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Synchronization of networked Lagrangian systems with input constraints ⋆

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Abstract: This paper addresses the synchronization problem of systems modeled by Euler-Lagrange equations and subject to input saturations. First, a new control design strategy, based on virtual systems, is proposed. This approach allows to generate control inputs that are *a priori* bounded in the presence of communication time-delays, regardless of the information flow topology between systems in the network. Second, we remove the requirement of the generalized velocities, leading to a velocity-free synchronization scheme with *a priori* bounded inputs. The effectiveness of the proposed control schemes is demonstrated through simulation examples on a network of four robot manipulator arms.

1. INTRODUCTION

Motion synchronization of multiple mechanical systems has received an increased interest in the last few years. Inspired by recent results in the cooperative control of multi-agent systems, interesting solutions have been proposed for the synchronization of the class of nonlinear systems modeled by Euler-Lagrange equations such as spacecraft formations (Chung et al., 2009; Kristiansen et al., 2008) and robot networks (Spong et al., 2006; Ren, 2009). The main idea in these works is to use local information exchange between the systems in the network to design each system's control input so that either state or output synchronization is achieved.

A practical problem frequent in the control of this type of mechanical systems is to design synchronization schemes that account for possible input torque saturations. This problem becomes more difficult when the number of systems in the network is large and the control input of each system depends on received information from a large number of neighboring systems. For a group of mobile robots, the work in Lawton et al. (2003) presents cooperative control schemes that account for input saturations, assuming a ring communication topology. The author in Ren (2008) extended the latter work to a general undirected communication topology, and proposed solutions to the consensus problem for second order dynamics that account for input constraints. This work was further extended to the consensus problem of networked Lagrangian systems with constrained inputs in Ren (2009).

Our interest in this paper is to design synchronization schemes for networked Lagrangian systems subject to input torque saturations. In the case where the states are available for feedback, we propose a new control method based on virtual systems. The input of each system is based on nonlinear bounded functions of the virtual states, and the virtual systems' inputs are constructed using the real systems' states and the interaction between the systems in the network. As a result, the control input of each system is guaranteed to be *a priori* bounded independently from the information flow in the network.

Another advantage of using these virtual systems is the ability to deal, simultaneously, with the problems of communication delays and input saturations. In fact, we show that under sufficient delay-dependent conditions, the proposed synchronization scheme achieves the control objectives with input saturations along with communication delays. To the best of the authors' knowledge, these two problems have not been studied simultaneously in the literature. The delay in the information transmission for a group of lagrangian systems has been considered in the literature both in the context of bilateral teleoperation (Nuño et al., 2010; Chopra et al., 2008) and the general framework of synchronization (Chung et al., 2009; Spong et al., 2006). In these works, a synchronization variable is defined using an appropriate change of variables, so that simplified closed loop dynamics are obtained, and the effects of the delayed communication is studied using Lyapunov-Krasovskii functionals. However, the extension of these works to account for input constraints is not trivial.

The second control law presented in this work removes the requirement of the generalized velocities measurements. We exploit the lead-filter approach, developed in Berghuis and Nijmeijer (1993), and propose a velocity-free synchronization scheme that accounts for the systems' constraints. This work can be considered as the application of the bounded velocity-free consensus algorithm proposed in Abdessameud and Tayebi (2010) for linear double integrators. The idea of using lead-filters has been considered in Lawton et al. (2003) and Ren (2009) to remove the requirement of velocity measurements in a network of Lagrangian systems, however, the proposed control schemes do not account for input constraints.

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2. SYSTEM MODEL AND PROBLEM STATEMENT

We consider n-systems governed by the dynamics

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{G}_i(\mathbf{q}_i) = \mathbf{\Gamma}_i, \quad (1)$$

for $i \in \mathcal{N} \triangleq \{1, ..., n\}$, with $\mathbf{q}_i \in \mathbb{R}^m$ is the vector of generalized coordinates, $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{m\times m}$ is the positive-definite inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i$ is the vector of coriolis/centrifugal forces, $\mathbf{G}_i(\mathbf{q}_i)$ is the vector of gravitational force, and $\mathbf{\Gamma}_i$ is the vector of torques associated with the i^{th} system. Each system is assumed to satisfy the following common properties;

- P.1 The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is lower and upper bounded as
  $0 < \lambda_{\min}\{\mathbf{M}_i(\mathbf{q}_i)\}I_n \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_{\max}\{\mathbf{M}_i(\mathbf{q}_i)\} < \infty$,
  where $\lambda_{\min}$ and $\lambda_{\max}$ denote respectively the minimum and maximum eigenvalue of a matrix.
- P.2 The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew symmetric, *i.e.,* $\mathbf{x}^\top(\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i))\mathbf{x} = 0$, for all $\mathbf{x} \in \mathbb{R}^m$. Note also that this property also implies that; $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i, \dot{\mathbf{q}}_i)$.
- P.3 For all $\mathbf{q}_i$, $\mathbf{x}$, $\mathbf{y} \in \mathbb{R}^m$, there exists $k_{c_i} \in \mathbb{R}^+$ such that $\|\mathbf{C}_i(\mathbf{q}_i, \mathbf{x})\mathbf{y}\| \leq k_{c_i}\|\mathbf{x}\|\|\mathbf{y}\|$.
- P.4 The vector of gravitational torques is bounded as; $\|\mathbf{G}_i(\mathbf{q}_i)\| \leq g_m$, for $\mathbf{q}_i \in \mathbb{R}^m$ and $g_m > 0$.

We assume that the communication flow between members of the team is fixed and undirected and is described by the weighted undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{K})$. $\mathcal{N}$ is the set of nodes or vertices, describing the set of systems in the network, $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$ is the set of unordered pairs of nodes, called edges, and $\mathcal{K} = [k_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix. An edge (i, j) indicates that the i^{th} and j^{th} systems are neighbors and can obtain information from one another. The weighted adjacency matrix is defined such that $k_{ii} \triangleq 0$, $k_{ij} = k_{ji} > 0$ for $(i, j) \in \mathcal{E}$, and $k_{ij} = k_{ji} = 0$ for $(i, j) \notin \mathcal{E}$. In addition, we assume that the systems are subject to input torque saturations such that $\|\mathbf{\Gamma}_i\|_\infty \leq \mathbf{\Gamma}_{\max}$, with $\mathbf{\Gamma}_{\max} > 0$.

