Attitude Synchronization of a Group of Spacecraft Without Velocity Measurements

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***Abstract*—We consider the coordinated attitude control problem for a group of spacecraft, without velocity measurements. Our approach is based on the introduction of auxiliary dynamical systems (playing the role of velocity observers in a certain sense) to generate the individual and relative damping terms in the absence of the actual angular velocities and relative angular velocities. Our main focus, in this technical note, is to address the following two problems: 1) Design a velocity-free attitude tracking and synchronization control scheme, that allows the team members to align their attitudes and track a time-varying reference trajectory (simultaneously). 2) Design a velocity-free synchronization control scheme, in the case where no reference attitude is specified, and all spacecraft are required to reach a consensus by aligning their attitudes with the same final time-varying attitude. In this work, one important and novel feature (besides the non-requirement of the angular velocity measurements), consists in the fact that the control torques are naturally bounded and the designer can arbitrarily assign the desired bounds on the control torques, *a priori*, through the control gains, regardless of the angular velocities. Throughout this technical note, the communication flow between spacecraft is assumed to be undirected. Simulation results of a scenario of four spacecraft are provided to show the effectiveness of the proposed control schemes.**

***Index Terms*—Attitude synchronization, attitude tracking, consensus, output feedback, spacecraft.**

I. Introduction

Cooperative and formation control of autonomous vehicles have received extensive interests in recent years leading to significant theoretical developments [3], [4]. In particular, the use of graph theory produced many interesting results [5], [6]. The above mentioned papers, mainly deal with simple dynamic models such as linear systems and single or double integrators, and hence they are often limited when it comes to dealing with rigid body dynamics. Recently, several papers have investigated the problem of controlling and maintaining the relative attitudes of formation flying spacecraft, or rigid bodies in general, and several approaches have been proposed, from which some common fundamental aspects can be extracted. Roughly, four main approaches can be found in the literature: Multiple input-multiple output (MIMO), leader-following, virtual structures and the behavioral methods, (see for instance [7]–[16] and references there in).

The above coordination control strategies are mainly based on the assumption that each spacecraft (vehicle) knows its own angular velocity, and the angular velocity of its neighbors. In this technical note, we consider the attitude synchronization problem of formation flying spacecraft and remove the requirement for the angular velocity and the relative angular velocities. In this context, the authors in [13] present a local passivity based control law for multi-spacecraft attitude alignment without velocity measurements, assuming a ring communication topology. In [17], the Modified Rodriguez Parameters have been used to extend the work of [13] to the case of a general undirected communication topology. In both works, the authors consider the case where the final angular velocity is zero, and the extension of the obtained results to the trajectory tracking case is not obvious.

In this technical note, we provide solutions to two different problems. The first problem consists of designing a control law that allows to achieve simultaneous attitude tracking and synchronization of a group of spacecraft without velocity measurements and without any restriction on the graph topology. In contrast to the velocity-free synchronization schemes available in the literature, our proposed control scheme allows to handle time-varying reference trajectories. This attitude tracking and synchronization scheme can be classified as a behavioral type in the sense that two different objectives (behaviors), namely tracking and synchronization, can be achieved simultaneously. A priority between the two objectives can be established through the choice of the control gains. In fact, this approach allows to synchronize a group of spacecraft before converging as a formation to the desired reference trajectory. This might be useful in several applications, such as spacecraft interferometry, where accurate spacecraft alignment is required while tracking a desired trajectory. We also show that the proposed control law can be simplified further by removing the condition on the gains as long as the graph topology is an undirected tree. This velocity-free result is quite similar to the results obtained in the full information case (*i.e.,* with velocity measurement) in [14]–[16].

The second problem solved in this technical note is the case where no leader and no reference trajectory are used to dictate the group's objective, and it is required that the spacecraft align their attitudes with the same (not necessarily constant) angular velocities, under an undirected, connected and acyclic graph. To the best of the knowledge of the authors, this technical note is the first dealing with the above aforementioned problems without velocity measurements.

To solve the above mentioned problems without requiring the angular velocity measurements, we rely on the auxiliary systems approach recently introduced in [18]. It consists of associating an auxiliary dynamic system to each spacecraft and to each pair of spacecraft with a communication link in order to recover and generate the necessary damping that would have been generated by the actual angular velocities and relative angular velocities. It is worth pointing out that by removing the velocity measurements for a formation with a large number of spacecraft, we reduce the cost related to the sensors and the communication flow between spacecraft, and guarantee a certain level of immunity against angular velocity sensors failure.

II. Spacecraft Dynamics and Problem Formulation

Consider a group of n spacecraft modeled as rigid bodies. The equations of motion of the j^{th} spacecraft are

$$I_{f_j}\dot{\omega}_j = \tau_j - S(\omega_j)I_{f_j}\omega_j, \tag{1}$$

$$\dot{\mathbf{q}}_j = \frac{1}{2}\mathbf{q}_j \odot \bar{\omega}_j = \frac{1}{2}\begin{pmatrix} \eta_j\, I_3 + S(q_j) \\ -q_j^T \end{pmatrix}\omega_j \tag{2}$$

where $\bar{\omega}_j^T = (\omega_j^T\ ,\ 0)$, and $\omega_j \in \mathbb{R}^3$ denotes the angular velocity of the j^{th} spacecraft expressed in the body-fixed frame $\mathcal{F}_j$. $I_{f_j} \in \mathbb{R}^{3\times 3}$ is a constant symmetric positive definite inertia matrix of the j^{th} spacecraft with respect to $\mathcal{F}_j$. The vector τ_j is the external torque applied to the j^{th} spacecraft expressed in $\mathcal{F}_j$. The unit-quaternion $\mathbf{q}_j = (q_j^T, \eta_j)^T$ is composed of a real part $\eta_j \in \mathbb{R}$ and a vector part $q_j \in \mathbb{R}^3$, and

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represents the orientation between the j^{th} spacecraft frame $\mathcal{F}_j$ and the inertial frame $\mathcal{F}_i$. The elements of a unit-quaternion $\mathbf{q}_j$ are subject to the constraint

$$\eta_j^2 + q_j^T q_j = 1. \tag{3}$$

The quaternion multiplication "$\odot$" of two unit-quaternion $\mathbf{q}_j = (q_j^T, \eta_j)^T$ and $\mathbf{q}_k = (q_k^T, \eta_k)^T$ is distributive and associative but not commutative, and is defined as

$$\mathbf{q}_j \odot \mathbf{q}_k = \begin{pmatrix} \eta_j q_k + \eta_k q_j + S(q_j) q_k \\ \eta_j \eta_k - q_j^T q_k \end{pmatrix} \tag{4}$$

where the matrix $S(\mathbf{x})$ is the skew-symmetric matrix such that $S(\mathbf{x})V = \mathbf{x} \times V$ for any vector $V \in \mathbb{R}^3$. The unit-quaternion inverse is given by $\mathbf{q}_j^{-1} = (-q_j^T, \eta_j)^T$. The orthogonal rotation matrix related to the unit-quaternion $\mathbf{q}_j$, that brings the inertial frame into the body frame, is defined as $R(\mathbf{q}_j)$, and can be obtained through the Rodriguez formula as

$$R(\mathbf{q}_j) = (\eta_j^2 - q_j^T q_j) I_3 + 2 q_j q_j^T - 2\eta_j S(q_j). \tag{5}$$

