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Attitude Synchronization of a Spacecraft Formation Without Velocity Measurement

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Abstract— **We consider the quaternion-based attitude synchronization problem of multiple spacecraft without velocity measurements. First, we propose a leader-follower-type control scheme for the attitude synchronization of a spacecraft formation to a desired constant attitude (available only to a single spacecraft called the leader), while maintaining the same attitude during formation maneuvers. Second, we consider the consensus seeking problem, where, no reference attitude is specified, and all spacecraft are required to align their attitudes with the same (possibly time varying) final angular velocity. The communication flow between spacecraft is assumed to be bidirectional. Simulation results of a scenario of four spacecraft are provided to show the effectiveness of the proposed control scheme.**

I. INTRODUCTION

Cooperative and formation control of autonomous vehicles have received extensive interests in recent years leading to significant results [1]-[3]. Closely related to formation control problems, the consensus seeking problem of multi-agent systems deals with the case where it is required that a group of agents agrees on a common decision based on local information exchange. In particular, the use of graph theory produced many interesting results, [4]-[7]. In fact, through an appropriate choice of the information states on which consensus is reached, consensus algorithms can be applied to formation control problems [8]. The above mentioned papers, mainly deal with simple dynamic models such as linear systems and single or double integrators, and hence they are often limited when it comes to dealing with rigid body dynamics.

Recently, several papers have investigated the problem of controlling and maintaining the relative attitudes of formation flying spacecraft, and several approaches are proposed, from which some common fundamental aspects can be extracted, [9]. Roughly, three approaches are found in the literature: Leader-following, virtual structures, and the behavioral methods. The leader-following approach defines a leader in the formation to be followed by the other members of the formation [10]. In the virtual structure approach, the entire formation is considered as a single virtual structure. In [11]-[12], centralized implementation of a virtual structure coordination strategy is presented, and in [13], the virtual structure approach is applied in a decentralized scheme. The behavioral approach prescribes a set of desired behaviors for each member in the team, and weights them such that desirable group behavior emerges. Possible behaviors include trajectory and neighbor tracking, collision and obstacle avoidance, and formation keeping [14]. In [15], the behavioral approach is used to maintain attitude alignment among a group of spacecraft and uses a ring communication topology. The results in [15] were extended to a more general communication graph in [16]. More recently, the passivity approach has been used to develop synchronization algorithms in [17], and in [18] it has been extended to the attitude synchronization problem of rigid bodies. The above coordination control strategies are based on the assumption that each spacecraft (vehicle) knows its own angular velocity, and the angular velocity of its neighbors.

In this paper, we consider the attitude synchronization problem of formation flying spacecraft and remove the requirement for angular velocity and relative angular velocities measurements. Based on the passivity based velocity-free attitude regulation scheme proposed in [19], [15] present a passivity based control law for multi-spacecraft attitude alignment without velocity measurements, where a communication ring topology is assumed, and local results are obtained. The author of [20] considers the Modified Rodriguez Parameters for orientation representation to extend the work of [15] and guarantee attitude synchronization under undirected connected communication topology.

Recently, based on the unit quaternion representation, a novel solution to the attitude tracking problem without velocity measurements has been proposed in [21], and almost global asymptotic stability is guaranteed. The latter has been considered to solve the spacecraft attitude alignment problem in [22], where a group of spacecraft are guaranteed to converge to their desired time varying attitudes. The basic idea in [22] is to introduce an auxiliary system for each spacecraft and for each pair of spacecraft with a communication link. The vector parts of the unit quaternion describing the discrepancy between the output of these auxiliary systems and spacecraft attitude errors as well as the relative attitude between spacecraft, are used in the control law to generate the necessary damping that would have been generated by the angular velocities and the relative angular velocities. The main contribution of this paper is to extend further the results obtained in [22], and design velocity-free decentralized attitude control schemes to solve two different problems related to the attitude synchronization problem of spacecraft within a formation under a bidirectional communication topology. First, we propose a velocity-free leader-follower based con-

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trol scheme that guarantees the convergence of multiple spacecraft to some constant desired attitude, available only to a single spacecraft acting as a leader, while maintaining the same attitude during formation maneuvers. Second, we consider the case where no leader and no external reference are used to dictate the group's behavior, and it is required that spacecraft align their attitudes with the same (possibly time varying) angular velocities. In both cases, almost global asymptotic stability results are obtained.

II. Background and Preliminaries

A. Graph Theory Preliminaries

The bidirectional information flow between spacecraft in the formation is described by undirected graphs. A weighted undirected graph $\mathscr{G}$ consists of $(\mathscr{N},\mathscr{E},\mathscr{W})$, with $\mathscr{N}$ being the set of nodes or vertices, describing the set of spacecraft in the formation, $\mathscr{E}$ the set of unordered pairs of nodes, called edges. An edge (j,k) in a weighted undirected graph indicates that nodes j and k are adjacent, or neighbors, and an undirected link exists between them. $\mathscr{W}$ is the set of weights associated to the links in the graph. If an orientation is assigned to the edges of the graph, we will obtain a weighted directed graph, $\tilde{\mathscr{G}} = (\mathscr{N},\tilde{\mathscr{E}},\mathscr{W})$, with $\tilde{\mathscr{E}}$ the set of ordered edges of the graph.