The control objective is to design control schemes for the class of systems modeled in (1) such that

$$\dot{\mathbf{q}}_i \to 0, \quad (\mathbf{q}_i - \mathbf{q}_j) \to 0, \quad \text{for } i, j \in \mathcal{N}. \quad (2)$$

To account for systems' saturations, we define for any vector $\mathbf{x} = \text{col}[x^k] \in \mathbb{R}^m$ the function χ as

$$\chi(\mathbf{x}) = \text{col}[\sigma(x^k)] \in \mathbb{R}^m, \quad \text{for} \quad k = 1, ...m, \quad (3)$$

with $\sigma : \mathbb{R} \to \mathbb{R}$, is a strictly increasing continuously differentiable function satisfying the following properties:

- P.5 $\sigma(0) = 0$ and $x\sigma(x) > 0$ for $x \neq 0$,
- P.6 $|\sigma(x)| \leq \sigma_b$, with $\sigma_b > 0$, for $x \in \mathbb{R}$.
- P.7 The function $\frac{\partial\sigma(x)}{\partial x}$ is uniformly bounded.

Examples of the function $\sigma(x)$ include: $\tanh(x)$ and $\frac{x}{\sqrt{1+x^2}}$.

3. STATE FEEDBACK SYNCHRONIZATION

In this section, we assume that the full state vector is available for feedback and propose a solution to the position synchronization with zero final velocities for networked Euler-Lagrange systems subject to input torque saturations. Let associate to each rigid body in the team the virtual system governed by

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{p}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{p}}_i + \mathbf{G}_i(\mathbf{q}_i) = \mathbf{\Gamma}_i + \boldsymbol{\eta}_i, \quad (4)$$

where $\mathbf{p}_i \in \mathbb{R}^m$ is a virtual variable that can be initialized arbitrarily and $\boldsymbol{\eta}_i$ is an additional input to be designed later. Note that the generalized coordinates of the systems are considered in (4) to evaluate the model parameters. In addition, we define the error between the states of each system and the states of its corresponding virtual system by $\tilde{\mathbf{p}}_i$ given as

$$\tilde{\mathbf{p}}_i = \mathbf{q}_i - \mathbf{p}_i. \quad (5)$$

The main idea from the above definitions is to design the input of each virtual system, $\boldsymbol{\eta}_i$, such that; $\dot{\tilde{\mathbf{p}}}_i \to 0$ and $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j) \to 0$ for all $i, j \in \mathcal{N}$. Thereafter, an appropriate design of the input torque of each system, $\mathbf{\Gamma}_i$, is applied for the purpose of driving the states of the virtual systems asymptotically to zero. In this way, the input $\mathbf{\Gamma}_i$ will be constructed using nonlinear saturation functions of only the virtual systems' states, and the additional input $\boldsymbol{\eta}_i$ can be designed without any consideration to the systems' constraints. This can be seen from the dynamics of the error signals (5) that can be derived, from (1) and (4), as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\tilde{\mathbf{p}}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\tilde{\mathbf{p}}}_i = -\boldsymbol{\eta}_i. \quad (6)$$

Following this idea, we propose the following control inputs to (1) and (4),

$$\mathbf{\Gamma}_i = \mathbf{G}_i(\mathbf{q}_i) - k_i^p\chi(\mathbf{p}_i) - k_i^d\chi(\dot{\mathbf{p}}_i), \quad (7)$$

$$\boldsymbol{\eta}_i = k_i^v\dot{\tilde{\mathbf{p}}}_i + \sum_{j=1}^{n} k_{ij}\left(\tilde{\mathbf{p}}_{ij} + \gamma\dot{\tilde{\mathbf{p}}}_{ij}\right), \quad (8)$$

with $\tilde{\mathbf{p}}_{ij} = (\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j)$, k_i^p, k_i^d, k_i^v and γ are positive scalar gains, k_{ij} is the $(i, j)^{th}$ entry of the adjacency matrix $\mathcal{K}$ of the weighted communication graph $\mathcal{G}$ describing the information flow between members of the network, and the function χ is defined in (3). One feature of this control structure can be seen from the upper bound of the systems' input torque that can be obtained as

$$\|\mathbf{\Gamma}_i\|_\infty \leq g_m + \sigma_b(k_i^p + k_i^d), \quad (9)$$

with g_m and σ_b are given respectively in P.4 and P.6. Therefore, it is easy to account for systems' actuator saturations by a simple choice of the two gains k_i^p and k_i^d without any consideration to the manner members of the network communicate with each other. This is interesting since no knowledge of the information topology between the systems is required to be known a priori, and the tuning difficulties of the control gains are considerably relaxed. Note that when the upper bounds on the systems inputs depend on the number of neighbors of each system, like in the work of Ren (2009), it is generally difficult to obtain a trade-off between achieving an acceptable/good transient performance while respecting the maximum allowed input values. This problem becomes more important when the number of neighbors of each system is large and the input of each system saturates for small control values.

Our main result in this section is stated in the next theorem after the following preliminary result proved in Appendix A.

Proposition 1. Consider the system

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{p}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{p}}_i = -k_i^p\chi(\mathbf{p}_i) - k_i^d\chi(\dot{\mathbf{p}}_i) + \boldsymbol{\varepsilon}_i, \quad (10)$$

with $\mathbf{p}_i \in \mathbb{R}^m$, the vector $\mathbf{q}_i$ is the generalized coordinates of system (1), the function χ is defined in (3), and k_i^p and k_i^d are positive scalars. If $\boldsymbol{\varepsilon}_i$ is bounded for all time and $\boldsymbol{\varepsilon}_i \to 0$, then $\mathbf{p}_i$ and $\dot{\mathbf{p}}_i$ are globally bounded and $\dot{\mathbf{p}}_i \to 0$. Furthermore, if $\dot{\mathbf{q}}_i$ is globally bounded, we have $\mathbf{p}_i \to 0$.

