u$ such that

$$q = \hat{k}\sin(\frac{\gamma}{2}), \quad q_0 = \cos(\frac{\gamma}{2}), \tag{5}$$

The rotation matrix R is related to the quaternion through the Rodriguez formula [5], [11]

$$R \ = \ I + 2q_0 S(q) + 2S(q)^2 \tag{6}$$

Algorithms allowing the extraction of q and q_0 from a rotation matrix R, can be found in [11].

In this paper, instead of using the rotation matrix R to describe the orientation of the rigid body in space, we will use the unit quaternion. The dynamical equation (2) can be replaced by the following dynamical equation in terms of the unit quaternion [5], [11]:

$$\dot{Q} = \frac{1}{2} Q \star Q_\Omega, \tag{7}$$

where $Q = (q_0, q) \in \mathbb{Q}_u$ and $Q_\Omega = (0, \Omega) \in \mathbb{Q}$.
We also define the unit quaternion error $\tilde{Q}$, which describes the deviation between two unit quaternion Q and $\hat{Q}$, as follows:

$$\tilde{Q} = \hat{Q}^{-1} \star Q = (\hat{q}_0 q_0 + \hat{q}^T q\ ,\ \hat{q}_0 q - q_0\hat{q} - \hat{q} \times q) := (\tilde{q}_0, \tilde{q}). \tag{8}$$

Note that the unit quaternion Q and $\hat{Q}$ coincide if $\tilde{Q} = (1, \mathbf{0})$.
It is also important to mention that the equilibrium point $(R = I, \Omega = 0)$ for (1) and (2) is equivalent to the equilibrium point $(q = 0, q_0 = \pm1, \Omega = 0)$ for (1) and (7). Since $q_0 = 1$ corresponds to $\gamma = 0$ and $q_0 = -1$ corresponds to $\gamma = 2\pi$, it is clear that $q_0 = \pm1$ correspond to the same physical point. Hence, the two equilibrium points $(q = 0, q_0 = \pm1, \Omega = 0)$ are in reality a unique physical equilibrium point corresponding to $(R = I, \Omega = 0)$.

IV. Observer-based quaternion feedback control design

Our result can be stated in the following theorem:
Theorem 1: Consider system (1) and (7) under the following control law

$$\tau = -\alpha_1 q - \alpha_2 \tilde{q} \tag{9}$$

$$\dot{\hat{Q}} = \frac{1}{2}\hat{Q} \star Q_\beta \tag{10}$$

$$\beta = \Gamma_1 \tilde{q} \tag{11}$$

where $\hat{Q} = (\hat{q}_0, \hat{q}) \in \mathbb{Q}_u$, $Q_\beta = (0, \beta) \in \mathbb{Q}$, $\Gamma_1 = \Gamma_1^T > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\tilde{q}$ is the vector part of the unit quaternion deviation defined in (8). Then, $\tilde{Q}$, Q and Ω are globally bounded[1] and $\lim_{t\to\infty} q(t) = \lim_{t\to\infty} \tilde{q}(t) = \lim_{t\to\infty} \Omega(t) = 0$, $\lim_{t\to\infty} \tilde{q}_0(t) = \pm1$ and $\lim_{t\to\infty} q_0(t) = \pm1$.

Proof: The time derivative of $\tilde{Q}$, in view of (7), (8) and (10), is given by

$$\begin{array}{rcl} \dot{\tilde{Q}} & = & \frac{d}{dt}(\hat{Q}^{-1} \star Q) \\ & = & -\frac{1}{2} Q_\beta \star \tilde{Q} + \frac{1}{2}\tilde{Q} \star Q_\Omega \\ & = & \left(\frac{1}{2}\tilde{q}^T(\beta - \Omega)\ ,\ \frac{1}{2}\tilde{q}_0(\Omega - \beta) + \frac{1}{2}\tilde{q} \times (\Omega + \beta)\right) \\ & := & (\dot{\tilde{q}}_0, \dot{\tilde{q}}). \end{array} \tag{12}$$