Assume that the desired trajectory is given by the unit quaternion $\mathbf{q}_d = (q_d^T, \eta_d)^T$ that represents the orientation of the desired frame, denoted by $\mathcal{F}_d$, and satisfies the unit-quaternion dynamics: $\dot{\mathbf{q}}_d = (1/2)\mathbf{q}_d \odot \bar{\omega}_d$, with $\bar{\omega}_d = (\omega_d^T, 0)^T$, and $\omega_d \in \mathbb{R}^3$ is the angular velocity of $\mathcal{F}_d$ expressed in $\mathcal{F}_d$, which is assumed to be bounded as well as its first and second time-derivatives. The discrepancy between the absolute attitude of the j^{th} spacecraft and the desired attitude defines the attitude tracking error for spacecraft j, namely $\tilde{\mathbf{q}}_j = (\tilde{q}_j^T, \tilde{\eta}_j)^T$, and is given by: $\delta_{qj}\tilde{\mathbf{q}}_j = \tilde{q}_d^{-1} \odot \mathbf{q}_j$, and obeys to the unit-quaternion dynamics

$$\dot{\tilde{q}}_j = \frac{1}{2}(\tilde{\eta}_j\, I_3 + S(\tilde{q}_j))\tilde{\omega}_j, \quad \dot{\tilde{\eta}}_j = -\frac{1}{2}\tilde{q}_j^T \tilde{\omega}_j, \tag{6}$$

$$\tilde{\omega}_j = \omega_j - R(\tilde{\mathbf{q}}_j)\, \omega_d \tag{7}$$

where $\tilde{\omega}_j$ is the angular velocity error vector describing the relative angular velocity of $\mathcal{F}_j$ with respect to $\mathcal{F}_d$ expressed in $\mathcal{F}_j$. Matrix $R(\tilde{\mathbf{q}}_j)$ is the rotation matrix, related to $\tilde{\mathbf{q}}_j$, that brings $\mathcal{F}^d$ onto $\mathcal{F}_j$ and is given by $R(\tilde{\mathbf{q}}_j) = R(\mathbf{q}_j)R(\mathbf{q}_d)^T$, [19].

Following the same steps as in [18], and using (1), (7) and the cross product properties, one can show that the angular velocity error dynamics for the j^{th} spacecraft satisfy

$$\tilde{\omega}_j^T I_{f_j} \dot{\tilde{\omega}}_j = \tilde{\omega}_j^T (\tau_j - \mathbf{F}(\dot{\omega}_d, \omega_d, \tilde{\mathbf{q}}_j)) \tag{8}$$

with $\mathbf{F}(\dot{\omega}_d, \omega_d, \tilde{\mathbf{q}}_j) = I_{f_j} R(\tilde{\mathbf{q}}_j)\dot{\omega}_d + S(R(\tilde{\mathbf{q}}_j)\omega_d) I_{f_j} R(\tilde{\mathbf{q}}_j)\omega_d$.

In the sequel, we say that the j^{th} and k^{th} spacecraft are neighbors, or connected by a communication link, if they have access to their relative information. In our case, two neighbors need to know their relative attitudes. The relative attitude between the j^{th} and k^{th} spacecraft can be either computed in each spacecraft, if their absolute attitudes are communicated to each other, or measured if each spacecraft is equipped with relative attitude sensors. The relative attitude between the j^{th} and k^{th} spacecraft, namely $\mathbf{q}_{jk} = (q_{jk}^T, \eta_{jk})^T$, is defined as: $\mathbf{q}_{jk} = \mathbf{q}_k^{-1} \odot \mathbf{q}_j$, and is governed by the following dynamics:

$$\dot{q}_{jk} = \frac{1}{2}(\eta_{jk}\, I_3 + S(q_{jk}))\omega_{jk}, \quad \dot{\eta}_{jk} = -\frac{1}{2} q_{jk}^T \omega_{jk} \tag{9}$$

$$\omega_{jk} = \omega_j - R(\mathbf{q}_{jk})\omega_k \tag{10}$$

where $\mathbf{q}_{jk}$ represents the rotation from $\mathcal{F}_k$ to $\mathcal{F}_j$, $R(\mathbf{q}_{jk})$ is the rotation matrix related to $\mathbf{q}_{jk}$, and the vector ω_{jk} is the relative angular velocity of $\mathcal{F}_j$ with respect to $\mathcal{F}_k$ expressed in $\mathcal{F}_j$. Using (4) and (5), the following relations can be easily verified:

$$R(\mathbf{q}_{kj})^T = R(\mathbf{q}_{jk}), \quad q_{kj} = -q_{jk} = -R(\mathbf{q}_{kj})\, q_{jk}. \tag{11}$$

With the above definitions, we can see that attitude tracking is achieved when $\mathbf{q}_j$ coincides with $\mathbf{q}^d$, such that $\tilde{\mathbf{q}}_j = (\mathbf{0}^T, \pm 1)^T$, and $\tilde{\omega}_j = \mathbf{0}$, which is equivalent, from (5), to $R(\tilde{\mathbf{q}}_j) = I_3$ and, from (7), to $\omega_j = \omega_d$. Note that due to the inherent redundancy of the quaternion representation, $\mathbf{q}$ and $-\mathbf{q}$ represent the same physical orientation however, one is rotated 2π relative to the other about an arbitrary axis. Accordingly, $\tilde{\mathbf{q}}_j = (\mathbf{0}^T, \pm 1)^T$ correspond to the same physical point. In addition, group alignment is attained, *i,e,* $\mathbf{q}_j$ coincides with $\mathbf{q}_k$ for all $j, k \in \{1, \ldots, n\}$, when $\mathbf{q}_{jk} = (\mathbf{0}^T, \pm 1)^T$ and $\omega_{jk} = \mathbf{0}$, and equivalently, $R(\mathbf{q}_{jk}) = I_3$ and $\omega_j = \omega_k$ for all $j, k \in \{1, \ldots, n\}$.

In this technical note, our main objective is to design coordinated attitude control laws without angular velocity measurements for each spacecraft to solve the following problems:

1) OBJ1: Design a velocity-free attitude tracking and synchronization scheme such that each spacecraft tracks the desired trajectory, and the relative attitudes and angular velocities between the team members converge to zero, simultaneously, *i.e.,* $\mathbf{q}_j(t) \to \mathbf{q}_k(t) \to \mathbf{q}_d(t)$ and $\omega_j(t) \to \omega_k(t) \to \omega_d(t)$, for all $j, k \in \{1, \ldots, n\}$.

2) OBJ2: We assume that no reference signal is available to any spacecraft, and we want to design a velocity-free synchronization scheme such that spacecraft align their attitudes, *i.e.,* $\mathbf{q}_j(t) \to \mathbf{q}_k(t)$ and $\omega_j(t) \to \omega_k(t)$, using only local information transmitted between neighbors among the group.

III. Simultaneous Attitude Tracking and Synchronization

In this section, we consider the first problem (OBJ1) which consists of the design of a simultaneous attitude tracking and synchronization scheme without velocity measurements, allowing a group of spacecraft to align their attitudes with a time-varying reference attitude, while maintaining the same relative attitude during formation maneuvers.