Theorem 1. Consider a network of n-systems modeled as in (1) with the control input given in (7) and the virtual variable $\mathbf{p}_i$ is governed by the dynamics (4) with (7) and (8). Let the undirected communication graph be connected[1]. If the control gains satisfy

$$\sigma_b(k_i^p + k_i^d) \leq \mathbf{\Gamma}_{\max} - g_m, \tag{11}$$

then $\|\mathbf{\Gamma}_i\|_\infty \leq \mathbf{\Gamma}_{\max}$, for $i \in \mathcal{N}$, the signals $\dot{\mathbf{q}}_i$ and $(\mathbf{q}_i - \mathbf{q}_j)$ are bounded and $\dot{\mathbf{q}}_i \to 0$ and $(\mathbf{q}_i - \mathbf{q}_j) \to 0$, for all $i, j \in \mathcal{N}$.

Proof. First, we can verify from (9) that $\|\mathbf{\Gamma}_i\|_\infty \leq \mathbf{\Gamma}_{\max}$ when the control gains are selected according to (11). The proof of the theorem relies on Matrosov's Theorem and Lemma 1, stated in Appendix B, and Proposition 1. Consider the following positive definite function

$$V = \frac{1}{2}\sum_{i=1}^{n}\left(\dot{\tilde{\mathbf{p}}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\tilde{\mathbf{p}}}_i + \frac{1}{2}\sum_{j=1}^{n} k_{ij}\tilde{\mathbf{p}}_{ij}^\top \tilde{\mathbf{p}}_{ij}\right). \tag{12}$$

The time derivative of V evaluated along the closed loop dynamics (6) is given by

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{n}\dot{\tilde{\mathbf{p}}}_i^\top\left(-\boldsymbol{\eta}_i - \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\tilde{\mathbf{p}}}_i + \frac{1}{2}\dot{\mathbf{M}}_i(\mathbf{q}_i)\dot{\tilde{\mathbf{p}}}_i\right) \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\dot{\tilde{\mathbf{p}}}_{ij}^\top \tilde{\mathbf{p}}_{ij} \\ =& -\sum_{i=1}^{n} k_i^v \dot{\tilde{\mathbf{p}}}_i^\top \dot{\tilde{\mathbf{p}}}_i - \frac{\gamma}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\dot{\tilde{\mathbf{p}}}_{ij}^\top \dot{\tilde{\mathbf{p}}}_{ij},\end{aligned} \tag{13}$$

where we have used property P.2, expression (8), and the relation $\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\dot{\tilde{\mathbf{p}}}_{ij}^\top \tilde{\mathbf{p}}_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\dot{\tilde{\mathbf{p}}}_i^\top \tilde{\mathbf{p}}_{ij}$, which can be shown using the symmetry property of the information flow, *i.e.,* $k_{ij} = k_{ji}$. Therefore, we have $\dot{V}$ is negative semi-definite and we conclude that $\dot{\tilde{\mathbf{p}}}_i$, for $i \in \mathcal{N}$, and $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j)$, for $(i,j) \in \mathcal{E}$, are globally bounded. Since the communication graph is connected, the above result is valid for all $i, j \in \mathcal{N}$.

First, we can see that the function V in (12) is decrescent with respect to $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$, where $\boldsymbol{\xi}$ is the stack vector of all $\tilde{\mathbf{p}}_{ij}$ for $(i,j) \in \mathcal{E}$ and $\boldsymbol{\zeta}$ is the stack vector of all $\dot{\tilde{\mathbf{p}}}_i$ for $i \in \mathcal{N}$. In addition, the time derivative of V is negative semi-definite. Therefore, conditions A.1 and A.2 in Theorem 4 (Matrosov's Theorem given in Appendix B) are satisfied. Inspired by the work of Ren (2009), we consider the function

$$W = \sum_{i=1}^{n}\dot{\tilde{\mathbf{p}}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\sum_{j=1}^{n} k_{ij}\tilde{\mathbf{p}}_{ij}. \tag{14}$$

It is clear that W satisfies condition A.3 in Theorem 4, *i.e.,* $|W|$ is bounded, since $\mathbf{M}_i(\mathbf{q}_i)$ is a bounded matrix and we have shown that $\dot{\tilde{\mathbf{p}}}_i$ and $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j)$ are bounded. We can see that on the set $\{(\boldsymbol{\zeta},\boldsymbol{\xi}) \mid \dot{V} = 0\}$, the time derivative of W in (14) along the trajectories of (6) is obtained as

$$\dot{W} = -\sum_{i=1}^{n}\left(\sum_{j=1}^{n} k_{ij}\tilde{\mathbf{p}}_{ij}\right)^\top\left(\sum_{j=1}^{n} k_{ij}\tilde{\mathbf{p}}_{ij}\right) \leq 0. \tag{15}$$

Note that $|\dot{W}|$ is positive definite with respect to $\boldsymbol{\xi}$, and hence it can be lower bounded by a class-$\mathcal{K}$ function θ. Also, since $|\dot{W}|$ does not explicitly depend on time, it follows from Lemma 1 that condition A.4 in Theorem 4 is satisfied. As a result, we conclude that the equilibrium of system (6), *i.e.* $(\boldsymbol{\zeta},\boldsymbol{\xi}) = (0,0)$, is asymptotically stable, which implies that $\dot{\tilde{\mathbf{p}}}_i \to 0$ for $i \in \mathcal{N}$ and $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j) \to 0$ for $(i,j) \in \mathcal{E}$. Since the communication graph is connected, the above result is valid for all $i, j \in \mathcal{N}$.

