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Synchronization of Heterogeneous Euler-Lagrange Systems with Time Delays and Intermittent Information Exchange

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Abstract: This paper studies the synchronization problem of networked uncertain Euler-Lagrange systems with intermittent communication in the presence of irregular communication delays and possible information loss. The interconnection between agents is described by a directed graph containing a spanning tree. Based on the small-gain framework, we propose an adaptive distributed control algorithm to steer all agents' positions to a common position with a prescribed desired velocity available to only some leaders. The communication between agents is intermittent in the sense that neighboring agents exchange their information in a discrete manner with possible packet dropout. Numerical simulation is provided to demonstrate the effectiveness of the proposed synchronization scheme.

Keywords: Synchronization; Euler-Lagrange systems; Leader-Follower; Intermittent communication; Communication delays.

1. INTRODUCTION

Motion coordination of mechanical systems modeled by Euler-lagrange dynamics has received an increased interest in the control community due to the potential applications involving groups of unmanned vehicles and robotic systems [Ren and Cao, 2011]. The coordinated control of these systems can be formulated as synchronization or consensus problems, where the goal is to drive the networked systems (or agents) to a common state. Recently, extensive efforts have been devoted to the design of control algorithms for the synchronization of networked Lagrangian systems [see, for instance, Spong and Chopra, 2007, Ren, 2009, Chung and Slotine, 2009, Mei et al., 2012, Wang, 2013b]. While various problems related to systems dynamics, such as uncertainties, and the communication topology between the team members have been addressed in these references, several problems remain unsolved especially in the presence of communication constraints.

In Spong and Chopra [2007], it has been shown that the passivity-based approach is robust to constant communication delays if the interconnection graph is directed, yet balanced and strongly connected. A similar property was shown in Chung and Slotine [2009] for unbalanced graphs using the contraction theorem. In Münz et al. [2011], a delay-robust control scheme is proposed for strictly passive relative-degree two systems with nonlinear interconnections. In Nuño et al. [2011], an adaptive synchronization algorithm is presented under a directed graph and constant communication delays. A virtual systems approach has been suggested for networked Lagrangian systems in Abdessameud and Tayebi [2011a] and for a different class of nonlinear systems in Abdessameud and Tayebi [2013] to account for input saturations in the presence of constant communication delays. With the same assumption on the delays, an adaptive cooperative tracking controller is proposed in Wang [2013a] for networked robotic systems in the task space. The more general case of time-varying irregular communication delays has been recently considered in Abdessameud et al. [2014] under a directed communication graph.

An important problem in the above mentioned papers dealing with communication delays is that position synchronization, *i.e.,* all positions converge to a common value, is achieved with zero final velocity. The only cases where the final velocities match a non-zero value assume a full access to a reference trajectory or to a leader's states (position and velocity). This problem can also be seen in the literature of nonlinear multi-agent systems [see, for instance, Abdessameud and Tayebi, 2011b, Abdessameud et al., 2012]. Another issue that can be observed in all the aforementioned results is the assumption that information is transmitted continuously between agents. In fact, it is not clear if these results still apply in situations where agents are allowed to communicate with their neighbors only on some disconnected time intervals or instants, due to environmental constraints, such as communication obstacles, or temporary sensor failure. In the ideal case of no communication delays, it has been shown that linear multi-agent systems [Wen et al., 2012b, 2013] and some globally Lipschitsz nonlinear systems [Wen et al., 2012a] can still achieve consensus with intermittent communication. Un-

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fortunately, these results cannot be applied to networked lagrangian systems, due to the nonlinear and generally uncertain systems dynamics, especially if one considers varying delays and possible packet loss.

In this paper, we consider the synchronization problem of networked uncertain heterogeneous Lagrangian systems with intermittent communication under a directed communication topology. Here, it is required that all systems achieve position synchronization with some non-zero desired velocity available to only some systems in the group acting as leaders. Based on the small-gain approach, we propose a continuous-time distributed adaptive control algorithm that allows agents to communicate with their neighbors only at some irregular discrete time-intervals and achieve our control objective. A discrete-time consensus algorithm is also used to handle the partial access of the desired velocity. In the case where no desired velocity is assigned to the team, the proposed synchronization algorithm achieves position synchronisation with some velocity agreed upon by all agents. In both cases, it is proved that, under some sufficient conditions, synchronization is achieved in the presence of unknown irregular communication delays and packet loss provided that the directed communication graph contains a spanning tree. The derived conditions impose a maximum allowable interval of time in which a particular agent does not receive information from some or all of its neighbors. This interval can be specified arbitrarily with a choice of the control gains. Simulation results on a team of ten robot manipulators are provided to show the effectiveness of our theoretical results.

2. PROBLEM FORMULATION

2.1 System model

Consider a network of n not necessarily identical systems described by Euler-Lagrange equations of the form

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{G}_i(\mathbf{q}_i) = \mathbf{u}_i, \qquad (1)$$

for $i \in \mathcal{N} \triangleq \{1, ..., n\}$, where $\mathbf{q}_i \in \mathbb{R}^m$ is the vector of generalized configuration coordinates, $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{m\times m}$ is the inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i \in \mathbb{R}^m$ is the vector of centrifugal/Coriolis forces, $\mathbf{G}_i(\mathbf{q}_i) \in \mathbb{R}^m$ is the vector of potential forces, and $\mathbf{u}_i \in \mathbb{R}^m$ is the vector of torques associated with the i^{th} system. The inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are symmetrical and positive definite uniformly with respect to $\mathbf{q}_i$. Other common properties of Euler-Lagrange systems (1) are as follows.