A. Auxiliary Systems

Instrumental in our approach, the concept of the auxiliary systems introduced in [18] to remove the angular velocity measurements. In fact, we associate a unit-quaternion auxiliary system to each individual spacecraft, defined as follows:

$$\dot{\mathbf{p}}_j = \frac{1}{2}\mathbf{p}_j \odot \bar{\beta}_j \tag{12}$$

with $\bar{\beta}_j = (\beta_j^T\ ,\ 0)^T$ and $\beta_j \in \mathbb{R}^3$ to be designed later. The mismatch between the auxiliary system output and the attitude tracking error for the j^{th} spacecraft is defined by the unit-quaternion $\tilde{\mathbf{p}}_j = (\tilde{p}_j^T, \tilde{\epsilon}_j)^T$ given by: $\tilde{\mathbf{p}}_j = \mathbf{p}_j^{-1} \odot \tilde{\mathbf{q}}_j$, satisfying the unit-quaternion dynamics

$$\dot{\tilde{p}}_j = \frac{1}{2}(\tilde{\epsilon}_j\, I_3 + S(\tilde{p}_j))\Omega_j, \quad \dot{\tilde{\epsilon}}_j = -\frac{1}{2}\tilde{p}_j^T \Omega_j \tag{13}$$

$$\Omega_j = \tilde{\omega}_j - R(\tilde{\mathbf{p}}_j)\beta_j \tag{14}$$

where $R(\tilde{\mathbf{p}}_j)$ is the rotation matrix related to $\tilde{\mathbf{p}}_j$.

We also associate a unit-quaternion auxiliary system to each pair of spacecraft (j, k), with a communication link, defined as follows:

$$\dot{\mathbf{p}}_{jk} = \frac{1}{2}\mathbf{p}_{jk} \odot \bar{\beta}_{jk} \tag{15}$$

with $\bar{\beta}_{jk} = (\beta_{jk}^T\ ,\ 0)^T$ and $\beta_{jk} \in \mathbb{R}^3$ to be designed later. We define the unit quaternion describing the discrepancy between this auxiliary system output and the relative attitude error between the j^{th} and k^{th} spacecraft, $\tilde{\mathbf{p}}_{jk} = (\tilde{p}_{jk}^T, \tilde{\epsilon}_{jk})^T$, as: $\tilde{\mathbf{p}}_{jk} = \mathbf{p}_{jk}^{-1} \odot \mathbf{q}_{jk}$, governed by the following dynamics:

$$\dot{\tilde{p}}_{jk} = \frac{1}{2}(\tilde{\epsilon}_{jk}\, I_3 + S(\tilde{p}_{jk}))\Omega_{jk}, \quad \dot{\tilde{\epsilon}}_{jk} = -\frac{1}{2}\tilde{p}_{jk}^T \Omega_{jk} \tag{16}$$

$$\Omega_{jk} = \omega_{jk} - R(\tilde{\mathbf{p}}_{jk})\beta_{jk}. \tag{17}$$

The main idea behind the introduction of the auxiliary systems is to provide an *indirect* asymptotic estimation of the angular velocities and relative angular velocities to generate the necessary damping for the overall closed loop stability. To explain the mechanism, let us consider the auxiliary system (12) associated to spacecraft j. Through an appropriate choice of the control input τ_j (that will be presented later), one can generate a passive mapping between the auxiliary system input β_j and the vector part of the unit-quaternion error $\tilde{\mathbf{p}}_j$, namely $\tilde{p}_j$. Hence, picking β_j as a simple feedback in terms of $\tilde{p}_j$, will guarantee the convergence of $\tilde{p}_j$ to zero, which in turns forces β_j towards $\tilde{\omega}_j$ asymptotically. Consequently, from this perspective, a particular asymptotic observer for $\tilde{\omega}_j$ is realized in the sense that $\tilde{\omega}_j$ can be replaced by $\tilde{p}_j$ to generate the necessary damping in the control law τ_j. A similar interpretation can be given for the relative auxiliary system (15) where the relative angular velocity ω_{jk} between spacecraft j and k can be estimated (asymptotically) through the relative auxiliary system input β_{jk} (which is taken proportional to the vector part of the unit quaternion $\tilde{\mathbf{p}}_{jk}$, namely $\tilde{p}_{jk}$). This will allow to replace the relative angular velocity ω_{jk} by $\tilde{p}_{jk}$ to generate the relative damping between spacecraft j and k.

B. Control Law Design for OBJ1

Based on the coupled dynamics controller proposed in [12], we propose a control scheme that consists of two terms in order to achieve two different objectives/behaviors. The first term aims to track a desired attitude and angular velocity, in order to achieve the goal-seeking behavior, and the second term is used to achieve the formation-keeping behavior by ensuring attitude synchronization of spacecraft in the formation while tracking the desired trajectory. Consider the following control action for the j^{th} spacecraft, given by

$$\tau_j = \underbrace{\mathbf{F}(\dot{\omega}_d, \omega_d, \tilde{\mathbf{q}}_j) - \alpha_{1j}\tilde{q}_j - \alpha_{2j}\tilde{p}_j}_{goal-seeking} \underbrace{- \sum_{k=1}^{n} k_{jk}^{p}\, q_{jk} - \sum_{k=1}^{n} k_{jk}^{d}\, (\tilde{p}_{jk} - R(\mathbf{q}_{jk})\tilde{p}_{kj})}_{formation-keeping} \tag{18}$$

where n is the number of spacecraft in the formation, α_{1j} and α_{2j} are strictly positive gains that we will call attitude tracking control gains and k_{jk}^p, k_{jk}^d are the formation-keeping behavior gains such that $k_{jj}^\star \triangleq 0$ and

$$\begin{cases} k_{jk}^\star = k_{kj}^\star > 0, & \text{if spacecraft } j \text{ and } k \text{ are connected} \\ k_{jk}^\star = k_{kj}^\star = 0, & \text{otherwise} \end{cases} \tag{19}$$

for $j, k \in \{1, \ldots, n\}$, $\star \in \{p, d\}$. The magnitude of a nonzero k_{jk}^p and/or k_{jk}^d determines the strength of the connection between spacecraft. Therefore, various coordination architectures can be used by different choices of these gains, [15]. In addition, by restrictions (19), we are assuming that the communication flow between spacecraft is undirected.

Our first result is stated in the following theorem.

Theorem 1: Consider the formation given in (1)–(2) under the control law (18), with (19), and let the inputs of the auxiliary systems (12) and (15) be, respectively

$$\beta_j = \Gamma_j \tilde{p}_j, \quad \beta_{jk} = \Gamma_{jk}\tilde{p}_{jk} \tag{20}$$

with $\Gamma_j = \Gamma_j^T > 0$ and $\Gamma_{jk} = \Gamma_{jk}^T > 0$. If the control gains satisfy

$$\alpha_{1j} > 2\sum_{k=1}^{n} k_{jk}^{p} \tag{21}$$

for $j \in \{1, \ldots, n\}$, then all the signals are globally bounded and $q_j(t) \to q_k(t) \to q_d(t)$ and $\omega_j(t) \to \omega_k(t) \to \omega_d(t)$ asymptotically, $\forall j, k \in \{1, \ldots, n\}$. Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$, for all $t \geq T$ and $j \in \{1, \ldots, n\}$, then the same convergence results are obtained without condition (21).