Exploiting the above results, we can see that $\boldsymbol{\eta}_i$ in (8) is globally bounded and converges asymptotically to zero. Therefore, the dynamics of the virtual system (4) with (7) and (8) can be rewritten as in (10) with $\boldsymbol{\varepsilon}_i = \boldsymbol{\eta}_i$, and we conclude from Proposition 1 that $\dot{\mathbf{p}}_i$ and $\mathbf{p}_i$ are globally bounded and $\dot{\mathbf{p}}_i \to 0$. Furthermore, since we have already shown that $\dot{\tilde{\mathbf{p}}}_i$ is bounded, we know from (5) that $\dot{\mathbf{q}}_i$ is globally bounded, and therefore we conclude from the result of Proposition 1 that $\mathbf{p}_i \to 0$. As a result, we conclude from (5) that $(\mathbf{q}_i - \mathbf{q}_j)$ are globally bounded and $\dot{\mathbf{q}}_i \to 0$ and $(\mathbf{q}_i - \mathbf{q}_j) \to 0$, for all $i, j \in \mathcal{N}$. □

3.1 Effects of Communication Delays

We study in this section the effects of communication delays on the control scheme proposed in Theorem 1. When the communication is delayed, the input torque $\mathbf{\Gamma}_i$ is not affected and is given in (7), but the virtual input $\boldsymbol{\eta}_i$ will be expressed as

$$\boldsymbol{\eta}_i = k_i^v \dot{\tilde{\mathbf{p}}}_i + \sum_{j=1}^{n} k_{ij}\left(\bar{\tilde{\mathbf{p}}}_{ij} + \gamma\dot{\bar{\tilde{\mathbf{p}}}}_{ij}\right), \tag{16}$$

with $\bar{\tilde{\mathbf{p}}}_{ij} = (\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j(t - \tau_{ij}))$ and $\dot{\bar{\tilde{\mathbf{p}}}}_{ij} = \left(\dot{\tilde{\mathbf{p}}}_i - \dot{\tilde{\mathbf{p}}}_j(t - \tau_{ij})\right)$, where the control gains are defined as in Theorem 1 and τ_{ij} is the constant communication delay between the i^{th} and j^{th} systems, with τ_{ij} is not necessarily equal to τ_{ji}. Our result is stated in the following theorem.

Theorem 2. Consider a network of n-systems modeled as in (1) with the control input given in (7) and the virtual variable $\mathbf{p}_i$ is governed by the dynamics (4) with (7) and (16). Let the undirected communication graph be connected. If the control gains satisfy condition (11) and

$$\bar{k}_i = k_i^v - \frac{1}{2}\sum_{j=1}^{n} k_{ij}\left(\epsilon + \frac{\tau^2}{\epsilon}\right) > 0, \tag{17}$$

for any arbitrary $\epsilon > 0$ and $\tau_{ij} \leq \tau$, for $(i,j) \in \mathcal{E}$, then $\|\mathbf{\Gamma}_i\|_\infty \leq \mathbf{\Gamma}_{\max}$, for $i \in \mathcal{N}$, the signals $\dot{\mathbf{q}}_i$ and $(\mathbf{q}_i - \mathbf{q}_j)$ are bounded and $\dot{\mathbf{q}}_i \to 0$ and $(\mathbf{q}_i - \mathbf{q}_j) \to 0$, for all $i, j \in \mathcal{N}$.

Proof. First, if condition (11) is satisfied, then the control input for each system is guaranteed to be bounded by $\mathbf{\Gamma}_{\max}$ in view of (9). Consider the following Lyapunov-Krasovskii functional

$$\begin{aligned}V =& \frac{1}{2}\sum_{i=1}^{n}\left(\dot{\tilde{\mathbf{p}}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\tilde{\mathbf{p}}}_i + \frac{1}{2}\sum_{j=1}^{n} k_{ij}\tilde{\mathbf{p}}_{ij}^\top \tilde{\mathbf{p}}_{ij}\right) \\ &+ \frac{\gamma}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\left(\int_{t-\tau_{ij}}^{t}\dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j ds\right) \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{k_{ij}\tau}{2\epsilon}\left(\int_{-\tau}^{0}\int_{t+s}^{t}\dot{\tilde{\mathbf{p}}}_j(\varrho)^\top \dot{\tilde{\mathbf{p}}}_j(\varrho)d\varrho ds\right),\end{aligned} \tag{18}$$

with $\tilde{\mathbf{p}}_{ij} = (\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j)$, $\epsilon > 0$ and τ is defined in the theorem. Note that V satisfies condition A.1 in Theorem 4. The time

[1] A graph is said to be connected if there is a path between any two distinct nodes in the graph, Jungnickel (2005).

derivative of V evaluated along the closed loop dynamics (6) with (16) is obtained as

$$\dot{V} =\sum_{i=1}^{n} \dot{\tilde{\mathbf{p}}}_i^\top \left(-k_i^v \dot{\tilde{\mathbf{p}}}_i - \sum_{j=1}^{n} k_{ij} (\bar{\tilde{\mathbf{p}}}_{ij} + \gamma \dot{\bar{\tilde{\mathbf{p}}}}_{ij} - \tilde{\mathbf{p}}_{ij}) \right)$$
$$+ \frac{\gamma}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij} \left(\dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j - \dot{\tilde{\mathbf{p}}}_j^\top (t-\tau_{ij}) \dot{\tilde{\mathbf{p}}}_j(t-\tau_{ij}) \right)$$
$$+ \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{k_{ij}\tau}{2\epsilon} \left(\tau \dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j - \int_{t-\tau}^{t} \dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j ds \right), \tag{19}$$

where we have used similar steps as in the proof of Theorem 1 to obtain this equality. Note that $(\bar{\tilde{\mathbf{p}}}_{ij} - \tilde{\mathbf{p}}_{ij}) = (\tilde{\mathbf{p}}_j - \tilde{\mathbf{p}}_j(t-\tau_{ij})) = \int_{t-\tau_{ij}}^{t} \dot{\tilde{\mathbf{p}}}_j ds$. Therefore, using young's and jensen's inequalities, we can verify that

