Proceedings of the
46th IEEE Conference on Decision and Control
New Orleans, LA, USA, Dec. 12-14, 2007

A velocity-free attitude tracking controller for rigid spacecraft

Abdelhamid Tayebi

***Abstract*— In this paper, we propose a quaternion-based dynamic output feedback for the attitude tracking problem of a rigid body without velocity measurement. Our approach consists of introducing an auxiliary dynamical system whose output (which is also a unit quaternion) is used in the control law together with the unit quaternion representing the attitude tracking error. Roughly speaking, the necessary damping that would have been achieved by the direct use of the angular velocity can be achieved, in our approach, by the vector part $\tilde{q}$ of the error signal between the output of the auxiliary system and the unit quaternion tracking error. The resulting velocity-free control scheme guarantees *almost* global [1] asymptotic stability which is as strong as the topology of the motion space can permit. Simulation results are provided to show the effectiveness of the proposed control scheme.**

I. INTRODUCTION

The attitude control problem of a spacecraft, or a rigid body in space in general, has been extensively studied during the past four decades. This is a particularly interesting problem in dynamics since the angular velocity of the body cannot be integrated to obtain the attitude of the body [8]. From a practical point of view, the design of efficient and low-cost attitude controllers is an important issue which is of great interest for aerospace industry for instance. The attitude stabilization of a rigid body in space, using the unit-quaternion and the angular velocity in the feedback control law, has been investigated by many researchers and a wide class of controllers has been proposed (see, for instance, [8], [14], [19], [20]). In [14], some quaternion based feedback controllers for the attitude stabilization have been proposed and tested experimentally on a quadrotor aircraft.

The attitude control of a rigid body with full states measurements (*i.e.,* quaternion and angular velocity), being relatively well understood, the research has been directed towards other performance and implementation-cost optimization issues, by removing the requirement of the velocity measurement. The passivity property, was the main idea behind the design of the attitude controllers, without velocity measurement, in [6], [10], [17]. In fact, in [6], the authors used the passivity-based adaptive control approach for robotic manipulators to derive their adaptive attitude control scheme without velocity measurement. In [10], a quite similar passivity argument has been used to develop a velocity-measurement-free attitude stabilization controller using a lead filter. In [15], an alternative solution to the attitude regulation problem without velocity measurement and without the use of a lead filter has been proposed. The author in [17] derives quite similar results as the results of [10] by using the Rodrigues Parameters instead of the quaternion [13]. The second approach that has also been used to avoid the velocity measurement is based on the use of nonlinear observers. In fact, in [12], a nonlinear velocity observer, using just the torque and orientation measurements, has been proposed based on the analogy to second-order linear systems, where a separation principle-like property was conjectured. In, [16], based on the work of [12] and [18], an estimation algorithm for the constant gyro bias has been proposed. This algorithm, using the orientation and gyroscopic measurements, has been combined with the attitude control scheme proposed in [6]. The extension of the velocity-free attitude regulation controllers to the tracking problem is not an obvious task especially when we aim for non-local results. In [3], two attitude tracking controllers without velocity measurement have been proposed. The first one is a locally exponentially stabilizing controller-observer scheme. The second scheme, guaranteeing also local exponential stability under an adequate choice of the control parameters, is a generalization of the lead filter based regulation scheme of [10] to the attitude tracking problem. In [4] a local velocity-free adaptive quaternion-based tracking controller for a rigid body with uncertainties has been proposed. Another alternative to the work of [3] has been proposed in [1] based on the results of [17] using the Rodrigues parameters instead of the unit-quaternion. Note that unlike the quaternion representation, the three-parameters (Rodrigues parameters) attitude representation suffers from singularity problems [13].

In the present paper, we use the four-parameters representation (quaternion), which is globally non-singular, to represent the attitude motion, and provide a new solution to the attitude tracking problem without velocity measurement. To the best of our knowledge, our result is the first velocity-free unit quaternion-based tracking controller guaranteeing *almost* global asymptotic stability. Our main idea is the introduction of an auxiliary unit-quaternion dynamical system having the same structure as the actual unit-quaternion attitude model. Under a feedback involving the unit quaternion tracking error and the vector part $\tilde{q}$ of the error signal between the output of

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

A. Tayebi is with the Department of Electrical Engineering, Lakehead University, Thunder Bay, Ontario, P7B 5E1, Canada. atayebi@lakeheadu.ca