Proof: See Appendix . ∎

It is worth noticing that condition (21) is restrictive in the sense that priority is given to the goal-seeking behavior over the formation-keeping behavior. This condition is not required if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$ for all $t > T$. From a practical point of view, this assumption can always be satisfied, and the scalar parts of unit-quaternion are ensured to be positive for all $t \geq 0$ if one restricts the rotation angle to be in $[-\pi, \pi]$.

Remark 1: It is important to note that the control law (18) consists of pure unit-quaternion feedback terms, and terms depending on the desired angular velocity, its derivative and the inertia matrix. Consequently, the control effort is bounded (regardless of the angular velocities) as follows: $\|\tau_j\| \leq \|I_{f_j}\|(\varrho + \rho^2) + \alpha_{1j} + \alpha_{2j} + \sum_{k=1}^{n}(k_{jk}^p + 2k_{jk}^d)$, with ϱ and ρ are the upper bounds of $\dot{\omega}_d(t)$ and $\omega_d(t)$, respectively. Hence, the designer can easily set the desired bounds on the control torques via an appropriate choice of the control gains.

In order to implement the proposed control scheme given in (18), spacecraft j must be able to compute the unit-quaternion $\tilde{\mathbf{q}}_j$ and the vector parts of the unit-quaternion $\mathbf{q}_{jk}$, $\tilde{\mathbf{p}}_j$, $\tilde{\mathbf{p}}_{jk}$ and $\tilde{\mathbf{p}}_{kj}$. The first four variables can be computed if the absolute attitudes of spacecraft j and k are available to spacecraft j. This can be realized either by relative sensing or by transmitting spacecraft absolute attitudes, $\mathbf{q}_j$, between neighbors in the team. The last variable, $\tilde{\mathbf{p}}_{kj}$, must also be transmitted via the communication channels. Therefore, the proposed control scheme does not increase the communication requirements as compared to the full information case where both attitudes and angular velocities are communicated between neighbors. In this case, The information flow between spacecraft can be described by the two undirected graphs $\mathcal{G}_1 = (\mathcal{N}, \mathcal{E}, \mathcal{K}_p)$ and $\mathcal{G}_2 = (\mathcal{N}, \mathcal{E}, \mathcal{K}_d)$. $\mathcal{N} = \{1, \ldots, n\}$ is the set of nodes or vertices, describing the set of spacecraft in the formation, $\mathcal{E}$ is the set of unordered pairs of nodes, called edges. An edge (j, k) indicates that spacecraft j and k are neighbors and can obtain information from one another. $\mathcal{K}_\star$ is the set of weights associated to the links in the graph. Note that $\mathcal{G}_1$ and $\mathcal{G}_2$ have the same set of nodes and set of edges, and they differ only by the sets of weights $\mathcal{K}_\star$ associated to every link of each graph respectively, containing the formation-keeping gains $k_{jk}^\star$, with $\star \in \{p, d\}$. Hence, $\mathcal{G}_1$ and $\mathcal{G}_2$ will have the same properties, and both describe the information flow graph between spacecraft in the formation. For more details on graph properties, the reader is referred to [20].

We can show that the above control law can be further simplified by allowing $\alpha_{1j} = 0$, under some conditions on the communication graph. Before we proceed, we state the following Lemma.

Lemma 1: Consider a group of n-spacecraft, with the relative attitudes between the group members defined as above by $\mathbf{q}_{jk} = \mathbf{q}_k^{-1} \odot \mathbf{q}_j$. If the communication graph between spacecraft is a tree[1], then the only solution to the set of equations

$$\sum_{k=1}^{n} k_{jk}^{p} q_{jk} = 0, \text{ for } j = 1, \ldots, n \tag{22}$$

is $q_{jk} = 0$ for $j, k \in \{1, \ldots, n\}$, where k_{jk}^p are defined as in (19). Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$, (or $\tilde{\eta}_j(t) < 0$), for all $t \geq T$ and $j \in \{1, \ldots, n\}$, then $q_{jk} = 0$ for $j, k \in \{1, \ldots, n\}$ is the only solution to (22) for any connected undirected graph.

[1]an undirected graph is a tree if there is a path between any two distinct nodes on the graph, and it contains no cycles, [20]

Proof: A similar proof can be found in [1] and [2]. ■

The result is stated as follows

Corollary 1: Given the formation (1)–(2) with the control law (18) with (19). Let $\alpha_{1j} = 0$, $\alpha_{2j} > 0$, and the inputs of the auxiliary systems (12) and (15) be given by (20). If the undirected communication graph between spacecraft is a tree, then all the signals are globally bounded and $q_j \to q_k, \forall j,k \in \{1,\ldots,n\}$, and $\omega_j \to R(\tilde{\mathbf{q}}_j)\omega_d(t)$, $\forall j \in \{1,\ldots,n\}$, asymptotically. Furthermore, if there exists a time $T > 0$ such that $\tilde{\eta}_j(t) > 0$, (or $\tilde{\eta}_j(t) < 0$), for all $t \geq T$ and $j \in \{1,\ldots,n\}$, then the above result holds for any connected undirected graph.

Proof: Following the same steps of the proof of theorem 1, we can conclude that $\tilde{\omega}_j \to 0$, and $\omega_{jk} \to 0$ for all $j,k \in \{1,\ldots,n\}$, and equation (A5), in this case, reduces to (22). Then using the result in Lemma 1, we conclude that $q_j \to q_k$ for all $j,k \in \{1,\ldots,n\}$. From (7), we have $\omega_j \to \omega'_d(t) \triangleq R(\tilde{\mathbf{q}}_j)\omega_d(t), \forall j \in \{1,\ldots,n\}$. Note that the final angular velocity, $\omega'_d(t)$, is the desired angular velocity expressed in the spacecraft body frame $\mathcal{F}_j$, and all spacecraft synchronize their attitudes to the common attitude $\bar{\mathbf{q}}_d$ satisfying the dynamics $\dot{\bar{\mathbf{q}}}_d = (1/2)\bar{\mathbf{q}}_d \odot \bar{\omega}'_d$, with $\bar{\omega}'_d = (\omega'_d(t)^T, 0)^T$. ■

The result in Corollary 1 extends the work of [14] and [16] to the velocity free case, where similar results were obtained in the full information case (*i.e.,* with velocity measurement) under the same sufficient condition on the communication graph.

One important requirement of the above control schemes is that the time-varying desired angular velocity must be available to all spacecraft in order to guarantee group alignment with a non-zero final angular velocity. The extension of the above control law to the case where the time-varying desired angular velocity is available to only one or some spacecraft is not straightforward. To the best of our knowledge, this problem is still open even with angular velocity measurements. A preliminary solution to this problem has been proposed in [16], with angular velocity measurements, where a time varying reference trajectory is known to a single spacecraft (the leader). The reference trajectory is assumed to be linearly parameterized in terms of some scalar time-varying functions known by all spacecraft, and unknown constant coefficients, and a classical adaptive control technique is used to recover these coefficients. In the approach of [16], some information on the reference velocity is still required to be available to all spacecraft and the type of reference trajectories is restricted. On the other hand, in [9], the author assumes that the desired angular velocity is available to some members of the team acting as leaders. A directed communication graph is considered, and attitude alignment is achieved provided that the directed graph can be reduced to a single node. In [17], the same author extends his result using the MRP parametrization for the attitude representation. In both papers, the author assumes that, in addition to their attitudes and angular velocities, spacecraft transmit their angular accelerations, which increases the cost and complexity in that more sensors and intensive communication are required especially if the number of spacecraft is increased.