Consider the following Lyapunov function candidate

$$\begin{array}{rcl} V & = & \alpha_2\left(\tilde{q}^T\tilde{q} + (\tilde{q}_0 - 1)^2\right) + \alpha_1\left(q^T q + (q_0 - 1)^2\right) \\ & & + \frac{1}{2}\Omega^T I_f \Omega \\ & = & 2\alpha_2(1 - \tilde{q}_0) + 2\alpha_1(1 - q_0) + \frac{1}{2}\Omega^T I_f \Omega \end{array} \tag{13}$$

whose time-derivative, in view of (1), (7) and (12) is given by

$$\begin{array}{rcl} \dot{V} & = & -2\alpha_2\dot{\tilde{q}}_0 - 2\alpha_1\dot{q}_0 + \Omega^T I_f \dot{\Omega} \\ & = & -\alpha_2\tilde{q}^T(\beta - \Omega) + \alpha_1\Omega^T q + \Omega^T(-\Omega \times I_f\Omega + \tau). \end{array} \tag{14}$$

[1] The global boundedness here is not used in the usual sense, but for any $Q(0), \tilde{Q}(0) \in \mathbb{Q}_u$ and any $\Omega(0) \in \mathbb{R}$.

Substituting (9) and (11) in (14), and using the fact that $\Omega^T(\Omega \times I_f\Omega) = 0$, we obtain

$$\dot{V} = -\alpha_2 \tilde{q}^T \Gamma_1 \tilde{q}. \tag{15}$$

Therefore, one can conclude that $\tilde{Q}$, Q and Ω are globally bounded. Hence, it is clear that $\ddot{V}$ is bounded. Using Barbalat lemma, one can conclude that $\lim_{t\to\infty} \tilde{q}(t) = 0$, which implies that $\lim_{t\to\infty} \tilde{q}_0(t) = \pm 1$. Since $\ddot{\tilde{Q}}$ is bounded, one can conclude that $\lim_{t\to\infty} \dot{\tilde{Q}}(t) = 0$, which in turns, from (12), implies that $\lim_{t\to\infty}(\Omega(t) - \beta(t)) = 0$. Since $\lim_{t\to\infty} \tilde{q}(t) = 0$, it is clear, from (11), that $\lim_{t\to\infty} \beta(t) = 0$. Consequently, one can conclude that $\lim_{t\to\infty} \Omega(t) = 0$, and hence, $\lim_{t\to\infty} \dot{\Omega}(t) = 0$ since $\ddot{\Omega}$ is bounded. Therefore, from (1), one can conclude that $\lim_{t\to\infty} \tau(t) = 0$, which in view of (9) and the fact that $\lim_{t\to\infty} \tilde{q}(t) = 0$, implies that $\lim_{t\to\infty} q(t) = 0$ and hence $\lim_{t\to\infty} q_0(t) = \pm 1$.

It is clear that, for the closed loop system, $\dot{V} = 0$ at the following four equilibrium points ($\tilde{q}_0 = \pm 1, q_0 = \pm 1, \Omega = 0$) and $\dot{V} < 0$ outside these equilibrium points. Note that these four equilibrium point represent the same physical equilibrium for the rigid body ($R = I$, $\Omega = 0$). If initially, the closed-loop system is at one of these four equilibrium points, it will remain there for all subsequent time. In the case where the closed-loop system is not at one of the four equilibrium points, it will converge to the attractive equilibrium point ($\tilde{q}_0 = 1, q_0 = 1, \Omega = 0$) for which $V = 0$ and $\dot{V} = 0$. The three isolated equilibrium points ($\tilde{q}_0 = 1, q_0 = -1, \Omega = 0$), ($\tilde{q}_0 = -1, q_0 = 1, \Omega = 0$) and ($\tilde{q}_0 = -1, q_0 = -1, \Omega = 0$) are not attractors, but repeller equilibria [6]. In fact, if the system is initially at one of the three repeller equilibria, and we apply a small disturbance at $q_0 = -1$ or $\tilde{q}_0 = -1$ (preserving the conditions $-1 \leq q_0 \leq 1$ and $-1 \leq \tilde{q}_0 \leq 1$), one can see from (13) that V decreases, and since $\dot{V} < 0$ outside these equilibrium points, one can conclude that q_0 and $\tilde{q}_0$ will converge to 1. □