If there is a path between any two distinct nodes of a weighted undirected graph $\mathscr{G}$, then $\mathscr{G}$ is said to be connected. If there exists such a path on a weighted directed graph ignoring the direction of the edge, then the graph is weakly connected. A cycle is a connected graph with each node having exactly two neighbors. An acyclic graph is a graph with no cycles. A weighted undirected graph which is connected and acyclic is called a tree, [23]. Having a weighted directed graph, we can define the weighted incidence matrix of the graph to be the matrix D with rows indexed by vertices and columns indexed by edges with the (u,f) entry equal to $+k_f$ if vertex u is the source of the directed edge f, $-k_f$ if u is the sink of f, and 0 if u is not in the edge f. The rank of D is $n-1$ if the graph $\tilde{\mathscr{G}}$ is weakly connected, and it is full column rank if this graph is weakly connected and acyclic.

B. Spacecraft Dynamics

Consider a group formation of n spacecraft modeled as rigid bodies. The equations of motion of the j^{th} spacecraft are

$$I_{f_j}\dot{\omega}_j = \tau_j - \omega_j \times I_{f_j}\omega_j, \tag{1}$$

$$\dot{Q}_j = \frac{1}{2}Q_j \odot \bar{\omega}_j \tag{2}$$

where $\bar{\omega}_j = (\omega_j^T \;,\; 0)^T$, and ω_j denotes the angular velocity of the j^{th} spacecraft expressed in the body-fixed frame $\mathscr{F}_j$. $I_{f_j} \in \mathbb{R}^{3\times 3}$ is the symmetric positive definite constant inertia matrix of the j^{th} spacecraft with respect to $\mathscr{F}_j$. The vector τ_j is the external torque applied to the j^{th} spacecraft expressed in $\mathscr{F}_j$. The unit quaternion $Q_j = (q_j^T, q_{j,4})^T$, composed of a vector component $q_j \in \mathbb{R}^3$ and a scalar component $q_{j,4} \in \mathbb{R}$, represents the orientation of the j^{th} spacecraft frame, $\mathscr{F}_j$, with respect to the inertial frame, $\mathscr{F}_i$, which are subject to the constraint

$$q_j^T q_j + q_{j,4}^2 = 1 \tag{3}$$

The rotation matrix that brings $\mathscr{F}_i$ onto $\mathscr{F}_j$, denoted by $R(Q_j) \in \mathbb{R}^{3\times 3}$, is defined as follows

$$R(Q_j) = (q_{j,4}^2 - q_j^T q_j)I_3 + 2q_j q_j^T - 2q_{j,4}q_j\times \tag{4}$$

where '$\times$' is the vector cross product and I_3 is the 3×3 identity matrix. The quaternion multiplication between two unit quaternions, Q_j and Q_k, is defined by the following non-commutative operation

$$Q_j \odot Q_k = \left((q_{j,4}q_k + q_{k,4}q_j + q_j \times q_k)^T, q_{j,4}q_{k,4} - q_j^T q_k\right)^T \tag{5}$$

The inverse or conjugate of a unit quaternion is defined by, $Q_j^{-1} = (-q_j^T, q_{j,4})^T$, with the quaternion identity given by $(0,0,0,1)^T$, [24]. The relative attitude between the j^{th} and the k^{th} spacecraft, represented by the unit quaternion Q_{jk}, is defined as

$$Q_{jk} = Q_k^{-1} \odot Q_j, \tag{6}$$

$$\dot{Q}_{jk} = \begin{pmatrix} \dot{q}_{jk} \\ \dot{q}_{jk,4} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} q_{jk,4}\; I_3 + q_{jk}\times \\ -\; q_{jk}^T \end{pmatrix}\omega_{jk} \tag{7}$$

$$\omega_{jk} = \omega_j - R(Q_{jk})\omega_k \tag{8}$$

The vector ω_{jk} is the relative angular velocity of $\mathscr{F}_j$ with respect to $\mathscr{F}_k$ expressed in $\mathscr{F}_j$, Q_{jk} represents the rotation from $\mathscr{F}_k$ to $\mathscr{F}_j$ and $R(Q_{jk})$ is the rotation matrix related to Q_{jk} defined in (4). The following equations relating the relative states of the j^{th} and k^{th} spacecraft can be derived easily

$$R(Q_{kj})^T = R(Q_{jk}) \tag{9}$$

$$q_{kj} = -q_{jk} = -R(Q_{kj})\; q_{jk} \tag{10}$$

From the above equations, our main objective is to guarantee that all spacecraft align their attitudes, *i,e.* $q_j \to q_k$, or, $q_{jk} = (0,0,0)^T$for $j,k = 1,...,n$, without velocity measurements.