[1] In this paper we use the term *almost* global (see, for instance, [9]) to indicate that the boundedness of the states is guaranteed over $\mathbb{Q}_u \times \mathbb{Q}_u \times \mathbb{R}^3$, where $\mathbb{Q}_u$ is the set of unit-quaternion. Furthermore, the closed-loop system has four equilibrium points (three repeller equilibria and one attractor) that are mathematically different but represent the same physical attitude of the rigid body. All trajectories starting in $\mathbb{Q}_u \times \mathbb{Q}_u \times \mathbb{R}^3$–except the three repeller equilibria— will converge to the unique attractor equilibrium. For more details about the topological obstruction to continuous global stabilization of rotational motion, the reader is referred to [2].

the auxiliary system and the unit quaternion tracking error, we show that the map between the auxiliary system input and $\tilde{q}$ is passive. Therefore, the auxiliary system input can be designed as a simple linear feedback in terms of $\tilde{q}$ achieving *almost* global asymptotic attitude tracking. Simulation results are also provided to support the theoretical developments.

II. Dynamical model and problem statement

The dynamical model of a spacecraft or a rigid body in space is given by

$$I_f\dot{\Omega} = -\Omega \times I_f\Omega + \tau, \tag{1}$$

$$\dot{R} = RS(\Omega), \tag{2}$$

where Ω denotes the angular velocity of the body expressed in the body-fixed frame $\mathcal{A}$. The orientation of the rigid body is given by the orthogonal rotation matrix $R \in SO(3)$. $I_f \in \mathbb{R}^{3\times3}$ is a symmetric positive definite constant inertia matrix of the body with respect to the frame $\mathcal{A}$ whose origin is at the center of mass. The vector τ is the torque applied the rigid body, considered as the input vector. The matrix $S(\Omega)$ is a skew-symmetric matrix such that $S(\Omega)V = \Omega \times V$ for any vector $V \in \mathbb{R}^3$, where $\times$ denotes the vector cross-product.
Our objective is to design a feedback controller, without velocity measurement, for the stabilization of the equilibrium point ($\tilde{R} := RR_d^T = I$, $\Omega - \Omega_d = 0$), where $R_d(t)$ is the desired orientation and $\Omega_d(t)$ is the desired angular velocity.

III. Unit-quaternion

The orientation of a rigid body with respect to the inertial frame can be described by a four-parameters representation, namely unit-quaternion [11]. A quaternion $Q = (q_0, q)$ is composed of a scalar component $q_0 \in \mathbb{R}$ and a vector $q \in \mathbb{R}^3$. The set of quaternion $\mathbb{Q}$ is a four-dimensional vector space over the reals, which forms a group with the quaternion multiplication denoted by "$\star$". The quaternion multiplication is distributive and associative but not commutative [11]. The multiplication of two quaternion $Q = (q_0, q)$ and $P = (p_0, p)$ is defined as [11], [13]

$$Q \star P = (q_0p_0 - q^T p \ , \ q_0p + p_0q + q \times p), \tag{3}$$

and has the quaternion $(1, \mathbf{0})$ as the identity element. Note that, for a given quaternion $Q = (q_0, q)$, we have $Q \star Q^{-1} = Q^{-1} \star Q = (1, \mathbf{0})$, where $Q^{-1} = \frac{(q_0,-q)}{\|Q\|^2}$.
The set of unit-quaternion $\mathbb{Q}_u$ is a subset of $\mathbb{Q}$ such that

$$\mathbb{Q}_u = \{Q = (q_0, q) \in \mathbb{R} \times \mathbb{R}^3 \mid q_0^2 + q^T q = 1\}. \tag{4}$$

Note that in the case where $Q = (q_0, q) \in \mathbb{Q}_u$, the unit-quaternion inverse is given by $Q^{-1} = (q_0, -q)$.

A rotation matrix R by an angle γ about the axis described by the unit vector $\hat{k} \in \mathbb{R}^3$, can be described by a unit-quaternion $Q = (q_0, q) \in \mathbb{Q}_u$ such that

$$q = \hat{k}\sin(\frac{\gamma}{2}), \quad q_0 = \cos(\frac{\gamma}{2}), \tag{5}$$

The rotation matrix R is related to the quaternion through the Rodriguez formula [7], [13]

$$\begin{aligned} R(Q) &= I + 2q_0S(q) + 2S^2(q) \\ &= (q_0^2 - q^T q)I + 2qq^T + 2q_0S(q). \end{aligned} \tag{6}$$

Algorithms allowing the extraction of q and q_0 from a rotation matrix R, can be found in [13].