- P.1 Each system in (1) admits a linear parametrization of the form $\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{x}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\mathbf{x}_i + \mathbf{G}_i(\mathbf{q}_i) = \mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \mathbf{x}_i, \dot{\mathbf{x}}_i)\boldsymbol{\theta}_i$, where $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \mathbf{x}_i, \dot{\mathbf{x}}_i)$ is a known regressor matrix and $\boldsymbol{\theta}_i \in \mathbb{R}^k$ is the vector of the system's parameters.
- P.2 The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew symmetric.
- P.3 There exists $k_{c_i} \geq 0$ such that $|\mathbf{C}_i(\mathbf{q}_i, \mathbf{x})\mathbf{y}| \leq k_{c_i}|\mathbf{x}| \cdot |\mathbf{y}|$ holds for all $\mathbf{q}_i,\ \mathbf{x},\ \mathbf{y} \in \mathbb{R}^m$. In addition, $\mathbf{M}_i(\mathbf{q}_i)$ and $\mathbf{G}_i(\mathbf{q}_i)$ are bounded uniformly with respect to $\mathbf{q}_i$. Here, $|\cdot|$ denotes the Euclidean norm of a vector.

2.2 Graph theory preliminaries

Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be a directed graph, with a set of nodes (or vertices) $\mathcal{N}$, and a set of ordered edges (pairs of nodes) $\mathcal{E} \subset \mathcal{N}\times\mathcal{N}$. An edge $(j, i) \in \mathcal{E}$ is represented by a directed link (arc) leaving node j and directed toward node i; in this case, node j is called a neighbor of node i. A directed graph $\mathcal{G}$ is said to contain a spanning tree if there exists at least one node that has a "directed path" to all the other nodes in the graph; by a directed path (of length q) from j to i is meant a sequence of edges in a directed graph of the form $(j, l_1), (l_1, l_2), \ldots, (l_{q-1}, l_q)$, with $l_q = i$, where for $q > 1$ the nodes $j, l_1, \ldots, l_{q-1} \in \mathcal{N}$ are distinct. Node r is called a root of $\mathcal{G}$ if it is the root of a directed spanning tree of $\mathcal{G}$; in this case, $\mathcal{G}$ is said to be rooted at r.

A weighted directed graph $\mathcal{G}_w$ consists of the triplet $(\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N}$ and $\mathcal{E}$ are, respectively, the sets of nodes and edges defined as above, and $\mathcal{A}$ is the weighted adjacency matrix defined such that $a_{ii} \triangleq 0$, $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Note that thus defined graph does not contain self-links at any node and will have the same properties as the unweighted graph with the same sets of nodes and edges. The Laplacian matrix $\mathbf{L} := [l_{ij}] \in \mathbb{R}^{n\times n}$ of the weighted directed graph $\mathcal{G}_w$ is defined such that: $l_{ii} = \sum_{j=1}^n a_{ij}$, and $l_{ij} = -a_{ij}$ for $i \neq j$.

2.3 Problem statement

It is assumed throughout the paper that the vectors of agents' parameters $\boldsymbol{\theta}_i \in \mathbb{R}^k$, $i \in \mathcal{N}$, are constant and unknown. Also, the interconnection topology in the network is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ describes the set of all agents (or systems modeled by (1)) in the network, and an edge $(j, i) \in \mathcal{E}$ indicates that the i-th agent *can* receive information from the j-th agent. While $\mathcal{G}$ is fixed, the communication process between neighbouring agents is not continuous but discrete in time and intermittent; in particular, it may be performed only over a certain sequence of discrete time instants and is subject to time-varying communication delays, information losses, and blackout intervals. Specifically, the communication process is described as follows. It is assumed that there exists a sequence of communication instants $t_k := kT \in \mathbb{R}_+$, $k \in \mathbb{Z}_+ = \{0, 1, \ldots\}$, where $T > 0$ is a fixed sampling period which is common for all agents, such that each agent is allowed to send information to all or some of its neighbors at instants t_k, $k = 0, 1, \ldots$. This discrete-time communication is subject to time-varying communication delays and information losses. Formally, for each pair $(j, i) \in \mathcal{E}$, there exist a sequence of communication delays $(\tau_k^{(j,i)})_{k\in\mathbb{Z}_+}$ that take values in $\{\mathbb{R}_+ \cup +\infty\}$ such that the information sent by agent j at instant t_k is available to agent i starting from the instant $t_k + \tau_k^{(j,i)}$. In particular, it is possible that $\tau_k^{(j,i)} = +\infty$ for some $k \in \mathbb{Z}_+$, which corresponds to a situation where the agent j has not sent information at instant t_k to neighbour i at all, or the corresponding information was never delivered possibly due to packet loss in the communication channel. The following assumption is imposed on the communication process between neighbouring agents.