IV. Consensus Seeking Without Reference Trajectory

In this section, we deal with the second problem (OBJ2). We consider the case where it is required to synchronize a group of spacecraft to reach an agreement on the final attitude without velocity measurements, and show that the auxiliary dynamics are instrumental in the control design. We assume that no desired reference trajectory is assigned, and spacecraft are required to converge to the same (not necessarily constant) angular velocity while maintaining the same attitudes during formation maneuvers, *i.e.,* $q_j \to q_k$ and $\omega_j \to \omega_k$. We assume that the communication between spacecraft is bidirectional and the spacecraft angular velocities are not available.

In order to solve this problem, we first redefine the unit quaternion $\tilde{\mathbf{p}}_j$ used in the previous section as follows: $\tilde{\mathbf{p}}_j = \mathbf{p}_j^{-1} \odot \mathbf{q}_j$, which is governed by the dynamics (13) with, $\Omega_j = \omega_j - R(\tilde{\mathbf{p}}_j)\beta_j$. Then, we consider a new unit-quaternion related to the output of the j^{th} and k^{th} auxiliary systems, defined as: $\bar{\mathbf{p}}_{jk} = \tilde{\mathbf{p}}_k^{-1} \odot \tilde{\mathbf{p}}_j \equiv (\bar{p}_{jk}^T, \bar{\epsilon}_{jk})^T$, satisfying the unit quaternion dynamics

$$\dot{\bar{p}}_{jk} = \frac{1}{2}(\bar{\epsilon}_{jk}I + S(\bar{p}_{jk}))\bar{\Omega}_{jk}, \quad \dot{\bar{\epsilon}}_{jk} = -\frac{1}{2}\bar{p}_{jk}^T\bar{\Omega}_{jk} \tag{23}$$

$$\bar{\Omega}_{jk} = \Omega_j - R(\bar{\mathbf{p}}_{jk})\Omega_k. \tag{24}$$

The following properties can be easily shown:

$$R(\bar{\mathbf{p}}_{kj})^T = R(\bar{\mathbf{p}}_{jk}), \quad \bar{p}_{kj} = -\bar{p}_{jk} = -R(\bar{\mathbf{p}}_{kj})\bar{p}_{jk}. \tag{25}$$

We propose the following control law for each individual spacecraft:

$$\tau_j = \sum_{k=1}^{n}\left(-k_{jk}^p\, q_{jk} - k_{jk}^d\, \left(\tilde{p}_{jk} - R(\mathbf{q}_{jk})\tilde{p}_{kj} + \bar{p}_{jk}\right)\right) \tag{26}$$

with the gains k_{jk}^p and k_{jk}^d are defined as in Theorem 1. Note that in order to implement (26), we assume that spacecraft can also communicate the unit quaternion $\tilde{\mathbf{p}}_j$ with one another via the information exchange topology. This will constitute an extra data to be transmitted through the information flow described by the weighted undirected graph $\mathcal{G}_2 = (\mathcal{N}, \mathcal{E}, \mathcal{K}_d)$. Now, we can state the following result.

Theorem 2: Consider the formation given in (1)–(2) under the control law (26), with restrictions (19), and let the inputs of the auxiliary systems (12) and (15) be, respectively

$$\beta_j = R(\tilde{\mathbf{p}}_j)^T\Gamma_j\left(\sum_{k=1}^{n} k_{jk}^d\bar{p}_{jk}\right), \quad \beta_{jk} = \Gamma_{jk}\tilde{p}_{jk} \tag{27}$$

with $\Gamma_j = \Gamma_j^T > 0$ and $\Gamma_{jk} = \Gamma_{jk}^T > 0$. If the information flow graph is a tree, then all the signals are globally bounded and $q_j(t) \to q_k(t)$ and $\omega_j(t) \to \omega_k(t)$ asymptotically, for all $j,k \in \{1,\ldots,n\}$. Furthermore, if there exists a time $T > 0$ such that $\tilde{\epsilon}_j(t) > 0$, (or $\tilde{\epsilon}_j(t) < 0$), for all $t \geq T$ and $j \in \{1,\ldots,n\}$, then the above result holds for any connected undirected graph.

Proof: Using the following Lyapunov function candidate:

$$V = \frac{1}{2}\sum_{j=1}^{n}\omega_j^T I_{f_j}\omega_j + \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p(1-\eta_{jk}) + \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\left(2(1-\tilde{\epsilon}_{jk}) + (1-\bar{\epsilon}_{jk})\right) \tag{28}$$

and following similar steps as in the proof of theorem 1, with the help of Lemma 1, the results of the theorem can be proven. Details of the proof are omitted due to space limitations and can be found in [2]. ■

It is important to mention that the proposed control law in this section ensures that all spacecraft converge to a final angular velocity which is guaranteed to be bounded, and not necessarily constant.

Remark 2: It is worth noticing that the control law (26) is a pure quaternion feedback, and consequently a natural saturation is achieved for the control effort as follows: $\|\tau_j\| \leq \sum_{k=1}^{n}(k_{jk}^p + 3k_{jk}^d)$.

V. Simulation Results

In this section, the performance of the proposed control schemes is investigated through numerical simulations. We consider the results obtained in Theorem 1 and Theorem 2. Using SIMULINK, we consider a scenario of four spacecraft under an undirected communication flow