III. Auxiliary Systems Design

Consider the following auxiliary systems

$$\dot{P}_j = \frac{1}{2}P_j \odot \bar{\beta}_j, \quad j = 1,...,n \tag{11}$$

with $\bar{\beta}_j = (\beta_j^T \;,\; 0)^T$ and $\beta_j \in \mathbb{R}^3$ to be defined later. The mismatch between the auxiliary system output and the attitude of the j^{th} spacecraft is defined by

$$\delta P_j = (P_j)^{-1} \odot Q_j, \tag{12}$$

$$\delta\dot{P}_j = \begin{pmatrix} \delta\dot{p}_j \\ \delta\dot{p}_{j,4} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \delta p_{j,4}\; I_3 + \delta p_j\times \\ -\; \delta p_j^T \end{pmatrix}\Omega_j \tag{13}$$

$$\Omega_j = \omega_j - R(\delta P_j)\beta_j \tag{14}$$

where $R(\delta P_j)$ is the rotation matrix related to δP_j. We also define the following set of auxiliary systems between each spacecraft j and its neighbor k as

$$\dot{P}_{jk} = \frac{1}{2}P_{jk} \odot \bar{\beta}_{jk} \tag{15}$$

with $\bar{\beta}_{jk}=(\beta_{jk}^T\ ,\ 0)^T$ and $\beta_{jk}\in\mathbb{R}^3$ to be defined later, and

$$\delta P_{jk}=(P_{jk})^{-1}\odot Q_{jk}, \tag{16}$$

$$\dot{\delta P}_{jk}=\begin{pmatrix}\dot{\delta p}_{jk}\\ \dot{\delta p}_{jk,4}\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\delta p_{jk,4}\ I_3+\delta p_{jk}\times\\ -\ \delta p_{jk}^T\end{pmatrix}\Omega_{jk} \tag{17}$$

$$\Omega_{jk}=\omega_{jk}-R(\delta P_{jk})\beta_{jk} \tag{18}$$

It is important to note that the purpose of the dynamical systems (11) and (15) is to use the error quaternion (12) and (16) in a decentralized control law to generate the necessary damping that would have been generated by the angular velocities and the relative angular velocities of spacecraft.

IV. Leader-Follower case

We consider the case where multiple spacecraft are required to converge to a final desired attitude with zero final angular velocity, while maintaining the same attitudes during formation maneuvers. We assume that the desired attitude, represented by $Q^d=(q^{d\ T},q_4^d)^T$, is available only to one spacecraft, the leader. The unit quaternion representing the attitude error for the leader is defined as

$$\delta Q_l=(Q^d)^{-1}\odot\ Q_l$$

$$\dot{\delta Q}_l=\begin{pmatrix}\dot{\delta q}_l\\ \dot{\delta q}_{l,4}\end{pmatrix}=\frac{1}{2}\begin{pmatrix}\delta q_{l,4}\ I_3+\delta q_l\times\\ -\ \delta q_l^T\end{pmatrix}\omega_l, \tag{19}$$

where subscript "l" stands for leader.

Consider the following control law for the j^{th} spacecraft

$$\tau_j=-\sigma_j\alpha_{1j}\delta q_j-\alpha_{2j}\delta p_j-\sum_{k=1}^{n}k_{jk}^p\ q_{jk}-\sum_{k=1}^{n}k_{jk}^d\ \left(\delta p_{jk}-R(Q_{jk})\delta p_{kj}\right) \tag{20}$$

for $j=1,...n$, where n is the number of spacecraft in the formation, $\alpha_{1l}>0$, $\alpha_{2j}>0$, and

$$k_{jk}^p=k_{kj}^p\geq 0,\quad k_{jk}^d=k_{kj}^d\geq 0,\quad k_{jj}^p=k_{jj}^d=0 \tag{21}$$

for j and $k=1,...n$, and

$$\sigma_j=\begin{cases}1, & \text{if spacecraft } j \text{ is the leader}\\ 0, & \text{if spacecraft } j \text{ is a follower}\end{cases}$$

The gains k_{jk}^p and k_{jk}^d are assumed to be strictly positive if spacecraft j and k are connected (have a communication link), for $j,k=1,...n$, otherwise they are equal to zero. Therefore, the choice of these gains determines the coordination architecture considered. Also, by restriction (21), we are assuming that the communication flow between spacecraft is bidirectional. We describe the information flow between spacecraft by the weighted undirected graphs $\mathscr{G}_1=(\mathscr{N},\mathscr{E},\mathscr{K}_p)$ and $\mathscr{G}_2=(\mathscr{N},\mathscr{E},\mathscr{K}_d)$. Note that $\mathscr{G}_1$ and $\mathscr{G}_2$ have the same set of nodes and set of edges, and they differ only by the sets of weights $\mathscr{K}_{p,d}$. Hence, $\mathscr{G}_1$ and $\mathscr{G}_2$ will have the same properties, and both describe the information flow graph between spacecraft in the formation.