In this paper, instead of using the rotation matrix R to describe the orientation of the rigid body in space, we will use the unit-quaternion. The dynamical equation (2) can be replaced by the following dynamical equation in terms of the unit-quaternion [7], [13]:

$$\dot{Q} = \frac{1}{2}Q \star Q_\Omega, \tag{7}$$

where $Q = (q_0, q) \in \mathbb{Q}_u$ and $Q_\Omega = (0, \Omega) \in \mathbb{Q}$. In the sequel, we will use $Q_\star$ to denote the quaternion $(0, \star)$. We also define the unit-quaternion error $E = (e_0, e)$, which describes the discrepancy between two unit-quaternion $Q = (q_0, q)$ and $\bar{Q}(\bar{q}_0, \bar{q})$, as follows:

$$E = \bar{Q}^{-1} \star Q = (\bar{q}_0q_0 + \bar{q}^T q \ , \ \bar{q}_0q - q_0\bar{q} - \bar{q} \times q). \tag{8}$$

Note that the unit-quaternion Q and $\bar{Q}$ coincide if $E = (1, \mathbf{0})$.
It is also important to mention that the equilibrium point $(R = I, \Omega = 0)$ for (1) and (2) is equivalent to the equilibrium point $(q = 0, q_0 = \pm1, \Omega = 0)$ for (1) and (7). Since $q_0 = 1$ corresponds to $\gamma = 0$ and $q_0 = -1$ corresponds to $\gamma = 2\pi$, it is clear that $q_0 = \pm1$ correspond to the same physical point. Hence, the two equilibrium points $(q = 0, q_0 = \pm1, \Omega = 0)$ are in reality a unique physical equilibrium point corresponding to $(R = I, \Omega = 0)$.

IV. Main results

Assume that the desired orientation to be tracked is given by

$$\dot{Q}^d = \frac{1}{2}Q^d \star Q_{\Omega_d}, \tag{9}$$

where Ω_d is the desired angular velocity, which is assumed to be bounded as well as its first and second time-derivatives.

Let us define the unit-quaternion tracking error Q^e, which describes the discrepancy between the actual unit-quaternion Q and the desired unit-quaternion Q^d, as follows $Q^e = (Q^d)^{-1} \star Q \equiv (q_0^e, q^e)$. Therefore, we have

$$Q^d \star Q^e = Q.$$

Differentiating both sides of the above equation with respect to time, we have

$$\dot{Q}^d \star Q^e + Q^d \star \dot{Q}^e = \dot{Q}.$$

Hence,

$$\dot{Q}^e = (Q^d)^{-1} \star (\dot{Q} - \dot{Q}^d \star Q^e).$$

Using (7) and (9), the error quaternion dynamics is given by

$$\begin{aligned} \dot{Q}^e &= -\tfrac{1}{2}Q_{\Omega_d} \star Q^e + \tfrac{1}{2}Q^e \star Q_\Omega \\ &= -\tfrac{1}{2}Q^e \star (Q^e)^{-1} \star Q_{\Omega_d} \star Q^e + \tfrac{1}{2}Q^e \star Q_\Omega. \end{aligned} \tag{10}$$

Using the fact that $(Q^e)^{-1} \star Q_{\Omega_d} \star Q^e = Q_{\bar{\Omega}_d}$, with $\bar{\Omega}_d = R^T(Q^e)\Omega_d$, where $R^T(Q^e)$ is obtained from (6) by substituting Q by Q^e, we have

$$\begin{aligned} \dot{Q}^e &= \tfrac{1}{2} Q^e \star Q_{\tilde{\Omega}} \\ &= \left(-\tfrac{1}{2}(q^e)^T\tilde{\Omega}, \tfrac{1}{2}(q_0^e I + S(q^e))\tilde{\Omega}\right) \\ &\equiv (\dot{q}_0^e, \dot{q}^e) \end{aligned} \tag{11}$$

where $\tilde{\Omega} = \Omega - \bar{\Omega}_d$.

Let us introduce the following auxiliary system:

$$\dot{\bar{Q}} = \frac{1}{2}\bar{Q} \star Q_\beta, \tag{12}$$

with $\bar{Q}(0) = (\bar{q}_0(0), \bar{q}(0)) \in \mathbb{Q}_u$, $Q_\beta = (0, \beta) \in \mathbb{Q}$, where the input β of (12) will be designed later. We define the unit-quaternion $\tilde{Q} = \bar{Q}^{-1} \star Q^e = (\tilde{q}_0, \tilde{q}) \in \mathbb{Q}_u$ describing the discrepancy between the unit-quaternion tracking error Q^e and the auxiliary unit-quaternion signal $\bar{Q}$.