Assumption 1. For each $(j, i) \in \mathcal{E}$ there exist numbers $k^* \in \mathbb{N}$, $h \geq 0$, and an infinite strictly increasing subsequence $\mathcal{K}^{(j,i)} := \{k_0^{(j,i)}, k_1^{(j,i)}, \ldots\} \subset \{0, 1, \ldots\}$ satisfying

- i) $k_0^{(j,i)} \leq k^*$, and $k_{l+1}^{(j,i)} - k_l^{(j,i)} \leq k^*$, $l \in \{0, 1, \ldots\}$,
- ii) $\tau_k^{(j,i)} \leq h$ for each $k \in \mathcal{K}^{(j,i)}$.

Assumption 1 essentially means that, for each pair $(j, i) \in \mathcal{E}$, and per any k^* consecutive sampling instants, there exists at least one sampling instant at which agent j has sent information to agent i, and this information has been successfully delivered with delay less than or equal to h. Assumption 1 also implies that, for each pair $(j,i) \in \mathcal{E}$, the maximal interval between two consecutive instants when agent i receives information from agent j is less than or equal to

$$h^* := k^*T + h. \tag{2}$$

The objective of this work is to design a control algorithm for (1) such that $\dot{\mathbf{q}}_i(t) \to \dot{\mathbf{q}}_d$ and $\mathbf{q}_i(t) - \mathbf{q}_j(t) \to 0$ for all $i,j \in \mathcal{N}$ as $t \to +\infty$, where $\dot{\mathbf{q}}_d \in \mathbb{R}^m$ is a constant desired velocity available to only a subset of systems in the group. For this, let $\mathcal{L} \subset \mathcal{N}$ denote the set of indices of the agents having access to the desired velocity, which play the role of leaders. The rest of agents, referred to as followers, do not know the desired velocity nor do they know who the leaders are.

3. MAIN RESULTS

In this section, we present synchronization schemes that achieve our objectives using discrete-time intermittent communication between the agents in the presence of time-varying communication delays and information losses. To solve this problem, a continuous-time control algorithm that takes into account the communication constraints will be developed for each agent in the team. In addition, since the desired velocity is not available to all agents, each follower (*i.e.*, each i-th agent with $i \in \mathcal{F} := \mathcal{N} \setminus \mathcal{L}$) will run a discrete-time consensus-seeking algorithm, updated at instants σT, $\sigma \in \mathbb{Z}_+$, that provides an estimate $\hat{\mathbf{v}}_i(\cdot)$ of the desired velocity vector $\dot{\mathbf{q}}_d$. This desired velocity estimate is transmitted through the communication channels to the neighbouring agents together with the position information and the corresponding time-stamp. Specifically, for each $(j,i) \in \mathcal{E}$ and each $k \in \mathbb{Z}_+$, let $\mathbf{Q}_j^i(k) := [\mathbf{q}_j(kT), \hat{\mathbf{v}}_j(k), k]$ denote the information that can be transmitted from agent j to agent i at $t = kT$, where $\mathbf{q}_j(kT)$ is the position of the j-th system at $t = kT$, $\hat{\mathbf{v}}_j(k)$ is the desired velocity estimate [1], and k is the time-stamp (*i.e.*, the sequence number at which information was sent).

For each $(j,i) \in \mathcal{E}$ and $\sigma \in \mathbb{Z}_+$, denote $l^{(j,i)}(\sigma) := \max\{l \in \mathbb{Z}_+ :\ Tl + \tau_l^{(j,i)} < \sigma T\}$. By definition, $l^{(j,i)}(\sigma)$ is the largest sequence number such that the data $\mathbf{Q}_j^i(l^{(j,i)}(\sigma))$ has been received by the i-th agent before the instant $t = \sigma T$. Consider the following consensus-seeking algorithm

$$\hat{\mathbf{v}}_i(\sigma+1) = \hat{\mathbf{v}}_i(\sigma), \qquad \text{for } i \in \mathcal{L}, \tag{3}$$

$$\hat{\mathbf{v}}_i(\sigma+1) = \frac{1}{\mid N_i(\sigma) \mid} \sum_{j \in N_i(\sigma)} \hat{\mathbf{v}}_{ij}(\sigma), \qquad \text{for } i \in \mathcal{F}, \tag{4}$$

where $\hat{\mathbf{v}}_i(0) = \dot{\mathbf{q}}_d$ for $i \in \mathcal{L}$, and $\hat{\mathbf{v}}_i(0)$ for $i \in \mathcal{F}$ can be set equal to arbitrary finite values, $N_i(\sigma) := \{i\} \cup N_{ji}(\sigma)$, where $N_{ji}(\sigma) := \{j : (j,i) \in \mathcal{E},\ l^{(j,i)}(\sigma) > l^{(j,i)}(\sigma-1)\}$ denotes the set of the neighbours of the i-th follower such that the most recent data from these neighbours has been received during the interval $[(\sigma-1)T, \sigma T)$, $\mid N_i(\sigma) \mid$ denotes the number of elements in $N_i(\sigma)$, and

$$\hat{\mathbf{v}}_{ij}(\sigma) := \begin{cases} \hat{\mathbf{v}}_j(l^{(j,i)}(\sigma)) & \text{if } j \neq i, \\ \hat{\mathbf{v}}_i(\sigma) & \text{if } j = i. \end{cases} \tag{5}$$

[1] For simplicity, we use throughout the paper the notation $x(k)$, $k \in \mathbb{Z}_+$, instead of $x(kT)$ for the discrete-time signals.