To this point, we can state our first result in the following theorem.

Theorem 1: Consider the formation given in (1)-(2) under the control law (20) , with restrictions (21), and let the inputs of the auxiliary systems (11) and (15) be respectively

$$\beta_j=\Gamma_j\delta p_j\quad,\quad\beta_{jk}=\Gamma_{jk}\delta p_{jk} \tag{22}$$

with $\Gamma_j=\Gamma_j^T>0$ and $\Gamma_{jk}=\Gamma_{jk}^T>0$. If the information flow graph is a tree, then all the signals are globally bounded and $q_j\rightarrow q_k\rightarrow q^d$ and $\omega_j\rightarrow\omega_k\rightarrow 0$ asymptotically, for $j,k=1,...,n$.

Proof: Consider the following positive definite Lyapunov function candidate

$$V=\sum_{j=1}^{n}\left(\frac{1}{2}\omega_j^T I_{f_j}\omega_j+2\sigma_j\alpha_{1j}(1-\delta q_{j,4})+2\alpha_{2j}(1-\delta p_{j,4})\right)+\sum_{j=1}^{n}\sum_{k=1}^{n}\left(k_{jk}^p(1-q_{jk,4})+2k_{jk}^d(1-\delta p_{jk,4})\right) \tag{23}$$

The time derivative of V evaluated along the dynamics (1)-(2) is given by

$$\dot{V}=\sum_{j=1}^{n}\left(\omega_j^T(\tau_j)+\sigma_j\alpha_{1j}\delta q_j^T\omega_j+\alpha_{2j}\Omega_j^T\delta p_j\right)+\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p q_{jk}^T\omega_{jk}+\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\delta p_{jk}^T\Omega_{jk} \tag{24}$$

Using equations (14) and (20) in (24), yields

$$\dot{V}=-\sum_{j=1}^{n}\alpha_{2j}\delta p_j^T R(\delta P_j)\beta_j+\sum_{j=1}^{n}\sum_{k=1}^{n}\left(\frac{1}{2}k_{jk}^p q_{jk}^T\omega_{jk}+k_{jk}^d\Omega_{jk}^T\delta p_{jk}\right)-\sum_{j=1}^{n}\sum_{k=1}^{n}\omega_j^T\left(k_{jk}^p\ q_{jk}+k_{jk}^d\left(\delta p_{jk}-R(Q_{jk})\delta p_{kj}\right)\right) \tag{25}$$

Using equations (8), (10) and (21), we can show that

$$\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p q_{jk}^T\omega_{jk}=\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p\omega_j^T q_{jk} \tag{26}$$

also, if we consider (21) and (18), we can write

$$\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\Omega_{jk}^T\delta p_{jk}=\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\omega_j^T\ \left(\delta p_{jk}-R(Q_{jk})\delta p_{kj}\right)-\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\beta_{jk}^T R(\delta P_{jk})^T\delta p_{jk} \tag{27}$$

From equations (22), (25)-(27) and using the fact that $q^T R(Q)=q^T$ for any unit quaternion Q, the time derivative of V is given by

$$\dot{V}=-\sum_{j=1}^{n}\alpha_{2j}\delta p_j^T\Gamma_j\delta p_j-\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\delta p_{jk}^T\Gamma_{jk}\delta p_{jk} \tag{28}$$

which is negative semi definite, which implies that $V(t)\leq V(0)$, and δQ_l, δP_j, ω_j, Q_{jk} and δP_{jk} are globally bounded. Invoking LaSalle's invariance theorem, one can show the following: $\delta p_j\rightarrow 0$ and $\delta p_{jk}\rightarrow 0$, as $t\rightarrow\infty$, which implies that $\delta p_{j,4}\rightarrow\pm 1$, $\delta p_{jk,4}\rightarrow\pm 1$, $\beta_j\rightarrow 0$, $\beta_{jk}\rightarrow 0$, $R(\delta P_j)\rightarrow I_3$ and $R(\delta P_{jk})\rightarrow I_3$. Consequently, $\Omega_j\rightarrow\omega_j$, and $\Omega_{jk}\rightarrow\omega_{jk}$.

Since $\delta p_j \to 0$ and $\delta p_{jk} \to 0$, it is clear that $\dot{\delta P}_j \to 0$ and $\dot{\delta P}_{jk} \to 0$, and hence, $\Omega_j \to 0$ and $\Omega_{jk} \to 0$, and consequently, $\omega_j \to 0$ and $\omega_{jk} \to 0$.