Remark 1: The result in Theorem 1 can also be interpreted in terms of passivity [3]. The introduction of the observer (10) allows to generate a passive map $-\beta \mapsto \tilde{q}$. In fact, this can be easily seen by substituting (9) in (14) to get

$$\int_0^T -\alpha_2 \tilde{q}^T \beta dt \geq V(X(T)) - V(X(0)), \tag{16}$$

with $X^T(t) = (\tilde{q}(t), \tilde{q}_0(t), q(t), q_0(t), \Omega(t))$. The passive system is shown in Figure 1. Therefore, the observer input β can be designed in a straightforward manner as in (11). The resulting closed-loop system is a feedback interconnection of a passive system and a constant gain as shown in Figure 2. This, guarantees global boundedness of $X(t)$ and the convergence of $\tilde{q}$ to zero. Finally, thanks to the fact that the largest positively invariant set $\{X|\ \dot{V} = 0\}$ is simply the set $\{X|\ \tilde{q} = 0, q = 0, \Omega = 0\}$.

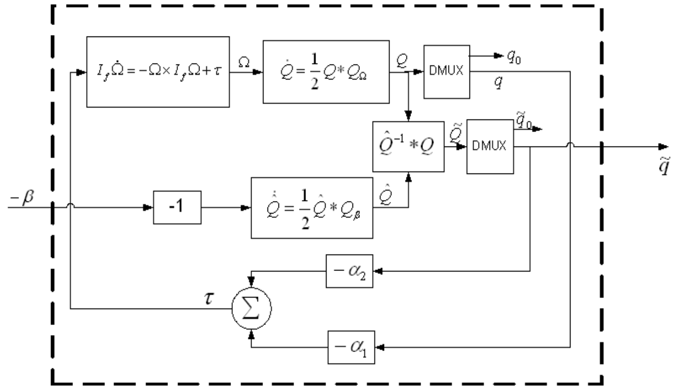

Fig. 1. Passive system

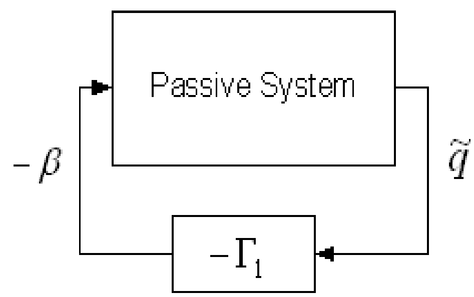

Fig. 2. Feedback interconnection

Remark 2: It is worth noting that the main purpose of the dynamical system (10) is not to estimate the actual quaternion which is assumed to be available for feedback, but to generate a passive mapping between the observer input and the vector part of the estimation-error quaternion. In fact, under the control law (9) and forcing the observer input to be $\beta = \Gamma_1 \tilde{q}$, we ensure asymptotic convergence of $\tilde{q}$ to zero. The convergence of $\tilde{q}$ to zero will guarantee the convergence of $(\Omega - \beta)$ to zero, and hence the convergence of Ω to zero (as shown in the proof of Theorem 1). The main idea in our approach is to force Ω to converge to zero by forcing $\tilde{q}$ to converge to zero. Once $\tilde{q}$ and Ω converge to zero, the convergence of q to zero follows from the system dynamics (1) and the structure of the control law (9).