Now, we can state the following theorem:

Theorem 1: Consider system (1) under the following control law

$$\tau = -\alpha_1 q^e - \alpha_2 \tilde{q} + I_f R^T(Q^e)\dot{\Omega}_d + S(\bar{\Omega}_d) I_f \bar{\Omega}_d, \tag{13}$$

with $\alpha_1 > 0$, $\alpha_2 > 0$, and let the input of the auxiliary system (12) be

$$\beta = \Gamma_1 \tilde{q}, \tag{14}$$

with $\Gamma_1 = \Gamma_1^T > 0$.

The vectors q^e and $\tilde{q}$ are the vector parts of the unit-quaternion Q^e and $\tilde{Q}$ respectively. Then, Q^e, $\tilde{Q}$ and Ω are globally bounded[2], and $\lim_{t\to\infty} q^e(t) = \lim_{t\to\infty} \tilde{q}(t) = \lim_{t\to\infty} \tilde{\Omega}^\star(t) = 0$, $\lim_{t\to\infty} q_0^e(t) = \pm 1$ and $\lim_{t\to\infty} \tilde{q}_0(t) = \pm 1$, where $\tilde{\Omega}^\star(t) \equiv \Omega(t) - \Omega_d(t)$.

Proof: The dynamical equation for the angular velocity tracking error is given by

$$I_f \dot{\tilde{\Omega}} = -(\tilde{\Omega} + \bar{\Omega}_d) \times I_f(\tilde{\Omega} + \bar{\Omega}_d) + I_f(\tilde{\Omega} \times \bar{\Omega}_d - R^T(Q^e)\dot{\Omega}_d) + \tau. \tag{15}$$

After some algebraic manipulations, one can show that

$$\begin{aligned} \tfrac{d}{dt}(\tfrac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega}) &= -\tilde{\Omega}^T S(\bar{\Omega}_d) I_f \bar{\Omega}_d \\ &- \tilde{\Omega}^T (S(\bar{\Omega}_d) I_f + I_f S(\bar{\Omega}_d))\tilde{\Omega} \\ &+ \tilde{\Omega}^T(\tau - I_f R^T(Q^e)\dot{\Omega}_d). \end{aligned} \tag{16}$$

Since $I_f = I_f^T > 0$, it is clear that $(S(\bar{\Omega}_d) I_f + I_f S(\bar{\Omega}_d))$ is a skew symmetric matrix and hence $\tilde{\Omega}^T(S(\bar{\Omega}_d) I_f + I_f S(\bar{\Omega}_d))\tilde{\Omega} = 0$. Therefore,

$$\frac{d}{dt}(\frac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega}) = \tilde{\Omega}^T(\tau - I_f R^T(Q^e)\dot{\Omega}_d - S(\bar{\Omega}_d) I_f \bar{\Omega}_d), \tag{17}$$

[2]The global boundedness here indicates that the states are bounded for any $(Q^e(0), \tilde{Q}(0), \Omega(0)) \in \mathbb{Q}_u \times \mathbb{Q}_u \times \mathbb{R}^3$. Note that the unit-quaternion $\tilde{Q}$ and Q^e are bounded by definition.

Using (11) and (12), one can show that

$$\begin{aligned} \dot{\tilde{Q}} &= \tfrac{d}{dt}(\bar{Q}^{-1} \star Q^e) \\ &= -\tfrac{1}{2} Q_\beta \star \tilde{Q} + \tfrac{1}{2}\tilde{Q} \star Q_{\tilde{\Omega}} \\ &= \left(\tfrac{1}{2}\tilde{q}^T(\beta - \tilde{\Omega}) \ , \ \tfrac{1}{2}\tilde{q}_0(\tilde{\Omega} - \beta) + \tfrac{1}{2}\tilde{q} \times (\tilde{\Omega} + \beta)\right) \\ &:= (\dot{\tilde{q}}_0, \dot{\tilde{q}}). \end{aligned} \tag{18}$$