$$2\dot{\tilde{\mathbf{p}}}_i^\top \int_{t-\tau_{ij}}^{t} \dot{\tilde{\mathbf{p}}}_j ds \le \epsilon_{ij} \dot{\tilde{\mathbf{p}}}_i^\top \dot{\tilde{\mathbf{p}}}_i + \frac{\tau_{ij}}{\epsilon_{ij}} \int_{t-\tau_{ij}}^{t} \dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j ds, \tag{20}$$

for some strictly positive ϵ_{ij}. Without loss of generality, we set $\epsilon_{ij} = \epsilon_{ji} = \epsilon$. Therefore, using the undirected property of the communication flow; $k_{ij} = k_{ji}$, we obtain

$$\dot{V} \le - \sum_{i=1}^{n} \bar{k}_i \dot{\tilde{\mathbf{p}}}_i^\top \dot{\tilde{\mathbf{p}}}_i - \frac{\gamma}{2} \sum_{j=1}^{n} k_{ij} \dot{\bar{\tilde{\mathbf{p}}}}_{ij}^\top \dot{\bar{\tilde{\mathbf{p}}}}_{ij}, \tag{21}$$

with $\bar{k}_i$ is given in (17) and we have used the inequality: $\tau_{ij} \int_{t-\tau_{ij}}^{t} \dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j ds \le \tau \int_{t-\tau}^{t} \dot{\tilde{\mathbf{p}}}_j^\top \dot{\tilde{\mathbf{p}}}_j ds$. Therefore, $\dot{V}$ is negative semi-definite and satisfies condition A.2 in Theorem 4. Let $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ be defined as in the proof of Theorem 1 and define the following negative semi-definite function $U = \left(-\sum_{i=1}^{n} \bar{k}_i \dot{\tilde{\mathbf{p}}}_i^\top \dot{\tilde{\mathbf{p}}}_i - \frac{\gamma}{2} \sum_{j=1}^{n} k_{ij} \dot{\bar{\tilde{\mathbf{p}}}}_{ij}^\top \dot{\bar{\tilde{\mathbf{p}}}}_{ij} \right)$. Following similar steps as in the proof of Theorem 1, with the same function W given in (14), we can show that the time derivative of W on the set $\{(\boldsymbol{\zeta}, \boldsymbol{\xi}) \mid U = 0\}$ is obtained as

$$\dot{W} = - \sum_{i=1}^{n} \left(\sum_{j=1}^{n} k_{ij} \bar{\tilde{\mathbf{p}}}_{ij} \right)^\top \left(\sum_{j=1}^{n} k_{ij} \tilde{\mathbf{p}}_{ij} \right), \tag{22}$$

which is equivalent to (15) in view of the following relation

$$\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j(t-\tau_{ij}) = \tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j - \int_{t-\tau_{ij}}^{t} \dot{\tilde{\mathbf{p}}}_j ds. \tag{23}$$

Therefore, using the same arguments as in the proof of Theorem 1, we can verify that condition A.4 in Theorem 4 is satisfied and we conclude that the equilibrium of system (6) is asymptotically stable, which implies that $\dot{\tilde{\mathbf{p}}}_i \to 0$ for $i \in \mathcal{N}$ and $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j(t-\tau_{ij})) \to 0$ for all $i, j \in \mathcal{N}$. Furthermore, in view of equation (23) and the convergence to zero of $\dot{\tilde{\mathbf{p}}}_i$, we conclude that $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j) \to 0$ for all $i, j \in \mathcal{N}$. To complete the proof, we notice that the signals $\dot{\tilde{\mathbf{p}}}_i$, $(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j)$ and the input $\boldsymbol{\eta}_i$ given in (16) are guaranteed to be bounded and converge asymptotically to zero. Therefore, the virtual system dynamics (4) can be rewritten as in (10) with $\boldsymbol{\varepsilon}_i = \boldsymbol{\eta}_i$, and using the same arguments as in the last part of the proof of Theorem 1, with the help of Proposition 1, the results of the theorem are obtained. □

4. SYNCHRONIZATION WITH PARTIAL STATE FEEDBACK

One of the major difficulties with the proposed control structure in the previous section is that the generalized coordinate derivatives are essential in the model of the introduced virtual system. This makes the extension of this method to the velocity-free case not straightforward. In this section, we present a velocity-free synchronization scheme that accounts for input torque saturations using the lead-filter approach introduced in Berghuis and Nijmeijer (1993) and used in Loria and Nijmeijer (1996) for the trajectory tracking of robot manipulators subject to input torque saturations. We propose the following velocity-free control scheme

$$\boldsymbol{\Gamma}_i = \mathbf{G}_i(\mathbf{q}_i) - k_i^v \chi(\mathbf{q}_i - \boldsymbol{\psi}_i) - \sum_{j=1}^{n} k_{ij} \chi(\mathbf{q}_i - \mathbf{q}_j), \tag{24}$$

$$\dot{\boldsymbol{\psi}}_i = k_i^\psi (\mathbf{q}_i - \boldsymbol{\psi}_i). \tag{25}$$

Note that the input torque for each system is guaranteed to be upper bounded as

$$\|\boldsymbol{\Gamma}_i\|_\infty \le g_m + \sigma_b (k_i^v + \sum_{j=1}^{n} k_{ij}). \tag{26}$$

Note that the above upper bound depends on each system's neighbors, and hence the proposed scheme in this section shares the same limitations of existent solutions in the full state information case in terms of control parameters tuning. Our results are stated in the following theorem.

Theorem 3. Consider a network of n-systems modeled as in (1) with the control input given in (24)-(25). Let the undirected communication graph be connected and the control gains satisfy

$$\sigma_b (k_i^v + \sum_{j=1}^{n} k_{ij}) \le \boldsymbol{\Gamma}_{\max} - g_m, \tag{27}$$

then $\|\boldsymbol{\Gamma}_i\|_\infty \le \boldsymbol{\Gamma}_{\max}$, for $i \in \mathcal{N}$, the signals $\dot{\mathbf{q}}_i$ and $(\mathbf{q}_i - \mathbf{q}_j)$ are bounded and $\dot{\mathbf{q}}_i \to 0$ and $(\mathbf{q}_i - \mathbf{q}_j) \to 0$, for all $i, j \in \mathcal{N}$.