Using the above concluding results, we have $\tau_j \to 0$, for $j = 1, ...n$, and hence

$$\sigma_j \alpha_{1j} \delta q_j + \sum_{k=1}^{n} k^p_{jk}\ q_{jk} = 0, \quad j = 1, ...n \tag{29}$$

For further analysis of (29), we assign a direction to the undirected links of the communication graph $\mathscr{G}_1$, by considering one of the nodes of each edge to be the positive end of the link, to obtain the directed graph $\tilde{\mathscr{G}}_1 = (\mathscr{N}, \tilde{\mathscr{E}}, \mathscr{K}_p)$, with $\tilde{\mathscr{E}}$ being the set of ordered edges of the graph. The positive end of a link can be chosen arbitrarily, since we are assuming a bidirectional communication topology between spacecraft. Let $m = |\tilde{\mathscr{E}}|$ be the total number of edges in the graph $\tilde{\mathscr{G}}_1$, which is also equal to the total number of undirected links in $\mathscr{G}_1$. With the above direction assignment, and the assumption that the communication graph $\mathscr{G}_1$ is a tree, the obtained directed graph $\tilde{\mathscr{G}}_1$ is weakly connected and acyclic, and $m = n - 1$.

In addition, we consider that the desired attitude is transmitted to the leader by a fictitious spacecraft, described by an additional $(n+1)^{th}$ node in the communication graph $\tilde{\mathscr{G}}_1$, via a directed communication link constituting a new n^{th} edge in $\tilde{\mathscr{G}}_1$, with weight α_{1l} that we assume equal to $k_{l,n+1} = k_{n+1,l}$. With these assumptions, we obtain a new directed graph $\tilde{\mathscr{G}}'_1 = (\mathscr{N}', \tilde{\mathscr{E}}', \mathscr{K}'_p)$, with $(n+1)$-nodes and n-edges. The weighted incidence matrix of $\tilde{\mathscr{G}}'_1$ is $D \in \mathbb{R}^{(n+1)\times n}$ defined as

$$d_{jl^{(u,v)}} = \begin{cases} +k^p_{uv} & \text{if node } j \text{ is node } u \\ -k^p_{uv} & \text{if node } j \text{ is node } v \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

where $l^{(u,v)} : \tilde{\mathscr{E}}' \to \{1, ..., n\}$ is a function that associates a single number from the set $\{1, ..., n\}$ to each edge $(u, v) \in \tilde{\mathscr{E}}'$. We can easily verify that the obtained directed graph, $\tilde{\mathscr{G}}'_1$, is also weakly connected and acyclic, and hence the matrix D is full column rank.

Let $Q_u \in \mathscr{R}^{3n}$ be the column vector containing the vectors q_{jk}, $\forall (j,k) \in \tilde{\mathscr{E}}$, and the vector δq_l. Using the fact that $q_{jk} = -q_{kj}$, equation (29) is equivalent to

$$(D' \otimes I_3) Q_u = 0 \tag{31}$$

We can see that the $n \times n$ matrix D' is constructed by deleting the last row of the above $(n+1)\times n$ incidence matrix D, and is full rank (corollary 4.2.6 in [25]). Hence, the only solution to (31) is the trivial solution $Q_u = 0$, that is $q_{jk} = 0$, $\forall (j,k) \in \tilde{\mathscr{E}}$, and $\delta q_l = 0$, or $q_l \to q^d$. Since the graph is connected, each spacecraft is communicating with at least one other spacecraft, we have $q_{jk} = 0$, $\forall j,k \in \{1, ...n\}$, and $R(Q_{jk}) \to I_3$. Finally, one can conclude that $q_j \to q_k \to q^d$, and $\omega_j \to \omega_k \to 0$, $\forall j,k \in \{1, ...n\}$. ■

Remark 1: The control law (20) is a pure quaternion feedback, and consequently a natural saturation is achieved for the control effort as follows $\|\tau_j\| \leq \sigma_j \alpha_{1j} + \alpha_{2j} + \sum_{k=1}^{n}(k^p_{jk} + 2k^d_{jk})$.

V. CONSENSUS SEEKING WITHOUT REFERENCE TRAJECTORY

In this section, we consider the case where no desired attitude is assigned, and spacecraft are required to converge to the same (not necessarily constant) angular velocity while maintaining the same attitudes during formation maneuvers, *i.e.,* $q_j \to q_k$ and $\omega_j \to \omega_k$. Again, we assume that the communication between spacecraft is bidirectional and the spacecraft angular velocities are not available.