Consider the following Lyapunov function candidate

$$\begin{aligned} V &= \alpha_2\left(\tilde{q}^T\tilde{q} + (\tilde{q}_0 - 1)^2\right) + \alpha_1\left((q^e)^T q^e + (q_0^e - 1)^2\right) \\ &+ \tfrac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega} \\ &= 2\alpha_2(1 - \tilde{q}_0) + 2\alpha_1(1 - q_0^e) + \tfrac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega} \end{aligned} \tag{19}$$

whose time-derivative, in view of (11), (17) and (18) is given by

$$\begin{aligned} \dot{V} &= -2\alpha_2\dot{\tilde{q}}_0 - 2\alpha_1\dot{q}_0^e + \tfrac{d}{dt}(\tfrac{1}{2}\tilde{\Omega}^T I_f \tilde{\Omega}) \\ &= -\alpha_2\tilde{q}^T(\beta - \tilde{\Omega}) + \alpha_1\tilde{\Omega}^T q^e \\ &+ \tilde{\Omega}^T(\tau - I_f R^T(Q^e)\dot{\Omega}_d - S(\bar{\Omega}_d) I_f \bar{\Omega}_d)), \end{aligned} \tag{20}$$

which in view of (13) and (14), leads to

$$\dot{V} = -\alpha_2\tilde{q}^T\Gamma_1\tilde{q}. \tag{21}$$

Therefore, one can conclude that $\tilde{Q}$, Q^e and $\tilde{\Omega}$ are globally bounded. Therefore, it is clear that $\ddot{V}$ is bounded. Hence, invoking Barbalat's lemma, one can conclude that $\lim_{t\to\infty} \tilde{q}(t) = 0$, which implies that $\lim_{t\to\infty} \tilde{q}_0(t) = \pm 1$. Consequently, one can show that $\ddot{\tilde{Q}}$ is bounded since $\dot{\Omega}_d$ is bounded, and hence $\lim_{t\to\infty} \dot{\tilde{Q}}(t) = 0$, which in turns, from (18), implies that $\lim_{t\to\infty}(\tilde{\Omega}(t) - \beta(t)) = 0$. Since $\lim_{t\to\infty} \tilde{q}(t) = 0$, it is clear, from (14), that $\lim_{t\to\infty} \beta(t) = 0$. Consequently, one can conclude that $\lim_{t\to\infty} \tilde{\Omega}(t) = 0$. Using the fact that $\ddot{\Omega}_d$ is bounded and the previous boundedness results, one can show that $\ddot{\tilde{\Omega}}$ is bounded, and hence, one can conclude that $\lim_{t\to\infty} \dot{\tilde{\Omega}}(t) = 0$. As t goes to infinity, from (15), we have $0 = -I_f R^T(Q^e)\dot{\Omega}_d - S(\bar{\Omega}_d) I_f \bar{\Omega}_d + \tau$. Therefore, from (13), it is clear that $\lim_{t\to\infty}(\alpha_1 q^e(t) + \alpha_2\tilde{q}(t)) = 0$, which implies that $\lim_{t\to\infty} q^e(t) = 0$ since $\lim_{t\to\infty} \tilde{q}(t) = 0$. Finally, $\lim_{t\to\infty} q_0^e(t) = \pm 1$. Since Q^e tends to $(\pm 1, 0)$, when t goes to infinity, it is clear $R(Q^e)$ goes to I and hence, $\bar{\Omega}_d$ tends to Ω_d. Consequently, $\lim_{t\to\infty}(\Omega(t) - \Omega_d(t)) = 0$. □

Remark 1: It is clear that our control scheme includes the attitude regulation problem as a particular case, i.e., for $R_d = I$ and $\Omega_d = 0$.

Remark 2: From the proof of Theorem 1, it is clear that for the closed loop system, $\dot{V} = 0$ at the following four equilibrium points $(\tilde{q}_0 = \pm 1, q_0^e = \pm 1, \tilde{\Omega}^\star = 0)$, and $\dot{V} < 0$ outside these equilibrium points. Note that these four equilibrium points represent the same physical equilibrium for the rigid body $(\tilde{R} := RR_d^T = I, \tilde{\Omega}^\star = 0)$. If initially, the closed-loop system is at one of these four equilibrium points, it will remain there for all subsequent time. In the case where the closed-loop system is not at one of the four equilibrium points, it will converge to the attractive equilibrium point