Proof. Similar to the proof of Theorem 1, if the control gains are selected according to (27), we can verify from (26) that $\|\boldsymbol{\Gamma}_i\|_\infty \le \boldsymbol{\Gamma}_{\max}$ for $i \in \mathcal{N}$.
Consider the following Lyapunov function candidate

$$V = \frac{1}{2} \sum_{i=1}^{n} \dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i) \dot{\mathbf{q}}_i + \sum_{i=1}^{n} k_i^v \sum_{k=1}^{m} \int_0^{\boldsymbol{\psi}_{ii}^k} \sigma(s) ds$$
$$+ \frac{1}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij} \sum_{k=1}^{m} \int_0^{\mathbf{q}_{ij}^k} \sigma(s) ds, \tag{28}$$

with $\boldsymbol{\psi}_{ii}^k$ and $\mathbf{q}_{ij}^k$ are the k^{th} elements of the vectors $(\mathbf{q}_i - \boldsymbol{\psi}_i)$ and $(\mathbf{q}_i - \mathbf{q}_j)$ respectively. The time derivative of V evaluated along the systems dynamics (1) is obtained as

$$\dot{V} = \sum_{i=1}^{n} \dot{\mathbf{q}}_i^\top \left(\boldsymbol{\Gamma}_i - \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i - \mathbf{G}_i(\mathbf{q}_i) + \frac{1}{2} \dot{\mathbf{M}}_i(\mathbf{q}_i) \dot{\mathbf{q}}_i \right)$$
$$+ \sum_{i=1}^{n} k_i^v (\dot{\mathbf{q}}_i - \dot{\boldsymbol{\psi}}_i)^\top \chi(\mathbf{q}_i - \boldsymbol{\psi}_i)$$
$$+ \frac{1}{2} \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij} (\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_j)^\top \chi(\mathbf{q}_i - \mathbf{q}_j), \tag{29}$$

which in view of (24) and (25), with property P.2, is obtained as

$$\dot{V} = - \sum_{i=1}^{n} k_i^\psi k_i^v (\mathbf{q}_i - \boldsymbol{\psi}_i)^\top \chi(\mathbf{q}_i - \boldsymbol{\psi}_i), \tag{30}$$

where we have used the properties of the undirected graph; $k_{ij} = k_{ji}$, and the properties of the function χ to deduce that

$$\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(\dot{\mathbf{q}}_i-\dot{\mathbf{q}}_j)^\top\chi(\mathbf{q}_i-\mathbf{q}_j) = \sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\dot{\mathbf{q}}_i^\top\chi(\mathbf{q}_i-\mathbf{q}_j).$$

Therefore, $\dot{V}$ is negative semi-definite, and we can conclude that $\dot{\mathbf{q}}_i$, $(\mathbf{q}_i-\boldsymbol{\psi}_i)$ and $(\mathbf{q}_i-\mathbf{q}_j)$ are bounded for all $i, j \in \mathcal{N}$, since the communication graph is connected. Also, we can see form (25) that $\dot{\boldsymbol{\psi}}_i$ is bounded. As a result, we know that $\ddot{V}$ is bounded and by invoking Barbălat Lemma, we conclude that $(\mathbf{q}_i - \boldsymbol{\psi}_i) \to 0$ and $\dot{\boldsymbol{\psi}}_i \to 0$.
Exploiting the above boundedness results, we can verify from (1), with (24)-(25) that $\ddot{\mathbf{q}}_i$ and $\ddot{\boldsymbol{\psi}}_i$ are bounded. Consequently, we conclude by Barbălat Lemma that $(\dot{\mathbf{q}}_i - \dot{\boldsymbol{\psi}}_i) \to 0$, and hence we know that $\dot{\mathbf{q}}_i \to 0$. In addition, we can verify that the time derivative of $\mathbf{M}_i^{-1}(\mathbf{q}_i)$ is bounded from the boundedness of $\dot{\mathbf{q}}_i$. Also, we know that $\frac{d\chi(\mathbf{q}_i-\mathbf{q}_j)}{dt} = h(\mathbf{q}_i-\mathbf{q}_j)(\dot{\mathbf{q}}_i-\dot{\mathbf{q}}_j)$ is bounded in view of property P.7, where the matrix $h(x)$ is defined as diag$[\frac{\partial\sigma(x^k)}{\partial x^k}]$, for any vector $\mathbf{x} = \text{col}[x^k]$, and $k = 1, \ldots, m$. Invoking the extended Barbălat Lemma (See for instance Lemma 2 in Hua et al. (2009)), we conclude that $\ddot{\mathbf{q}}_i \to 0$ since $\chi(\mathbf{q}_i - \mathbf{q}_j)$ is uniformly continuous. As a result, the closed loop dynamics reduces to: $\sum_{j=1}^{n} k_{ij}\chi(\mathbf{q}_i - \mathbf{q}_j) \to 0$, for $i \in \mathcal{N}$, which is equivalent to: $\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\mathbf{q}_i^\top\chi(\mathbf{q}_i - \mathbf{q}_j) \to 0$. Using the properties of the undirected graph, we can show that $\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}\mathbf{q}_i^\top\chi(\mathbf{q}_i - \mathbf{q}_j) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij}(\mathbf{q}_i - \mathbf{q}_j)^\top\chi(\mathbf{q}_i - \mathbf{q}_j)$, and hence we conclude that $(\mathbf{q}_i - \mathbf{q}_j) \to 0$ for all $i, j \in \mathcal{N}$. □