We assume that spacecraft communicate the output of their individual auxiliary systems δP_j, defined in (12), and consider the new unit quaternion that describes the discrepancy between the output of the j^{th} and k^{th} auxiliary systems as

$$\delta \tilde{P}_{jk} = (\delta P_k)^{-1} \odot \delta P_j \tag{32}$$

$$\dot{\delta \tilde{P}}_{jk} = \begin{pmatrix} \dot{\delta \tilde{p}}_{jk} \\ \dot{\delta \tilde{p}}_{jk,4} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \delta \tilde{p}_{jk,4} I + \delta \tilde{p}_{jk} \times \\ -\delta \tilde{p}^T_{jk} \end{pmatrix} \tilde{\Omega}_{jk} \tag{33}$$

$$\tilde{\Omega}_{jk} = \Omega_j - R(\delta \tilde{P}_{jk}) \Omega_k \tag{34}$$

The following properties can easily be shown

$$R(\delta \tilde{P}_{kj})^T = R(\delta \tilde{P}_{jk}) \tag{35}$$

$$\delta \tilde{p}_{kj} = -\delta \tilde{p}_{jk} = -R(\delta \tilde{P}_{kj}) \delta \tilde{p}_{jk} \tag{36}$$

We propose the following control law for the j^{th} spacecraft

$$\tau_j = -\sum_{k=1}^{n} k^p_{jk} q_{jk} - \sum_{k=1}^{n} k^d_{jk} \left(\delta p_{jk} - R(Q_{jk}) \delta p_{kj} + \delta \tilde{p}_{jk} \right) \tag{37}$$

with the gains k^p_{jk} and k^d_{jk} are defined as in (21), and state the following result

Theorem 2: Consider the formation given in (1)-(2) under the control law (37) , with restrictions (21), and let the inputs of the auxiliary systems (11) and (15) be respectively

$$\beta_j = R(\delta P_j)^T \Gamma_j \left(\sum_{k=1}^{n} k^d_{jk} \delta \tilde{p}_{jk} \right) \quad , \quad \beta_{jk} = \Gamma_{jk} \delta p_{jk} \tag{38}$$

with $\Gamma_j = \Gamma^T_j > 0$ and $\Gamma_{jk} = \Gamma^T_{jk} > 0$. If the information flow graph is a tree, then all the signals are globally bounded and $q_j \to q_k$ and $\omega_j \to \omega_k$ asymptotically, for all $j,k = 1, ..., n$.

Proof: Consider the Lyapunov function candidate

$$\begin{aligned} V \quad = \quad & \frac{1}{2} \sum_{j=1}^{n} \omega^T_j I_{f_j} \omega_j + \sum_{j=1}^{n} \sum_{k=1}^{n} k^p_{jk} (1 - q_{jk,4}) \\ & + \sum_{j=1}^{n} \sum_{k=1}^{n} k^d_{jk} \left(2(1 - \delta p_{jk,4}) + (1 - \delta \tilde{p}_{jk,4}) \right) \end{aligned} \tag{39}$$

Using equations (34)-(36) and (14), we can show that

$$\begin{aligned} \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} k^d_{jk} \delta \tilde{p}^T_{jk} \tilde{\Omega}_{jk} &= \sum_{j=1}^{n} \sum_{k=1}^{n} k^d_{jk} \delta \tilde{p}^T_{jk} \Omega_j \\ &= \sum_{j=1}^{n} \sum_{k=1}^{n} k^d_{jk} \delta \tilde{p}^T_{jk} \omega_j - \sum_{j=1}^{n} \sum_{k=1}^{n} k^d_{jk} \delta \tilde{p}^T_{jk} R(\delta P_j) \beta_j \end{aligned}$$

Following a similar analysis as in the proof of theorem 1, the time derivative of V in (39) evaluated along the system dynamics (1)-(2), with (37) and (38), is obtained as

TABLE I

SIMULATION PARAMETERS

$Q_1(0)=(0,0,1,0)$, $Q_2(0)=(1,0,0,0)$, $Q_3(0)=(0,1,0,0)$, $Q_4(0)=(0,0,sin(-\pi/4),\cos(-\pi/4))$, $\omega_1(0)=(-0.5,0.5,-0.45)$, $\omega_2(0)=(0.5,-0.3,0.1)$, $\omega_3(0)=(0.1,0.6,-0.1)$, $\omega_4(0)=(0.4,0.4,-0.5)$, $\Gamma_j=\Gamma_{jk}=diag(6,6,6)$, $P_j(0)=P_{jk}(0)=\tilde{P}_{jk}(0)=(1,0,0,0)$, $\alpha_{1l}=70$, $\alpha_{2j}=90$, $k^p_{jk}=50$, $k^d_{jk}=25$, $for\ j,k\in\mathscr{E}$.