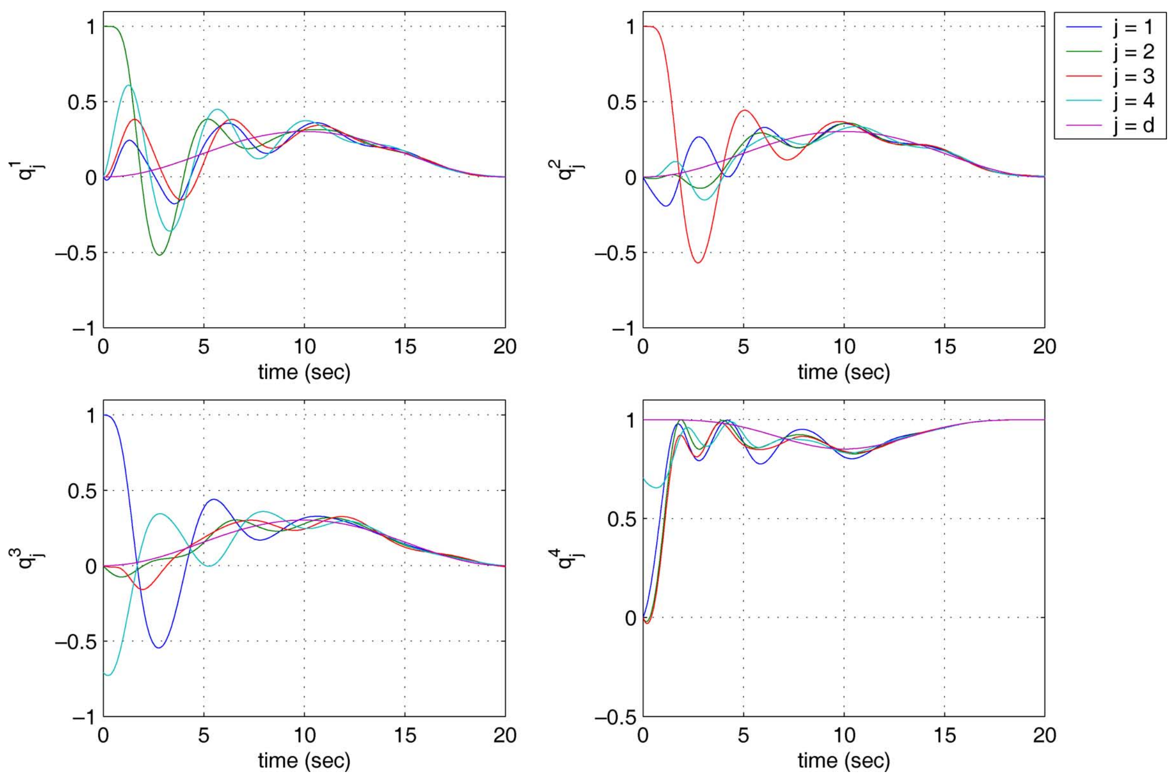

Fig. 1. Spacecraft attitudes in case of Theorem 1.

TABLE I
SIMULATION PARAMETERS

$\mathbf{q}_1(0) = (0,0,1,0)^T$, $\mathbf{q}_2(0) = (1,0,0,0)^T$, $\mathbf{q}_3(0) = (0,1,0,0)^T$,
$\mathbf{q}_4(0) = (0,0,sin(-\pi/4), cos(-\pi/4))^T$, $\mathbf{p}_j(0) = \mathbf{p}_{jk}(0) = \mathbf{q}_2(0)$,
$\omega_1(0) = (-0.5, 0.5, -0.45)^T$, $\omega_2(0) = (0.5, -0.3, 0.1)^T$, $\Gamma_j = 6I_3$,
$\omega_3(0) = (0.1, 0.6, -0.1)^T$, $\omega_4(0) = (0.4, 0.4, -0.5)^T$, $\Gamma_{jk} = \Gamma_j$,
Theorem 1: $\alpha_{1j} = 60$, $\alpha_{2j} = 60$, $k_{jk}^p = 5$, $k_{jk}^d = 5$, for $(j,k) \in \mathcal{E}_1$,
Theorem 2: $k_{jk}^p = 30$, $k_{jk}^d = 25$, for $(j,k) \in \mathcal{E}_2$.

graph. The spacecraft are modeled as rigid bodies whose inertia matrices are taken as $I_{f_j} = \mathrm{diag}(20, 20, 30)$.

First, we consider the results in Theorem 1, where spacecraft are required to align their attitudes while tracking the desired reference trajectory defined by $\omega_d(t) = 0.1\sin(0.1\pi t)(1,1,1)^T$ and $\mathbf{q}_d(0) = (0,0,0,1)^T$. We consider the communication flow between spacecraft described by the graph whose set of edges is given by; $\mathcal{E}_1 = \{(1,2),(1,3),(1,4),(2,3)\}$, and the controller gains are selected as in Table I in order to satisfy condition (21). Fig. 1 shows the spacecraft attitudes represented by the unit quaternion $\mathbf{q}_j^i$, $j = 1,\ldots,4$, and $j = d$ for the desired attitude. We use the superscript "i" to denote the i^{th} component of a vector. It is clear that the four spacecraft converge to the same specified attitude.

We consider next the Consensus seeking problem with $\mathcal{E}_2 = \{(1,2),(1,4),(2,3)\}$. We assume that no reference trajectory is assigned and spacecraft are required to synchronize their attitudes to a common final attitude (not necessarily constant). In Fig. 2 we can see that spacecraft reach an agreement and converge to the same final time varying attitude. Note that the final trajectory depends on the initial conditions and the weights assigned to each link of the communication graph.

VI. Conclusion

We addressed the problem of quaternion-based attitude tracking and synchronization of a group of spacecraft without velocity measurements, under an undirected communication graph. Instrumental in our approach, the introduction of the so-called "auxiliary systems" playing the role of velocity observers allowing to generate the necessary damping in the absence of the actual spacecraft angular velocities and relative angular velocities. We proposed a behavioral-type approach (Theorem 1) guaranteeing simultaneous group synchronization and trajectory tracking. Almost global asymptotic stability results are obtained in the sense that the closed loop system has several equilibria, that represent the same physical configuration, but only one of them is an attractor [21]. In Theorem 2, we solved the velocity-free consensus-seeking problem, where global attitude agreement can be reached between spacecraft provided that the communication graph is a tree. In this last result, spacecraft attitudes are guaranteed to converge to a common bounded time varying trajectory. The prediction of this final angular velocity will be examined in our future work. It is important to mention that although we consider the velocity-free attitude synchronization problem in the context of a group of spacecraft, our results are applicable to the attitude synchronization problem among rigid bodies in general satisfying the rotational dynamics. Moreover, we believe that the control schemes derived in this work carry an important and novel feature (besides the non-requirement of the angular velocity measurements), which consists in the fact that the control torques are naturally bounded since all feedback terms involved in the control laws are unit quaternion. This feature allows the designer to arbitrarily set the desired bounds for the control torques, *a priori*, using the control gains, regardless of the angular velocities. The extension of the present work to dynamically switching and/or directed communication topologies is a challenging topic that will be part of our future work.