5. SIMULATION RESULTS

We consider in this section the example of a group of four two degrees of freedom rigid robot manipulator arms (with revolute joints). The four arms are governed by the same dynamic model described in Section 4 in Tayebi (2004), with the initial conditions: $\mathbf{q}_1(0) = (\pi/6, \pi/5)^\top$ rad, $\mathbf{q}_2(0) = (\pi/4, \pi/3)^\top$ rad, $\mathbf{q}_3(0) = (\pi/2, \pi/7)^\top$ rad, $\mathbf{q}_4(0) = (\pi/5, \pi)^\top$ rad, $\dot{\mathbf{q}}_1(0) = (-0.3, 0.4)^\top$ rad/sec, $\dot{\mathbf{q}}_2(0) = (0.2, -0.3)^\top$ rad/sec, $\dot{\mathbf{q}}_3(0) = (-0.1, 0.1)^\top$ rad/sec, $\dot{\mathbf{q}}_4(0) = (0, 0.5)^\top$ rad/sec. It is assumed that the inputs of all systems are constrained such that $\|\boldsymbol{\Gamma}_i\|_\infty \le \boldsymbol{\Gamma}_{\max} = 13$. The communication flow between systems in the network is represented by undirected graph $\mathcal{G}$ having the set of edges $\mathcal{E} = \{(1,2), (1,4), (2,3), (2,4)\}$, and adjacency matrix $\mathcal{K} = [k_{ij}]_{4\times4}$, with $k_{ij} = 6$ for $(i, j) \in \mathcal{E}$, and zero otherwise.

In the full state information case, we implement for each manipulator the virtual system (4) with the inputs (7) and (16) with the initial states: $\mathbf{p}_i(0) = \dot{\mathbf{p}}_i(0) = (0, 0)^\top$, the control gains: $(k_i^p, k_i^d, k_i^v) = (1.5, 1.5, 20)$, and the constant communication delays: $\tau_{1j} = 0.5$ sec, $\tau_{2j} = 0.4$ sec, $\tau_{3j} = 0.7$ sec, and $\tau_{4j} = 0.3$ sec, for $j \in \mathcal{N} := \{1, ..., 4\}$. Note that the control gains are selected so that conditions (11) and (17) are satisfied with $g_m = 9.81$, $\tau = 0.8$ and $\epsilon = 1$. The obtained result are illustrated in Fig.1 to Fig.2, where we can see that all robot arms synchronize their joint angles to the same constant final value, and $\|\boldsymbol{\Gamma}_i\|_\infty \le \boldsymbol{\Gamma}_{\max}$ for $i \in \mathcal{N}$. Similar results have been obtained when the velocity-free synchronization scheme in Theorem 3 is considered, and are omitted due to space limitations.

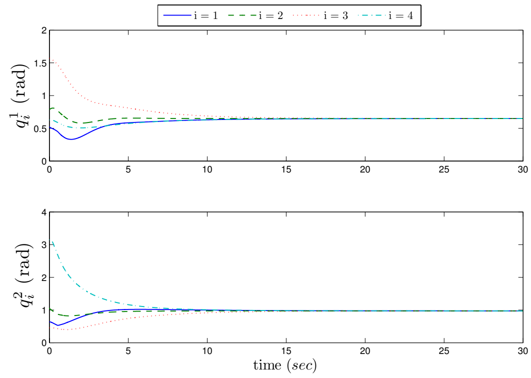

Fig. 1. Joint angles with $\mathbf{q}_i = (q_i^1, q_i^2)^\top$, Theorem 2.

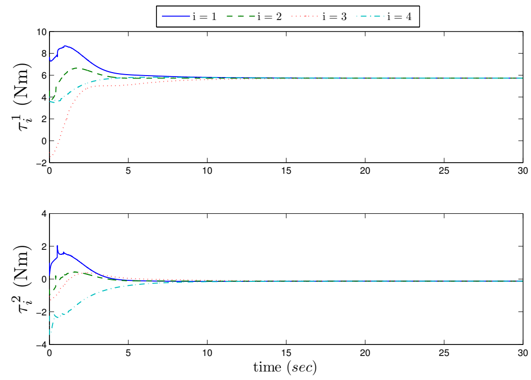

Fig. 2. Input torques with $\boldsymbol{\Gamma}_i = (\tau_i^1, \tau_i^2)^\top$, Theorem 2.

6. CONCLUSION

The synchronization problem of networked Euler-Lagrange systems subject to input torque constraints was addressed in this paper. Two control schemes have been presented in the full and partial state information cases. The proposed state feedback control scheme is based on the implementation, in each member of the group, of a virtual system governed by similar dynamics with an additional virtual input. This design method was shown to provide an input upper bound that can be set independently from the information topology between the systems in the network. Moreover, the effects of delays often present in communication systems can be studied without any consideration of the boundedness constraint of the input torque for each system. In the partial state feedback case, a velocity-free synchronization law accounting for the systems' input constraints was proposed, where lead-filters have been used to remove the requirement of velocity measurements. It should be noted that the last control law is not based on virtual systems, and hence does not enjoy the properties of the proposed state feedback synchronization scheme.

Appendix A. PROOF OF PROPOSITION 1

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}\dot{\mathbf{p}}_i^\top\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{p}}_i + k_i^p\sum_{k=1}^{m}\int_0^{\mathbf{p}_i^k}\sigma(s)ds, \qquad \text{(A.1)}$$

with $\mathbf{p}_i = \text{col}[p_i^k]$, for $k = 1, ..., m$. We can easily verify that V_1 is radially unbounded form the definition of σ. The time derivative of V_1 along (10) is given as

$$\begin{aligned}\dot{V}_1 &= \dot{\mathbf{p}}_i^\top(-\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{p}}_i - k_i^p\chi(\mathbf{p}_i) - k_i^d\chi(\dot{\mathbf{p}}_i) + \boldsymbol{\varepsilon}_i)\\ &\quad + \frac{1}{2}\dot{\mathbf{p}}_i^\top\dot{\mathbf{M}}_i(\mathbf{q}_i)\dot{\mathbf{p}}_i + k_i^p\dot{\mathbf{p}}_i^\top\chi(\mathbf{p}_i)\\ &\le -\sum_{k=1}^{m}|\dot{\mathbf{p}}_i^k|\left(k_i^d\sigma(|\dot{\mathbf{p}}_i^k|) - |\boldsymbol{\varepsilon}_i^k|\right), \end{aligned} \tag{A.2}$$