$$\begin{aligned}\dot{V} \;\; = \;\; & -\sum_{j=1}^{n}\sum_{k=1}^{n}k^d_{jk}\delta p^T_{jk}\Gamma_{jk}\delta p_{jk}\\ & -\sum_{j=1}^{n}\left(\sum_{k=1}^{n}k^d_{jk}\delta\tilde{p}_{jk}\right)^T\Gamma_j\left(\sum_{k=1}^{n}k^d_{jk}\delta\tilde{p}_{jk}\right)\end{aligned}$$

from which we can conclude that $\dot{V}$ is negative semi definite, and hence, Q_{jk}, δP_{jk}, $\delta\tilde{P}_{jk}$ and ω_j are globally bounded. Invoking LaSalle's invariance theorem, one can show the following (through standard signal chasing): $\delta p_{jk}\to 0$ and

$$\sum_{k=1}^{n}k^d_{jk}\delta\tilde{p}_{jk}\to 0, \quad j=1,...n \tag{40}$$

which implies that $\beta_j\to 0$ and $\beta_{jk}\to 0$, allowing to conclude that $\Omega_{jk}\to\omega_{jk}$ and $\Omega_j\to\omega_j$. In order to determine the solutions of (40), we consider the same direction assignment to the communication graph, as in the proof of Theorem 1, to obtain the directed graph $\tilde{\mathscr{G}}_2$, which is weakly connected and acyclic, since $\mathscr{G}_2$ is assumed to be a tree. From (36), we can see that (40) can be rewritten as

$$(M\otimes I_3)P_u=0 \tag{41}$$

with $P_u\in\mathscr{R}^{3(n-1)}$ being the column vector containing all $\delta\tilde{p}_{jk}$, $\forall(j,k)\in\tilde{\mathscr{E}}$, and $M\in\mathbb{R}^{n\times(n-1)}$ is the weighted incidence matrix of $\tilde{\mathscr{G}}_2$, defined as in (30), with the superscript "p" in the weights replaced by "d". The matrix M is full column rank, from which one can conclude that the only solution to (41) is $P_u=0$, that is $\delta\tilde{p}_{jk}\to 0$, $\forall(j,k)\in\tilde{\mathscr{E}}$. Since the communication graph is connected, we conclude that $\delta\tilde{p}_{jk}\to 0$, and $R(\delta\tilde{P}_{jk})\to I_3$, for all $j,k=1,...n$. Furthermore, since $\delta p_{jk}\to 0$ and $\delta\tilde{p}_{jk}\to 0$, we have $\delta\dot{p}_{jk}\to 0$ and $\delta\dot{\tilde{p}}_{jk}\to 0$, and hence $\Omega_{jk}\to 0$ and $\tilde{\Omega}_{jk}\to 0$. Since $\Omega_{jk}\to\omega_{jk}$ and from (34), it is straightforward to write

$$\begin{cases}\omega_{jk}\to 0,\\ \Omega_j-R(\delta\tilde{P}_{jk})\Omega_k\to 0 \;\Rightarrow\; \Omega_j\to\Omega_k\end{cases}$$

Hence, from (8) and since $\Omega_j\to\omega_j$, we have

$$\begin{cases}\omega_j\to R(Q_{jk})\omega_k,\\ \omega_j\to\omega_k\end{cases}$$

which leads us to conclude that $R(Q_{jk})\to I_3$, that is $q_j\to q_k$ and $\omega_j\to\omega_k$, for all $j,k=1,...n$. ■

The sufficient condition used in Theorem 2 (*i.e.,* the communication graph is a tree), can be relaxed to remove the acyclic requirement, under certain conditions on the scalar parts of δP_j, $j\in\{1,...n\}$, as stated in the following corollary.

Corollary 1: If there exists $T>0$ such that $\delta p_{j,4}(t)>0$ (or $\delta p_{j,4}(t)<0$), for all $t\geq T$ and for all $j\in\{1,...n\}$, then the result of Theorem 2 holds if the communication graph is connected.

Proof: From equation (40), using (3) and (5), we can write

$$\begin{aligned}&\textstyle\sum_{j=1}^n\delta p^T_j\sum_{k=1}^n k^d_{jk}\delta\tilde{p}_{jk}\\ &=\textstyle\sum_{j=1}^n\sum_{k=1}^n k^d_{jk}\left(\delta p_{k,4}\delta p^T_j\delta p_j-\delta p_{j,4}\delta p^T_j\delta p_k\right)\\ &=\textstyle\sum_{j=1}^n\sum_{k=1}^n k^d_{jk}\left(\delta p_{k,4}(1-\delta p^2_{j,4})-\delta p_{j,4}\delta p^T_j\delta p_k\right)\\ &=\textstyle\sum_{j=1}^n\sum_{k=1}^n k^d_{jk}\left(\delta p_{j,4}-\delta p_{j,4}(\delta p_{k,4}\delta p_{j,4}+\delta p^T_j\delta p_k)\right)\\ &=\textstyle\sum_{j=1}^n\sum_{k=1}^n k^d_{jk}\delta p_{j,4}\left(1-\delta\tilde{p}_{jk,4}\right)=0\end{aligned} \tag{42}$$