It should be noted that the update law for the followers, given in (4)-(5), is based on the agent's own velocity estimate and its neighbours' most recent velocity estimates that have been successfully delivered and have not been used in the update law at the previous update instants. On the other hand, the leaders do not update their estimates (according to (3)). Also, the parameter $l^{(j,i)}(\sigma)$ and the set $N_i(\sigma)$ can be evaluated easily, at each σT, by simple comparison between the successfully received time stamps.

Now, define the following filtered reference velocity estimate

$$\begin{cases} \mathbf{v}_i^r = \dot{\mathbf{q}}_d, & \text{for } i \in \mathcal{L}, \\ \ddot{\mathbf{v}}_i^r(t) = -k_i^d \dot{\mathbf{v}}_i^r - k_i^p \left(\mathbf{v}_i^r(t) - \hat{\mathbf{v}}_i\left(\lfloor t/T \rfloor\right)\right), & \text{for } i \in \mathcal{F} \end{cases} \tag{6}$$

where $\mathbf{v}_i^r(0)$ and $\dot{\mathbf{v}}_i^r(0)$, for $i \in \mathcal{F}$, can take arbitrary finite values, and k_i^p, k_i^d are strictly positive gains. Also, for each pair $(j,i) \in \mathcal{E}$ and each time instant $t \geq 0$, let $k_t^{(j,i)}$ denote the largest integer number such that $\mathbf{Q}_j^i(k_t^{(j,i)})$ is the most recent information of agent j that is already delivered to agent i at t, *i.e.*,

$$k_t^{(j,i)} := \max\{k \in \mathbb{Z}_+ :\ kT + \tau_k^{(j,i)} \leq t\}, \tag{7}$$

and define

$$\mathbf{q}_j^{(i)}(t) := \mathbf{q}_j(k_t^{(j,i)}T) + \epsilon_t^{(j,i)}, \tag{8}$$

where $\epsilon_t^{(j,i)}$ can be seen as a corrective term for the received position of the j-th agent, which has changed since its last information has been sent (and successfully received). This term is selected as

$$\epsilon_t^{(j,i)} = \hat{\mathbf{v}}_j(k_t^{(j,i)}) \cdot (t - k_t^{(j,i)}T), \tag{9}$$

where $\hat{\mathbf{v}}_j(k_t^{(j,i)})$ is maintained constant in the interval $[k_t^{(j,i)}T, t)$ (until the next most recent information is received). Again, $k_t^{(j,i)}$ can be obtained by a simple comparison of the received time stamps.

Consider the following control algorithm for each system

$$\begin{aligned} \mathbf{u}_i &= \mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \boldsymbol{\eta}_i + \mathbf{v}_i^r, \dot{\boldsymbol{\eta}}_i + \dot{\mathbf{v}}_i^r)\hat{\boldsymbol{\theta}}_i - k_i^s \mathbf{s}_i, \\ \dot{\hat{\boldsymbol{\theta}}}_i &= -\Pi_i \mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \boldsymbol{\eta}_i + \mathbf{v}_i^r, \dot{\boldsymbol{\eta}}_i + \dot{\mathbf{v}}_i^r)^\top \mathbf{s}_i, \end{aligned} \tag{10}$$

where $\hat{\boldsymbol{\theta}}_i \in \mathbb{R}^k$ can take arbitrary initial values, the matrix Π_i is symmetric positive definite and $k_i^s > 0$ is a scalar gain. The variable $\mathbf{s}_i$ is defined as

$$\mathbf{s}_i = \dot{\mathbf{q}}_i - \mathbf{v}_i^r - \boldsymbol{\eta}_i, \tag{11}$$

with $\boldsymbol{\eta}_i$ satisfying

$$\begin{cases} \dot{\boldsymbol{\eta}}_i = -k_i^\eta \boldsymbol{\eta}_i - \lambda_i\left(\kappa_i \mathbf{q}_i - \boldsymbol{\psi}_i\right) \\ \dot{\boldsymbol{\psi}}_i = -\boldsymbol{\psi}_i + \kappa_i \mathbf{v}_i^r + \sum_{j=1}^n a_{ij}\mathbf{q}_j^{(i)}(t) \end{cases}, \tag{12}$$

for $i \in \mathcal{N}$, where $\boldsymbol{\eta}_i(0)$, $\boldsymbol{\psi}_i(0)$ can be selected arbitrarily, k_i^η, λ_i are strictly positive scalar gains, and $\kappa_i := (\sum_{j=1}^n a_{ij})$. The coefficients $a_{ij} \geq 0$ are defined such that the matrix $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{n\times n}$ is the weighted adjacency matrix of the weighted directed graph, denoted $\mathcal{G}_w = (\mathcal{N}, \mathcal{E}, \mathcal{A})$, having the same vertex and edges sets as $\mathcal{G}$. Our result is given in the following theorem and is proved in Section 4.2.