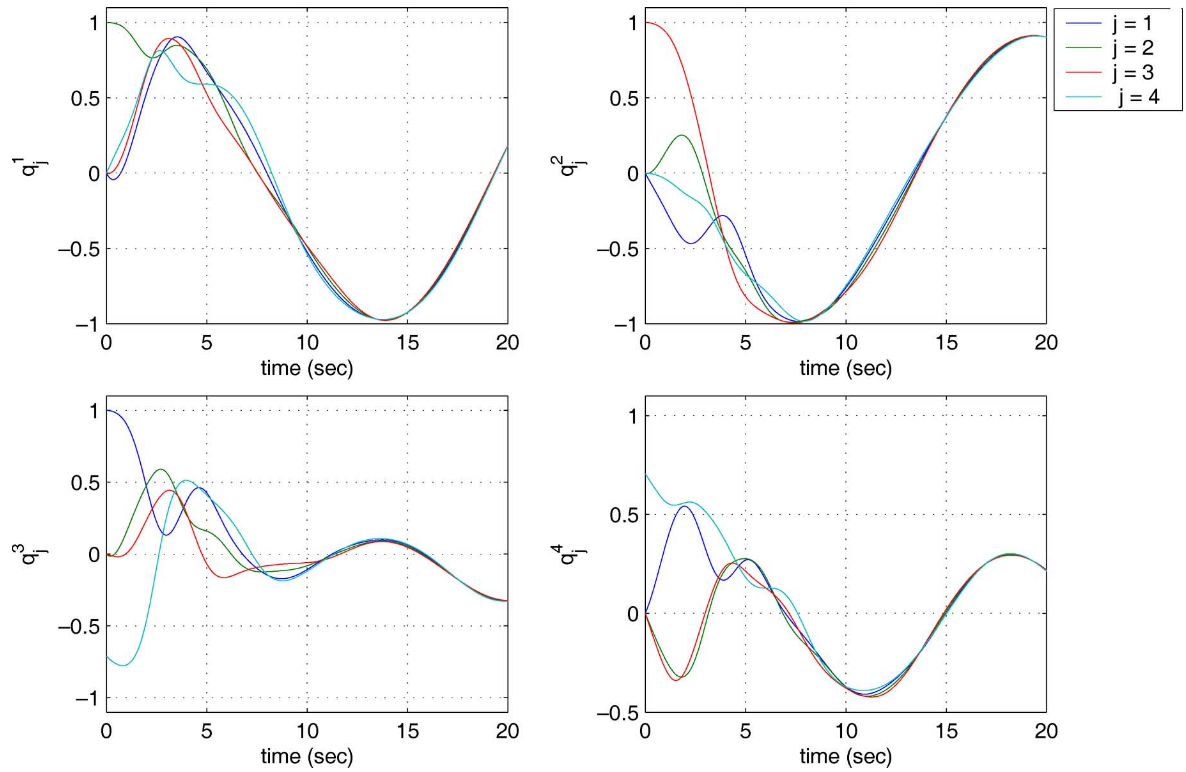

Fig. 2. Spacecraft attitudes in case of Theorem 2.

APPENDIX
PROOF OF THEOREM 1

Consider the following Lyapunov function candidate:

$$V = \sum_{j=1}^{n}\left(\frac{1}{2}\tilde{\omega}_j^T I_{f_j}\tilde{\omega}_j + 2\alpha_{1j}(1-\tilde{\eta}_j) + 2\alpha_{2j}(1-\tilde{\epsilon}_j)\right) + \sum_{j=1}^{n}\sum_{k=1}^{n}\left(k_{jk}^p(1-\eta_{jk}) + 2k_{jk}^d(1-\tilde{\epsilon}_{jk})\right).$$

Note that: $2(1-\tilde{\eta}_j) = \tilde{q}_j^T\tilde{q}_j + (1-\tilde{\eta}_j)^2$, and is valid for $\tilde{\mathbf{q}}_{jk}$, $\tilde{\mathbf{p}}_j$ and $\tilde{\mathbf{p}}_{jk}$. The time derivative of V evaluated along the closed loop dynamics of the j^{th} spacecraft (8), using (18) and (14), is given by

$$\begin{aligned}\dot{V} = &\sum_{j=1}^{n}\sum_{k=1}^{n}\tilde{\omega}_j^T\left(-k_{jk}^p q_{jk} - k_{jk}^d\left(\tilde{p}_{jk} - R(\mathbf{q}_{jk})\tilde{p}_{kj}\right)\right)\\ &-\sum_{j=1}^{n}\alpha_{2j}\tilde{p}_j^T R(\tilde{\mathbf{p}}_j)\beta_j\\ &+\sum_{j=1}^{n}\sum_{k=1}^{n}\left(\frac{1}{2}k_{jk}^p q_{jk}^T\omega_{jk} + k_{jk}^d\tilde{p}_{jk}^T\Omega_{jk}\right).\end{aligned}$$

Using the fact that spacecraft are required to align their attitudes to the same desired angular velocity, the following equations relating the relative attitude of the j^{th} and k^{th} spacecraft can be derived easily:

$$\mathbf{q}_{jk} = \tilde{\mathbf{q}}_k^{-1}\odot\tilde{\mathbf{q}}_j,\quad \omega_{jk} = \tilde{\omega}_j - R(\mathbf{q}_{jk})\tilde{\omega}_k \qquad \text{(A1)}$$

Using (A1), (11) and (19) we can write

$$\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p q_{jk}^T\omega_{jk} = \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^p\tilde{\omega}_j^T q_{jk}. \qquad \text{(A2)}$$

Similarly, using the expression of Ω_{jk}, given in (17), with (11), (19) and (A1), we get

$$\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\Omega_{jk}^T\tilde{p}_{jk} = -\sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\beta_{jk}^T R(\tilde{\mathbf{p}}_{jk})^T\tilde{p}_{jk} + \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\tilde{\omega}_j^T\left(\tilde{p}_{jk} - R(\mathbf{q}_{jk})\tilde{p}_{kj}\right). \qquad \text{(A3)}$$

Then, from (A2)–(A3), and using (20) with the fact that $q^T R(\mathbf{q}) = q^T$ for any quaternion $\mathbf{q} = (q^T, \eta)^T$, we obtain

$$\dot{V} = -\sum_{j=1}^{n}\alpha_{2j}\tilde{p}_j^T\Gamma_j\tilde{p}_j - \sum_{j=1}^{n}\sum_{k=1}^{n}k_{jk}^d\tilde{p}_{jk}^T\Gamma_{jk}\tilde{p}_{jk} \qquad \text{(A4)}$$

which implies that $V(t)\le V(0)$, and $\tilde{\mathbf{q}}_j, \tilde{\mathbf{p}}_j, \tilde{\omega}_j, \mathbf{q}_{jk}$ and $\tilde{\mathbf{p}}_{jk}$ are globally bounded. In addition, we can verify that $\dot{\tilde{p}}_j$ and $\dot{\tilde{p}}_{jk}$ are bounded, and so is $\ddot{V}$. Hence, invoking Barbalat's lemma, [22], we can conclude that $\tilde{p}_j \to 0$ and $\tilde{p}_{jk}\to 0$, as $t\to\infty$, which implies that $\epsilon_j \to \pm 1$, $\epsilon_{jk}\to\pm 1$, $\beta_j \to 0$, $\beta_{jk}\to 0$, $R(\tilde{\mathbf{p}}_j)\to I_3$ and $R(\tilde{\mathbf{p}}_{jk})\to I_3$. Now, since $\dot{\omega}_d$ is bounded, one can show that $\ddot{\tilde{\mathbf{p}}}_j$ and $\ddot{\tilde{\mathbf{p}}}_{jk}$ are bounded, and hence $\dot{\tilde{\mathbf{p}}}_j \to 0$ and $\dot{\tilde{\mathbf{p}}}_{jk}\to 0$, and from (13)–(14) and (16)–(17) we can conclude that $\Omega_j\to 0$ and $\Omega_{jk}\to 0$, and consequently $\tilde{\omega}_j \to 0$ and $\omega_{jk}\to 0$. Furthermore, one can easily verify that $\ddot{\tilde{\omega}}_j$ is bounded since $\ddot{\omega}_d$ is bounded, and so we conclude that $\dot{\tilde{\omega}}_j\to 0$. Using the above results, the closed loop dynamics (8), with (18) reduces to

$$\alpha_{1j}\tilde{q}_j + \sum_{k=1}^{n}k_{jk}^p\, q_{jk} = 0,\quad \text{for } j = 1,\ldots n. \qquad \text{(A5)}$$