$(\tilde{q}_0 = 1, q_0^e = 1, \tilde{\Omega}^\star = 0)$ for which $V = 0$ and $\dot{V} = 0$. The three isolated equilibrium points $(\tilde{q}_0 = 1, q_0^e = -1, \tilde{\Omega}^\star = 0)$, $(\tilde{q}_0 = -1, q_0^e = 1, \tilde{\Omega}^\star = 0)$ and $(\tilde{q}_0 = -1, q_0^e = -1, \tilde{\Omega}^\star = 0)$ are not attractors, but repeller equilibria [8]. In fact, if the system is initially at one of the three repeller equilibria, and we apply a small disturbance at $q_0^e = -1$ or/and $\tilde{q}_0 = -1$ (preserving the conditions $-1 \le q_0^e \le 1$ and $-1 \le \tilde{q}_0 \le 1$), one can see from (19) that V decreases, and since $\dot{V} < 0$ outside these equilibrium points, one can conclude that q_0^e and $\tilde{q}_0$ will converge to 1.

Remark 3: The introduction of the auxiliary system (12) allows to generate a passive map $-\beta \mapsto \tilde{q}$ [5]. In fact, this can be easily seen by substituting (13) in (20) to get

$$\int_0^T -\alpha_2 \tilde{q}^T \beta dt \ge V(X(T)) - V(X(0)), \qquad (22)$$

with $X^T(t) = (\tilde{q}(t), \tilde{q}_0(t), q^e(t), q_0^e(t), \tilde{\Omega}(t))$. Therefore, the auxiliary system input β can be designed in a straightforward manner as in (14). The resulting closed-loop system is a feedback interconnection of a passive system and a constant gain. This, guarantees global boundedness of $X(t)$ and the convergence of $\tilde{q}$ to zero. Finally, thanks to the fact that the largest positively invariant set $\{X | \dot{V} = 0\}$ is simply the set $\{X | \tilde{q} = 0, q^e = 0, \tilde{\Omega}^\star = 0\}$.

Remark 4: It is worth noting that the main purpose of the auxiliary dynamical system (12) is to generate a passive mapping between $(-\beta)$ and the vector part of the unit quaternion error $\tilde{q}$. In fact, under the control law (13) and forcing the input of system (12) to be proportional to $\tilde{q}$, we ensure asymptotic convergence of $\tilde{q}$ to zero. The convergence of $\tilde{q}$ to zero will guarantee the convergence of $\tilde{\Omega}^\star$ to zero (as shown in the proof of Theorem 1). Once $\tilde{q}$ and $\tilde{\Omega}^\star$ converge to zero, the convergence of q to zero is guaranteed in view of the system dynamics (1) and the structure of the control law (13).

Remark 5: Our result in Theorem 1 differs from the result of [3] mainly in two aspects: 1) In [3], the authors show the existence of control gains guaranteeing local exponential stability while in our case, using a different controller and a different proof, the choice of the control gains is straightforward resulting in *almost* global asymptotic stability. 2) Theorem 2 presented in [3], using a lead filter, is an extention of the regulation controller proposed in [10] where the necessary damping, that would have been achieved by the use of the angular velocity in the feedback controller, is achieved by a filtered derivative of the actual unit quaternion. In our approach, the lead filter is not used and hence the designer does not have to worry about the cut-off frequency of the lead filter in the presence of noise. The necessary damping is achieved, roughly speaking, by substituting the angular velocity by $\tilde{q}$.

V. Simulation Results

In this section, we present some simulation results showing the effectiveness of the proposed controller. The inertia matrix has been taken as $I_f = \text{diag}(20, 20, 30)$. We applied the control law of Theorem 1, with $\alpha_1 = \alpha_2 = 20$ and $\Gamma_1 = \text{diag}(3, 3, 3)$. The initial conditions have been taken as follows: $Q(0) = (0, 0, 1, 0)$ and $\bar{Q}(0) = (0, 1, 0, 0)$. The reference trajectory is given by (9) with $Q^d(0) = (1, 0, 0, 0)$ and $\Omega_d = 0.1 \sin(0.2\pi t)[1, 1, 1]^T$. The simulation was performed with Simulink for a time span of 50 seconds. Figure 1 shows the evolution of the three components of the angular velocity tracking error $\Omega - \Omega_d$ with respect to time. Figure 2, shows the evolution of the unit-quaternion tracking error Q^e, describing the deviation between the orientation of the body and the desired orientation, with respect to time. Figure 3, shows the time evolution of the unit-quaternion error $\tilde{Q}$, describing the deviation between Q^e and $\bar{Q}$. Figure 4, shows the control input versus time.