with $\boldsymbol{\varepsilon}_i = \text{col}[\boldsymbol{\varepsilon}_i^k]$, for $k = 1, ..., m$, and we have used the property; $x\sigma(x) = |x|\sigma(|x|)$, for any $x \in \mathbb{R}$, and the skew symmetric property P.2. First of all, let us show that $\mathbf{p}_i$ and $\dot{\mathbf{p}}_i$ cannot escape in finite time. In fact, it is clear from (A.2) that $\dot{V}_1 \le \|\dot{\mathbf{p}}_i\|\|\boldsymbol{\varepsilon}_i\|$. Using the fact that $2V_1 \ge \lambda_{\min}\{\mathbf{M}_i(\mathbf{q}_i)\}\|\dot{\mathbf{p}}_i\|^2$, we have $\dot{V}_1 \le \bar{\varepsilon}_i\sqrt{V_1}$, with $\sqrt{\frac{2}{\lambda_{\min}\{\mathbf{M}_i(\mathbf{q}_i)\}}}\|\boldsymbol{\varepsilon}_i\| \le \bar{\varepsilon}_i$, which can be rewritten as $\frac{dV_1}{\sqrt{V_1}} \le \bar{\varepsilon}_i dt$. Integrating this last inequality over the interval $[t_0, t]$ yields to: $2\left(\sqrt{V_1(t)} - \sqrt{V_1(t_0)}\right) \le \bar{\varepsilon}_i(t-t_0)$, which shows that V_1 cannot go to infinity in finite time. Now, it is easily seen that the right hand side of (A.2) is negative as long as:

$$\sigma(|\dot{\mathbf{p}}_i^k|) > \frac{|\boldsymbol{\varepsilon}_i^k|}{k_i^d}, \quad \text{for } k = 1, ..., m. \tag{A.3}$$

Due to the fact that σ is bounded, inequality (A.3) cannot be satisfied when $|\boldsymbol{\varepsilon}_i^k| > \sigma_b k_i^d$, for $k = 1, ..., m$. However, since $\boldsymbol{\varepsilon}_i$ is bounded and converges asymptotically to zero, it is clear that there exists a finite time t_1 such that $|\boldsymbol{\varepsilon}_i^k(t)| \le \sigma_b k_i^d$ for all $t \ge t_1$. Note that $\mathbf{p}_i$ and $\dot{\mathbf{p}}_i$ remain bounded on the interval $[0, t_1]$ as there is no finite-escape time. Consequently, for all $t \ge t_1$, one can conclude that $\dot{V}_1 < 0$, and $\mathbf{p}_i$ and $\dot{\mathbf{p}}_i$ are bounded outside the set

$$\mathcal{S} = \left\{\dot{\mathbf{p}}_i \;\middle|\; \sigma(|\dot{\mathbf{p}}_i^k|) \le \frac{|\boldsymbol{\varepsilon}_i^k|}{k_i^d}, \text{ for } k = 1, ..., m\right\}.$$

Since $\sigma(|.|)$ is a class $\mathcal{K}$ function, $\dot{\mathbf{p}}_i$ is ultimately bound to reach the set $\mathcal{S}$ and will be driven to zero as $\boldsymbol{\varepsilon}_i \to 0$. If $\dot{\mathbf{q}}_i$ is globally bounded, we can conclude that $\frac{d}{dt}(\mathbf{M}_i(\mathbf{q}_i)^{-1})$ is bounded. This can be seen from properties P.1-P.3 and the expression: $\frac{d}{dt}(\mathbf{M}_i(\mathbf{q}_i)^{-1}) = -\mathbf{M}_i(\mathbf{q}_i)^{-1}\dot{\mathbf{M}}_i(\mathbf{q}_i)\mathbf{M}_i(\mathbf{q}_i)^{-1}$. As a result, using the extended Barbălat Lemma (See Lemma 2 in Hua et al. (2009)) and property P.7, we conclude form (10) that $\ddot{\mathbf{p}}_i \to 0$, and hence we have $\mathbf{p}_i \to 0$.

Appendix B. MATROSOV'S THEOREM

The following theorem and Lemma are reported from Ren (2009) and Paden and Panja (1988).

Theorem 4. (Matrosov's theorem) Given the system

$$\dot{x} = f(x, t), \tag{B.1}$$

where $f(t, 0) = 0$ and f is such that solutions exist and are unique. Let $V(t, x)$ and $W(t, x)$ be continuous functions on domain $\mathbb{D}$ and satisfy the following four conditions:

- A.1 V(t,x) is positive definite and decrescent.
- A.2 $\dot{V}(x, t) \le U(x) \le 0$, where $U(x)$ is continuous.
- A.3 $|W(t, x)|$ is bounded.
- A.4 $\max(d(x, M), |\dot{W}(t, x)|) \ge \alpha(\|x\|)$, where $M = \{\ x \mid U(x) = 0\ \}$, $d(x, M)$ denotes the distance from x to set M, and $\alpha(\cdot)$ is a class-$\mathcal{K}$ function.

Then the equilibrium of (B.1) is uniformly asymptotically stable on $\mathbb{D}$.

Lemma 1. (Paden and Panja (1988)) Condition A.4 in Theorem 4 is satisfied if the following two conditions are satisfied:

(1) The function $\dot{W}(x, t)$ is continuous in both arguments and $\dot{W}(x, t) = g(x, \beta(t))$, where g is continuous in both arguments and $\beta(t)$ is continuous and bounded.
(2) There exists a class $\mathcal{K}$ function, θ, such that $|\dot{W}(x, t)| \ge \theta(\|x\|)$ for all $x \in M$, where M is the set defined in Theorem 4.

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