Theorem 1. Consider the network of n-systems described by (1), where the interconnection between the systems is described by the directed communication graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, and suppose Assumption 1 holds. Consider the control algorithm (10)-(12) with (6)-(9) and the discrete-time observer (3)-(5). If the directed communication graph $\mathcal{G}$ contains a spanning tree with a root $r \in \mathcal{L} \neq \emptyset$, then $\mathbf{v}_i^r(t) \to \dot{\mathbf{q}}_d$ as $t \to +\infty$ for $i \in \mathcal{F}$. In addition, if

$$\frac{\mu_i}{\kappa_i} > 1 + 2 \cdot h^*, \quad \text{for } i \in \bar{\mathcal{N}}, \tag{13}$$

where $\bar{\mathcal{N}} = \{i : i \in \mathcal{N} \text{ and } \kappa_i \neq 0\}$ and $\mu_i := -\max(\mathcal{R}e(\mu_{i,1}), \mathcal{R}e(\mu_{i,2}))$, $\mu_{i,1}$, $\mu_{i,2}$ are the roots of $p^2 + k_i^\eta p + \lambda_i \kappa_i = 0$, then $\dot{\mathbf{q}}_i(t) \to \dot{\mathbf{q}}_d$ and $(\mathbf{q}_i(t) - \mathbf{q}_j(t)) \to 0$ for all $i, j \in \mathcal{N}$ as $t \to +\infty$.
Furthermore, if $\mathcal{L} \equiv \emptyset$ and $\mathcal{G}$ contains a spanning tree, then $(\dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_j(t)) \to 0$ and $(\mathbf{q}_i(t) - \mathbf{q}_j(t)) \to 0$ for all $i, j \in \mathcal{N}$ as $t \to +\infty$. □

Theorem 1 gives a solution to the synchronization problem of the class of nonlinear systems (1) with relaxed communication requirements. In fact, each agent needs to send its information to its prescribed neighbors only at some instants of time. This information transfer is also subject to constraints inherent to the communication channels such as irregular communication delays and packet loss. An important feature of the above result is that it gives sufficient conditions for synchronization, given in (13), that can be easily satisfied with an appropriate choice of the control gains. Notice that the constant $h^* := (k^*T + h)$ can be easily estimated in practice, and is simply defined as the maximum blackout interval of time an individual agent does not receive information from each one of its neighbors. Then, the control gains, namely k_i^η, λ_i and κ_i, can be freely selected to satisfy (13), which is imposed for all agents that receive information. In particular, we can show that $\frac{\mu_i}{\kappa_i}$ can be made arbitrarily large with some choice of these gains. On the other hand, condition (13) is equivalent to $0 < h^* < \frac{\mu_i}{2 \cdot \kappa_i} - \frac{1}{2}$, which specifies the maximal allowable time interval during which each agent can run its control algorithm without receiving information from its neighbors. Then, this allowable interval can be made arbitrarily large.

4. PROOF OF MAIN RESULT

4.1 Definitions and Preliminary result

Before we present proof of our main result, we give a preliminary result that will be used in the subsequent analysis. Consider an affine nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)u_1 + \ldots + g_p(x)u_p, \\ y_1 &= h_1(x), \\ &\vdots \\ y_q &= h_q(x), \end{aligned} \tag{14}$$

where $x \in \mathbb{R}^N$, $u_i \in \mathbb{R}^{\tilde{m}_i}$ for $i \in \mathcal{N}_p := \{1, \ldots, p\}$, $y_j \in \mathbb{R}^{\bar{m}_j}$ for $j \in \mathcal{N}_q := \{1, \ldots, q\}$, and $f(\cdot)$, $g_i(\cdot)$, for $i \in \mathcal{N}_p$, and $h_j(\cdot)$, for $j \in \mathcal{N}_q$, are locally Lipschitz functions of the corresponding dimensions, $f(0) = 0$, $h(0) = 0$. We assume that for any initial condition $x(t_0)$ and any inputs $u_1(t), \ldots, u_p(t)$ that are uniformly essentially bounded on $[t_0, t_1)$, the corresponding solution $x(t)$ is well defined for all $t \in [t_0, t_1]$.

Theorem 2. Consider a system of the form (14). Suppose the system is weakly input-to-output stable (WIOS) [2] with linear IOS gains $\gamma_{ij}^0 \geq 0$. Suppose also that each input $u_j(\cdot)$, $j \in \mathcal{N}_p$, is a Lebesgue measurable function satisfying

$$u_j(t) \equiv 0 \quad \text{for } t < 0, \tag{15}$$

and

$$|u_j(t)| \leq \sum_{i \in \mathcal{N}_q} \mu_{ji} \cdot \sup_{s \in [t - \vartheta_{ji}(t), t]} |y_i(s)| + |\delta_j(t)|, \tag{16}$$

for almost all $t \geq 0$, where $\mu_{ji} \geq 0$, all $\vartheta_{ji}(t)$ are Lebesgue measurable uniformly bounded nonnegative functions of time, and $\delta_j(t)$ is an uniformly essentially bounded signal that satisfy $|\delta_j(t)| \to 0$ at $t \to +\infty$. Let $\Gamma := \Gamma^0 \cdot \mathcal{M} \in \mathbb{R}^{q \times q}$, where $\Gamma^0 := \{\gamma_{ij}^0\}$, $\mathcal{M} := \{\mu_{ji}\}$, $i \in \mathcal{N}_q$, $j \in \mathcal{N}_p$. If $\rho(\Gamma) < 1$, where $\rho(\Gamma)$ is the spectral radius of the matrix Γ, then the trajectories of the system (14) with input-output constraints (15), (16) are well defined for all $t \geq 0$ and such that all the outputs $y_i(t)$, $i \in \mathcal{N}_q$, and all the inputs $u_j(\cdot)$, $j \in \mathcal{N}_p$, are uniformly bounded and satisfy $|y_i(t)| \to 0$, $|u_j(t)| \to 0$ as $t \to +\infty$. □

Theorem 2 is a version of [Theorem 1 Abdessameud et al., 2014]; in particular, the proof follows the same lines and, therefore, is omitted. Both Theorem 2 above and [Theorem 1 Abdessameud et al., 2014] are special cases of a more general result given in Polushin et al. [2013].