V. Simulation Results

In this section, we present some simulation results showing the effectiveness of the proposed controller. The inertial matrix has been taken as $I_f = \text{diag}(20, 20, 30)$ and the control parameters have been chosen as follows: $\alpha_1 = \alpha_2 = 1000$ and $\Gamma_1 = \text{diag}(15, 15, 15)$. The initial conditions have been taken as follows: $Q = (0, 0, 1, 0)$ and $\hat{Q} = (0, 1, 0, 0)$.

Figure 3 shows the evolution of the three components of the angular velocity with respect to time, and Figure 4 shows the evolution of the Euclidian norm of the angular velocity with respect to time. Figure 5, shows the evolution of the quaternion, describing the orientation of the body, with respect to time. Figure 6, shows the evolution of the error quaternion $\tilde{Q}$, describing the deviation between the actual quaternion Q and the observed quaternion $\hat{Q}$, with respect to time.

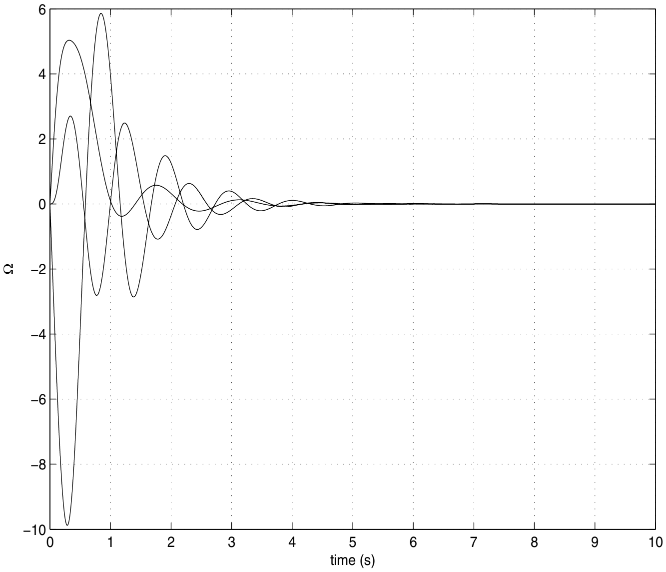

Fig. 3. The three components of the angular velocity Ω versus time

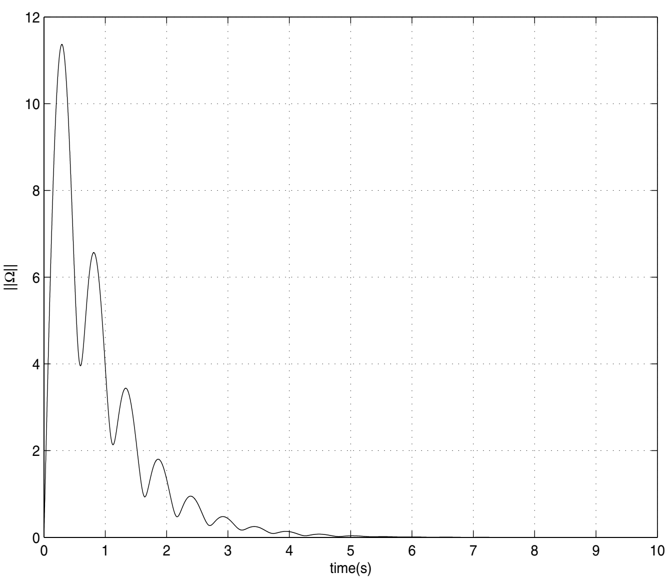

Fig. 4. The Euclidian norm of the angular velocity Ω versus time

VI. Conclusion

An alternative solution to the global attitude stabilization problem, without velocity measurement, has been proposed. Our approach, does not involve the use of a lead filter as in [7], [14], but uses a unit quaternion observer whose input is related to the vector part of the estimation-error quaternion via a passive map, under a linear feedback control law involving the vector parts of the actual unit quaternion and the estimation-error