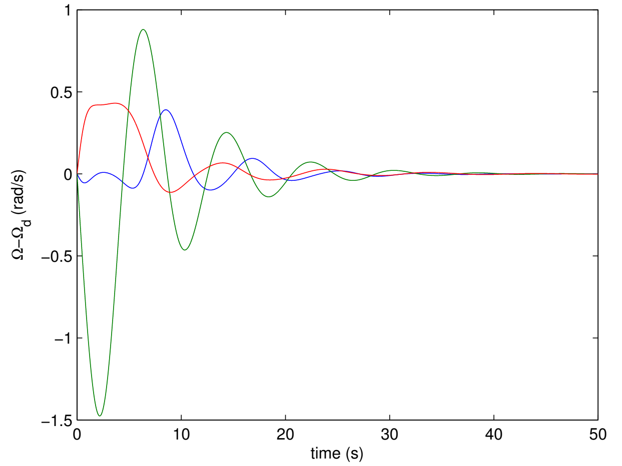

Fig. 1. The three components of the angular velocity error $\Omega - \Omega_d$ versus time

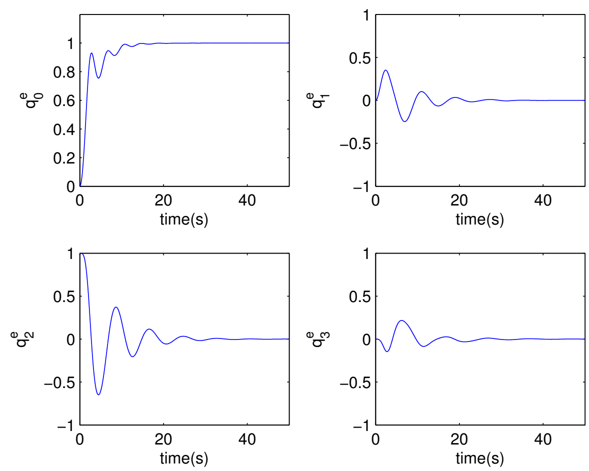

Fig. 2. Unit quaternion tracking error $Q^e = (q_0^e, q_1^e, q_2^e, q_3^e)$ versus time

VI. Conclusion

A new quaternion-based solution to the attitude tracking problem, without velocity measurement, has been proposed. Our approach is based on the use of a unit-quaternion

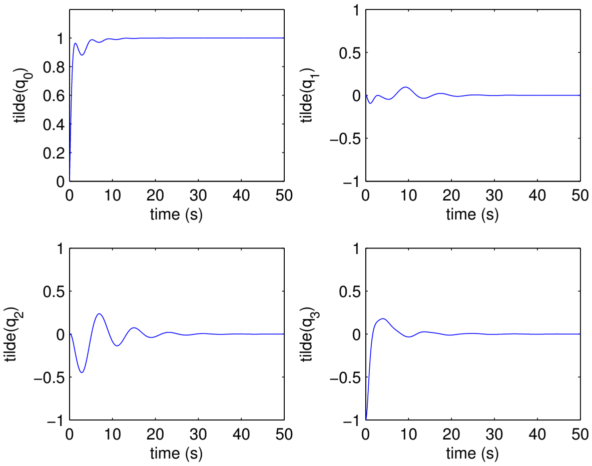

Fig. 3. $\tilde{Q} = (\tilde{q}_0, \tilde{q}_1, \tilde{q}_2, \tilde{q}_3)$ versus time

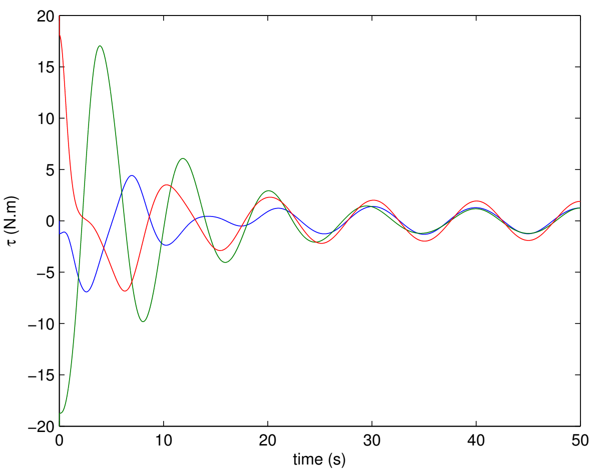

Fig. 4. Control input τ versus time

auxiliary system whose input is related to the vector part of the unit quaternion error $\tilde{q}$ via a passive map, under an appropriate unit quaternion-based feedback. The proposed control scheme includes the attitude regulation problem as a particular case, and guarantees *almost* global asymptotic stability of the equilibrium point ($\tilde{R} := RR_d^T = I$, $\tilde{\Omega}^\star = 0$).