4.2 Proof of Theorem 1

Consider first the consensus algorithm (3)-(5). The interaction between the agents in the system (3)-(5) is described by a directed graph $\mathcal{G}_s = (\mathcal{N}, \mathcal{E}_s)$, which can be formally obtained from the directed graph $\mathcal{G}$ by modifying some of its links, as follows: (1) removing the incoming arcs to each leader node (or agent), (2) adding a directed link from any leader node to any other leader node, and (3) adding a self arc to each node in the graph. It can be verified that, if the $\mathcal{G}$ is rooted at $r \in \mathcal{L}$, then $\mathcal{G}_s$ is also rooted at $r \in \mathcal{L}$. In the case of no leaders ($\mathcal{L} = \emptyset$), the above modifications reduce to adding a self arc to each node; in this case, $\mathcal{G}_s$ is rooted if $\mathcal{G}$ is rooted. Now, in view of the above definition, the consensus algorithm (3)-(5) can be formally written as

$$\hat{\mathbf{v}}_i(\sigma + 1) = \frac{1}{|\bar{N}_i(\sigma)|} \sum_{j \in \bar{N}_i(\sigma)} \hat{\mathbf{v}}_j(\sigma - \hat{\tau}^{(j,i)}(\sigma)), \tag{17}$$

for all $i \in \mathcal{N}$, where $\bar{N}_i(\sigma) = N_i(\sigma)$, for $i \in \mathcal{F}$, and ,$\bar{N}_i(\sigma) = \mathcal{L}$, for $i \in \mathcal{L}$, and $\hat{\tau}^{(j,i)}(\sigma)$ is a delay that takes some integer value at σT and, in view of Assumption 1 and (2), satisfies $\hat{\tau}^{(j,i)}(\sigma) \leq h_\sigma^* := \lceil h^*/T \rceil$ for all $\sigma = 0, 1, \ldots$. Note that $\hat{\tau}^{(i,i)}(\sigma) = 0$ and $\hat{\tau}^{(j,i)}(\sigma) = 0$ for all σ if $i, j \in \mathcal{L}$.

Proposition 1. Under the assumptions of Theorem 1, the discrete-time consensus algorithm (17) achieves consensus in the sense that $\hat{\mathbf{v}}_i(k) \to \mathbf{v}_c$ as $k \to +\infty$ for all $i \in \mathcal{N}$ for some constant $\mathbf{v}_c \in \mathbb{R}^m$. In particular, $\mathbf{v}_c = \dot{\mathbf{q}}_d$ if $\mathcal{L} \neq \emptyset$ and $\mathcal{G}$ is rooted at $r \in \mathcal{L}$. Furthermore, $\mathbf{v}_i^r$ is uniformly bounded and $\mathbf{v}_i^r \to \mathbf{v}_c$ as $t \to +\infty$, for all $i \in \mathcal{N}$. □

The next step in the proof is to use the small-gain theorem (Theorem 2) to show that synchronization is achieved. Let

[2] A definition of input-to-output stability (IOS) and WIOS can be found, respectively, in [Sontag, 2008] and [Polushin et al., 2013, Abdessameud et al., 2014].

$\tilde{\boldsymbol{\theta}}_i = (\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i)$, $\tilde{\mathbf{q}}_i := \kappa_i \mathbf{q}_i - \boldsymbol{\psi}_i$, $\tilde{\boldsymbol{\psi}}_i := \boldsymbol{\psi}_i - \sum_{j=1}^n a_{ij}\mathbf{q}_j$, $\Delta\mathbf{q}_i := \sum_{j=1}^n a_{ij}(\mathbf{q}_j - \mathbf{q}_j^{(i)})$, and $\dot{\hat{\mathbf{q}}}_i := \sum_{j=1}^n a_{ij}\left(\dot{\mathbf{q}}_j - \mathbf{v}_i^r\right)$, where $\mathbf{v}_i^r$ and $\mathbf{q}_j^{(i)}$ are defined in (6) and (8), respectively.