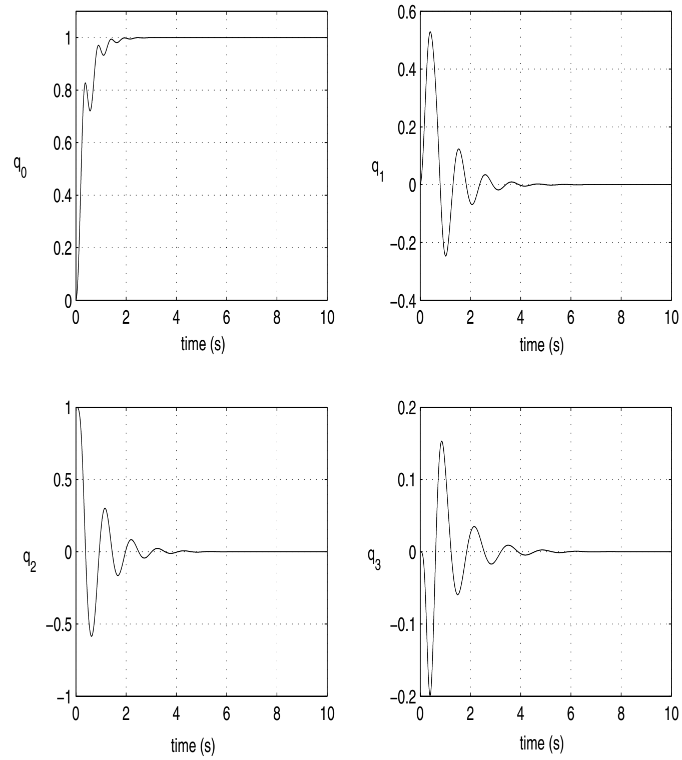

Fig. 5. Quaternion versus time

quaternion. This allows to chose the observer input as a simple linear feedback in terms of the vector part of the estimation-error quaternion leading to the global asymptotic stability of the equilibrium point $(R = I, \Omega = 0)$.

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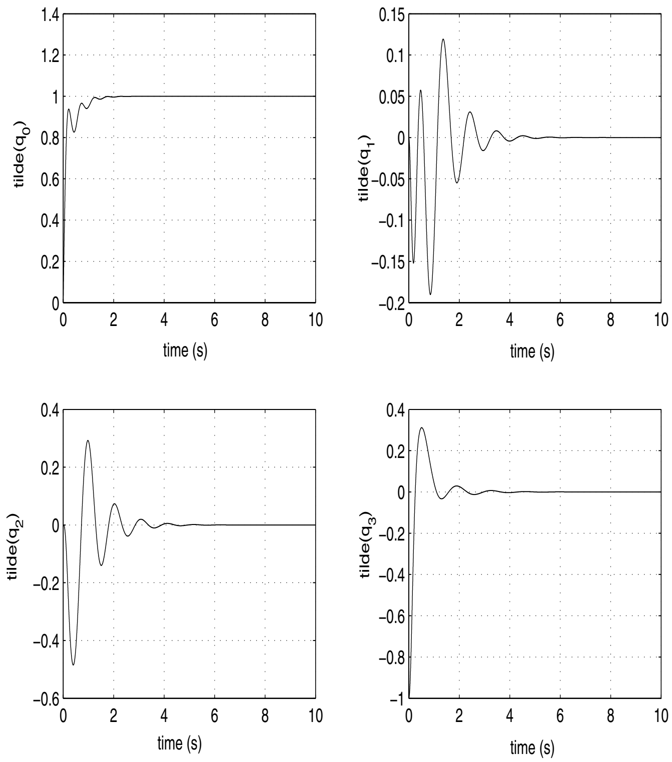

Fig. 6. Quaternion error $\tilde{q}_0$, $\tilde{q}_1$, $\tilde{q}_2$ and $\tilde{q}_3$ versus time

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