REFERENCES

[1] M.R. Akella, J.T. Halbert and G.R. Kotamraju, "Rigid body attitude control with inclinometer and low-cost gyro measurements," *Systems & Control Letters*, Vol. 49, pp. 151-159, 2003.

[2] S. P. Bhat and D. S. Bernstein "Atopological obstruction to continuous global stabilization of rotational motion and the unwinding phenomenon," *Systems & Control Letters*, Vol. 39, pp. 63-70, 2000.

[3] F. Caccavale and L. Villani, "Output feedback control for attitude tracking ," *Systems & Control Letters*, Vol. 38, No. 2, pp. 91-98, 1999.

[4] B.T. Costic, D.M. Dawson, M.S. de Queiroz, and V. Kapila, "A quaternion-based adaptive attitude tracking controller without velocity Measurements," In proc. of the *39th IEEE CDC*, Sydney, Australia, 2000, pp. 2424-2429.

[5] C.A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. Electrical Science Series, Academic Press, New York, 1975.

[6] O. Egeland and J.M. Godhavn, "Passivity-based adaptive attitude control of a rigid spacecraft," *IEEE Transactions on Automatic Control*, Vol. 39, pp. 842-846, 1994.

[7] P.C. Hughes, *Spacecraft attitude dynamics*. New York: Wiley, 1986.

[8] Joshi S. M., A.G. Kelkar and J. T-Y Wen, "Robust Attitude stabilization of spacecraft using nonlinear quaternion feedback," *IEEE Transactions on Automatic Control*, Vol. 40, No. 10, pp. 1800-1803, 1995.

[9] D. E. Koditschek, "Application of a new Lyapunov function to global adaptive attitude tracking," In proc. of the *27th IEEE CDC*, Austin, Texas, USA, pp. 63-68,1988.

[10] F. Lizarralde and J. T. Wen, "Attitude control without angular velocity measurement: A passivity approach," *IEEE Transactions on Automatic Control*, Vol. 41, No. 3, pp. 468-472, 1996.

[11] R.M. Murray, Z. Li and S. Satry.*A mathematical introduction to robotic manipulation*. CRC Press, 1994.

[12] S. Salcudean, "A globally convergent angular velocity observer for rigid body motion," *IEEE Transactions on Automatic Control*, Vol. 36 , No. 12, pp. 1493-1497, 1991.

[13] M.D. Shuster, "A survey of attitude representations," *J. Astronautical Sciences*, Vol. 41 , No. 4, pp. 439-517, 1993.

[14] A. Tayebi and S. McGilvray, "Attitude stabilization of a quadrotor aircraft," *IEEE Transactions on Control Systems Technology*, Vol. 14, No. 3, pp. 562-571, 2006.

[15] A. Tayebi, "Unit quaternion observer based attitude stabilization of a rigid spacecraft without velocity measurement," In proc. of the *45th IEEE Conference on Decision and Control*, San Diego, California, USA, 2006, pp. 1557-1561.

[16] J. Thienel and R.M. Sanner "A coupled nonlinear spacecraft attitude controller and observer with an unknown constant gyro bias and gyro noise," *IEEE Transactions on Automatic Control*, Vol. 48, No. 11, pp. 2011-2015, 2003.

[17] P. Tsiotras, "Further results on the attitude control problem," *IEEE Transactions on Automatic Control*, Vol. 43, No. 11, pp. 1597-1600, 1998.

[18] B. Vik, T.A. Fossen, "Nonlinear observer design for integration of DGPS and INS," *New Directions in Nonlinear Observer Design. Lecture notes in Control and Information Sciences*, Springer-Verlag, New York, pp. 135-159, 1999.

[19] J. T-Y. Wen and K. Kreutz-Delgado, "The attitude control problem," *IEEE Transactions on Automatic Control*, Vol. 36, No. 10, pp. 1148-1162, 1991.

[20] B. Wie, H. Weiss and A. Arapostathis, "Quaternion feedback regulator for spacecraft eigenaxis rotations," *AIAA J. Guidance Control*, Vol. 12, No. 3, pp. 375-380, 1989.