The closed loop dynamics of each agent $i \in \bar{\mathcal{N}}$, *i.e.*, $\kappa_i \neq 0$, can be written as follows

$$\dot{\mathbf{s}}_i = \mathbf{M}_i^{-1}(\mathbf{q}_i)\left(\mathbf{Y}_i\tilde{\boldsymbol{\theta}}_i - \mathbf{C}_i\mathbf{s}_i - k_d^s\mathbf{s}_i\right) \quad (18)$$

$$\dot{\tilde{\boldsymbol{\theta}}}_i = -\Pi_i \mathbf{Y}_i^\top \mathbf{s}_i \quad (19)$$

$$\dot{\tilde{\mathbf{q}}}_i = \kappa_i(\boldsymbol{\eta}_i + \mathbf{s}_i) + \tilde{\boldsymbol{\psi}}_i + \Delta\mathbf{q}_i \quad (20)$$

$$\dot{\boldsymbol{\eta}}_i = -k_i^\eta \boldsymbol{\eta}_i - \lambda_i \tilde{\mathbf{q}}_i \quad (21)$$

$$\dot{\tilde{\boldsymbol{\psi}}}_i = -\tilde{\boldsymbol{\psi}}_i - \Delta\mathbf{q}_i - \dot{\hat{\mathbf{q}}}_i \quad (22)$$

where the arguments of $\mathbf{Y}_i$ and $\mathbf{C}_i$ are omitted, the vectors $\mathbf{s}_i$, $\tilde{\boldsymbol{\eta}}_i$, $\tilde{\mathbf{q}}_i$, $\boldsymbol{\eta}_i$, and $\tilde{\boldsymbol{\psi}}_i$ are the states, $\dot{\hat{\mathbf{q}}}_i$ and $\Delta\mathbf{q}_i$ are the inputs and the output is given by: $\boldsymbol{\xi}_i := \mathbf{s}_i + \boldsymbol{\eta}_i$.

Since it is assumed that $\mathcal{G}$ is directed and contains a spanning tree, there is at most one agent with no incoming links. In this case, the closed loop dynamics of the i-th agent, with $i \in \mathcal{N} \setminus \bar{\mathcal{N}}$, can be obtained as

$$\dot{\mathbf{s}}_i = \mathbf{M}_i^{-1}(\mathbf{q}_i)\left(\mathbf{Y}_i\tilde{\boldsymbol{\theta}}_i - \mathbf{C}_i\mathbf{s}_i - k_d^s\mathbf{s}_i\right), \quad \dot{\tilde{\boldsymbol{\theta}}}_i = -\Pi_i \mathbf{Y}_i^\top \mathbf{s}_i$$

$$\dot{\boldsymbol{\eta}}_i = -k_i^\eta \boldsymbol{\eta}_i + \lambda_i \boldsymbol{\psi}_i, \quad \dot{\boldsymbol{\psi}}_i = -\boldsymbol{\psi}_i$$

which is similar to (18)-(22) with the output $\boldsymbol{\xi}_i$ and $\kappa_i = 0$, $\tilde{\mathbf{q}}_i = -\tilde{\boldsymbol{\psi}}_i = -\boldsymbol{\psi}_i$, and $\dot{\hat{\mathbf{q}}}_i = \Delta\mathbf{q}_i = 0$.

Proposition 2. Consider the overall closed loop system that consists of the subsystems (18)-(22), for $i \in \mathcal{N}$, with n outputs, given by $y_l := \boldsymbol{\xi}_l$ for $l \in \mathcal{N}$, and $2n$ inputs ordered as: $u_{2l-1} := \dot{\hat{\mathbf{q}}}_l$, $u_{2l} := \Delta\mathbf{q}_l$ for $l \in \mathcal{N}$, where the signals $\mathbf{v}_i^r$ are obtained from (3)-(6). Then, the overall system is WIOS, and the closed-loop gain matrix $\Gamma := \Gamma^0 \cdot \mathcal{M} = \{\bar{\gamma}_{ij}\}$ in Theorem 2, with $\Gamma^0 \in \mathbb{R}^{n\times 2n}$, $\mathcal{M} \in \mathbb{R}^{2n\times n}$, is obtained as

$$\bar{\gamma}_{ij} = \begin{cases} \dfrac{a_{ij}}{\mu_i}\left(1 + 2\cdot h^*\right), & \text{if } \kappa_i \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

where μ_i and h^* are defined in Theorem 1. In addition, all the inputs u_j, $j \in \{1,\ldots,2n\}$, satisfy (16) for some $\delta_j(t)$, $j \in \{1,\ldots,2n\}$, that are uniformly bounded and satisfy $|\delta_j(t)| \to 0$ as $t \to +\infty$. $\Box$

Using the result of Proposition 2, one can show that $\rho(\Gamma) < 1$ if $\sum_{j=1}^n \bar{\gamma}_{ij} < 1$, for $i \in \mathcal{N}$, which is satisfied by (13). Therefore, all the conditions of Theorem 2 are satisfied and one can conclude that $\boldsymbol{\xi}_i$, $\dot{\hat{\mathbf{q}}}_i^s$ and $\Delta\hat{\mathbf{q}}_i$ are uniformly bounded and $\boldsymbol{\xi}_i(t) \to 0$, $\dot{\hat{\mathbf{q}}}_i^s(t) \to 0$, $\Delta\hat{\mathbf{q}}_i(t) \to 0$ as $t \to +\infty$. Since $\boldsymbol{\xi}_i = (\boldsymbol{s}_i + \boldsymbol{\eta}_i) = (\dot{\mathbf{q}}_i - \mathbf{v}_i^r) \to 0$ as $t \to +\infty$, the result of Proposition 1 leads one to conclude that $\dot{\mathbf{q}}_i(t) \to \dot{\mathbf{q}}_d$ as $t \to +\infty$ if $\mathcal{L} \neq \emptyset$, and $(\dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_j(t)) \to 0$ as $t \to +\infty$ if $\mathcal{L} = \emptyset$.