Following a similar procedure as in [13] and [1], and using (4) and (A1), we can write (A5) in matrix form, using the Kronecker product $\otimes$, as

$$(M(t)\otimes I_3)\,\tilde{Q} = 0 \qquad \text{(A6)}$$

where $\tilde{Q} \in \mathbb{R}^{3n}$ is the column vector composed of all the vectors $\tilde{q}_j$, for $j = 1, \ldots, n$, and the matrix $M(t) = [m_{jk}(t)] \in \mathbb{R}^{n \times n}$ is given by; $m_{jj}(t) = \alpha_{1j} + \sum_{k=1}^{n} k_{jk}^{p} \tilde{\eta}_k$, and $m_{jk}(t) = -k_{jk}^{p} \tilde{\eta}_j$. We can see that the formation has converged to the desired trajectory and consequently all spacecraft are aligned only if $\tilde{Q} = 0$. A necessary and sufficient condition for this is that the matrix $M(t)$ has full rank. We can easily verify that matrix $M(t)$ is strictly diagonally dominant if condition (21) is satisfied, [1]. This implies that the only solution of (A6) is $\tilde{Q} = 0$, that is $\tilde{q}_j = 0$ for $j \in \{1, \ldots, n\}$. Finally, we can conclude that $\tilde{q}_j \to 0$ and $\tilde{\eta}_j \to \pm 1$, or equivalently $q_j \to q_k \to q_d$. Moreover, since $\tilde{\omega}_j \to 0, \omega_{jk} \to 0, R(\tilde{\mathbf{q}}_j) \to I_3$ and $R(\mathbf{q}_{jk}) \to I_3$, we conclude that $\omega_j \to \omega_k \to \omega_d(t), \forall j, k \in \{1, \ldots, n\}$.

Furthermore, we can see from the definition of matrix $M(t)$ that if the scalar parts $\tilde{\eta}_j(t)$ for $j \in \{1, \ldots, n\}$ are positive, then matrix $M(t)$ is strictly diagonally dominant, [1]. Also, note that equation (A5) holds when t tends to infinity. Then, if there exists a time $T > 0$, such that $\tilde{\eta}_j(t) > 0$ for $t > T$ and $j \in \{1, \ldots, n\}$, the only solution to (A6) is $\tilde{q}_j = 0$ for $j \in \{1, \ldots, n\}$ without any condition and the same convergence results hold.

References

[1] A. Abdessameud and A. Tayebi, "Decentralized attitude alignment control of spacecraft within a formation without velocity measurements," in *Proc. 17th IFAC World Congress*, Seoul, Korea, 2008, pp. 1766–1771.

[2] A. Abdessameud and A. Tayebi, "Attitude synchronization of a spacecraft formation without velocity measurement," in *Proc. 47th IEEE CDC*, Cancun, Mexico, 2008, pp. 3719–3724.

[3] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1465–1476, Sep. 2004.

[4] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbour rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.

[5] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *Special Issue of the IEEE Trans. Autom. Control Networked Control Syst.*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[6] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multivehicle cooperative control: Collective group behavior through local interaction," *IEEE Cont. Syst. Mag.*, vol. 27, no. 2, pp. 71–82, 2007.

[7] D. P. Scharf, F. Y. Hadeagh, and S. R. Ploen, "A survey of spacecraft formation flying guidance and control (Part II): Control," in *Proc. Amer. Control Conf.*, Boston, MA, 2004, pp. 2976–2985.

[8] P. Wang, F. Hadaegh, and K. Lau, "Synchronized formation rotation and attitude control of multiple free-flying spacecraft," *J. Guid., Cont. Dynam.*, vol. 22, pp. 28–35, 1999.

[9] W. Ren, "Formation keeping and attitude alignment for spacecraft through local interactions," *J. Guid., Cont. Dynam.*, vol. 30, no. 2, pp. 633–638, 2007.

[10] W. Ren and R. W. Beard, "Decentralized scheme for spacecraft formation flying via the virtual structure approach," *J. Guid., Cont. Dynam.*, vol. 27, no. 1, pp. 73–82, 2004.

[11] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Trans. Robot. Autom.*, vol. 14, pp. 926–939, 1998.

[12] J. Lawton, R. W. Beard, and F. Y. Hadaegh, "Elementary attitude formation maneuvers via leader-following and behavior-based control," in *Proc. Guid., Navig., Control Conf. Exhibit*, Denver, CO, 2000, pp. 2000–4442.

[13] J. Lawton and R. W. Beard, "Synchronized multiple spacecraft rotations," *Automatica*, vol. 38, no. 8, pp. 1359–1364, 2002.

[14] W. Ren, "Distributed attitude alignment in spacecraft formation flying," *Int. J. Adapt. Control Signal Processing*, vol. 21, pp. 95–113, 2007.

[15] M. C. Vandyke and C. D. Hall, "Decentralized coordinated attitude control within a formation of spacecraft," *J. Guid., Control. Dynam.*, vol. 29, no. 5, pp. 1101–1109, 2006.

[16] H. Bai, M. Arcak, and J. T. Wen, "A decentralized design for group alignment and synchronous rotation without inertial frame information," in *Proc. 46th IEEE CDC*, New Orleans, LA, 2007, pp. 2552–2557.

[17] W. Ren, "Distributed attitude synchronization for multiple rigid bodies with Euler-Lagrange equations of motion," in *Proc. 46th IEEE CDC*, New Orleans, LA, 2007, pp. 2363–2368.

[18] A. Tayebi, "Unit quaternion based output feedback for the attitude tracking problem," *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1516–1520, 2008.

[19] M. D. Shuster, "A survey of attitude representations," *J. Astronaut. Sci.*, vol. 41, pp. 439–517, 1993.

[20] D. Jungnickel, *Graphs, Networks and Algorithms*, second ed. New York: Springer, 2005, vol. 5.

[21] S. P. Bhat and D. S. Bernstein, "A topological obstruction to continuous global stabilization of rotational motion and the unwinding phenomenon," *Syst. Control Lett.*, vol. 39, pp. 63–70, 2000.

[22] H. K. Khalil, *Nonlinear Systems*, Third Edition ed. Englewood Cliffs, NJ: Prentice Hall, 2002.

New Expressions of 2 × 2 Block Matrix Inversion and Their Application

Youngjin Choi and Joono Cheong

***Abstract*—A 2 × 2 block matrix inversion is a tool that is frequently used in areas of control, estimation theory and signal processing. However, one of the two diagonal entries of the block matrix should be invertible to carry out a conventional block matrix inversion. In this technical note, we show that this assumption can be partially released with three new types of symbolic block matrix inversion. Also, an application example of an inverse plant model of a multi-inputs and multi-output (MIMO) plant, which cancels plant noise and disturbance, is suggested to show the effectiveness of these new types of matrix inversion.**

***Index Terms*—Block matrix inversion, inverse plant model, multi-inputs and multi-output (MIMO).**

I. Introduction

In optimal filtering, optimization, inverse model-based control, and a disturbance canceling method, a 2 × 2 block matrix inversion is often needed [1], [2]. The conventional 2 × 2 block matrix inversion, which is well established in [1], [3], requires an assumption that at least one of diagonal entries of the block matrix should be invertible. If this assumption is not satisfied even when the entire matrix is full rank, we

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