Note that (42) holds when t tends to infinity. Hence, it is clear that if there exists $T>0$ such that $\delta p_{j,4}(t)>0$ (or $\delta p_{j,4}(t)<0$), for all $t\geq T$ and for all $j\in\{1,...n\}$, then the only solution to (42) is $\delta\tilde{p}_{jk,4}=1$, that is $\delta p_j\to\delta p_k$, for all $j,k\in\mathscr{E}$, and if the graph is connected, this is verified for all $j,k\in\{1,...n\}$. The rest of the proof is similar to the proof of Theorem 2. ■

Remark 2: It is worth noticing that the control law (37) is a pure quaternion feedback, and consequently a natural saturation is achieved for the control effort as follows $\|\tau_j\|\leq\sum_{k=1}^n(k^p_{jk}+3k^d_{jk})$.

VI. SIMULATION RESULTS

Using SIMULINK, we consider a scenario where four spacecraft are required to align their attitudes under a bidirectional communication flow graph satisfying the conditions in Theorems 1 and 2. The spacecraft are modeled as rigid bodies whose inertia matrices are taken as $I_{f_j}=diag(20,20,30)$. The simulation parameters are illustrated in table I.

The obtained results are illustrated in Figures 1-4. Figure 1 shows the components of the unit quaternion, Q^i_j, $i=1,...4$, representing the attitude of the four spacecraft in the formation (we use the superscript (i) to denote the i^{th} component of a vector), in the leader-follower case, where we consider that the desired attitude is available only to spacecraft one, and is $q^d(0)=(0,0,0,1)^T$. Note that all spacecraft converge to the same desired attitude. In Figure 2 we illustrate the elements of spacecraft angular velocity vectors, from which the convergence to zero is clear. Figures 3 and 4 illustrate the obtained results in the consensus seeking case without leader and without reference trajectory, where we can see that spacecraft reach an agreement and converge to the same final time varying attitude and angular velocity.

VII. CONCLUSION

We addressed the problem of quaternion-based attitude synchronization of spacecraft flying formation without velocity measurements. First, we proposed a velocity-free leader-follower control strategy that guarantees attitude synchronization with zero angular velocities, under a connected acyclic undirected communication graph. We showed that a modification of the proposed scheme solves the agreement seeking problem, where the attitude alignment of the group within a formation is ensured with (not necessarily constant)

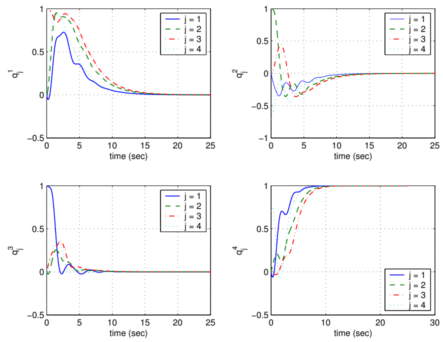

Fig. 1. Spacecraft attitudes in the leader-follower control scheme

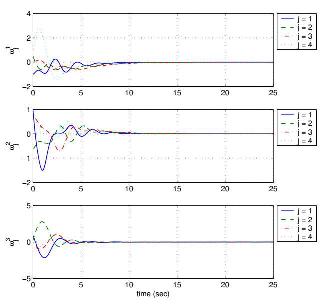

Fig. 2. The three elements of the spacecraft angular velocities in the leader-follower scheme

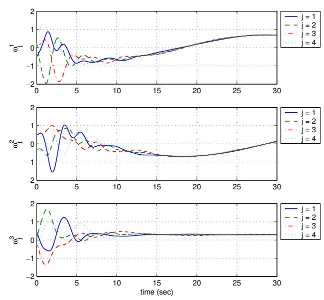

Fig. 3. The three elements of the spacecraft angular velocity vectors in consensus seeking case

final attitudes and angular velocities. Almost global asymptotic stability results are obtained [26]. It is important to mention that although we consider the velocity-free attitude synchronization problem in the context of spacecraft within a formation, our results are applicable to the attitude synchronization problem among rigid bodies satisfying the rotational dynamics. Simulation results have shown the effectiveness of the proposed control schemes.

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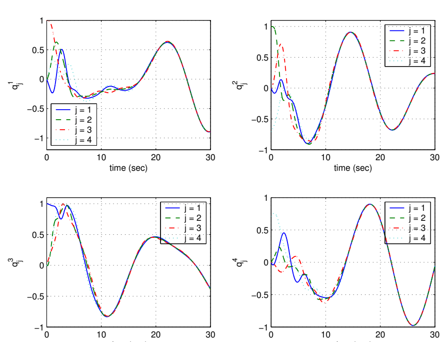

Fig. 4. Spacecraft attitudes in the consensus seeking case

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