In addition, it is easy to verify, from Proposition 2 and the properties of systems (18)-(22) for $i \in \mathcal{N}$, that $\mathbf{s}_i$, $\tilde{\boldsymbol{\theta}}_i$, $\tilde{\mathbf{q}}_i$, $\boldsymbol{\eta}_i$ and $\tilde{\boldsymbol{\psi}}_i$ are uniformly bounded and $\boldsymbol{s}_i(t) \to 0$, $\tilde{\mathbf{q}}_i(t) \to 0$, $\boldsymbol{\eta}_i(t) \to 0$ and $\tilde{\boldsymbol{\psi}}_i(t) \to 0$ as $t \to +\infty$. This implies that $(\kappa_i\mathbf{q}_i - \sum_{j=1}^n a_{ij}\mathbf{q}_j)$ is uniformly bounded and $\sum_{j=1}^n a_{ij}(\mathbf{q}_i(t) - \mathbf{q}_j(t)) \to 0$ as $t \to +\infty$ for all $i \in \mathcal{N}$, which is equivalent to $(\mathbf{L}\otimes\mathbf{I}_m)\mathbf{Q} \to 0$, where $\mathbf{L}$ is the Laplacian matrix of the communication graph

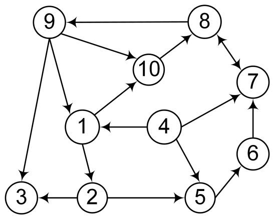

Fig. 1. Communication graph $\mathcal{G}$.

$\mathcal{G}_w$, $\mathbf{I}_m$ is the $m \times m$ identity matrix, $\mathbf{Q} \in \mathbb{R}^{nm}$ is the vector containing all $\mathbf{q}_i$ for $i \in \mathcal{N}$, and $\otimes$ is the Kronecker product. Finally, since $(\mathbf{L}\otimes\mathbf{I}_m)\mathbf{Q} = 0$ implies that $\mathbf{q}_1 = \ldots = \mathbf{q}_n$ if the directed communication graph contains a spanning tree [Ren and Beard, 2005], we conclude that $(\mathbf{q}_i - \mathbf{q}_j) \to 0$ as $t \to +\infty$, for all $i, j \in \mathcal{N}$. The proof is complete.

5. SIMULATION RESULTS

In this section, we apply the proposed synchronization scheme to a network of ten planar two-degrees of freedom rigid manipulator arms (with revolute joints). The systems models are described in Abdessameud et al. [2014]. The interconnection between the systems in the network is described by the directed graph $\mathcal{G}$, given in Fig. 1, and the communication process is described in Section 2.3 with $T = 0.1$ sec and the parameter h^* in (2) being estimated to be smaller than or equal to 1.3 sec.

We implement the control scheme in Theorem 1 with $\mathcal{L} = \{1, 4\}$; the systems labeled 1 and 4 are the only systems having access to the desired velocity $\dot{\mathbf{q}}_d = (0.3, 0.6)^\top$ rad/sec. The observer (3)-(5) is updated at T, and the control gains are selected as: $k_i^p = k_i^d = 2$, $\Pi_i = 0.3\mathbf{I}_5$, $k_i^s = 10$, $\lambda_i = 8$, $k_i^\eta = 2\sqrt{\lambda_i \cdot \kappa_i}$, and all the weights on the communication links of $\mathcal{G}_w$, which is the same as $\mathcal{G}$, are set such that $\kappa_i = 1$. Note that this choice of the gains satisfies condition (13).

Fig. 2 and Fig. 3 illustrate the relative positions and relative velocities defined as $\mathbf{q}_{1i} := (\mathbf{q}_1 - \mathbf{q}_i)$, for $i \in \tilde{\mathcal{N}} := \mathcal{N} - \{1\}$, and $\dot{\mathbf{q}}_{1i} := (\dot{\mathbf{q}}_1 - \dot{\mathbf{q}}_i)$ for $i \in \tilde{\mathcal{N}} \cup \{d\}$, where subscript '$d$' is used for the desired velocity. It is clear that all agents synchronize their positions and velocities with the desired velocity. The output of the discrete-time system (3)-(5) is given in Fig. 4, where it can be seen that the desired velocity estimate of each agent converges to the desired velocity available to the leader agents.

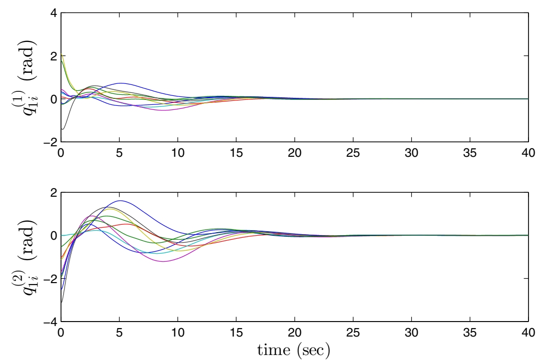

Fig. 2. Relative position vectors, $\mathbf{q}_{1i} = (q_{1i}^{(1)}, q_{1i}^{(2)})^\top$.

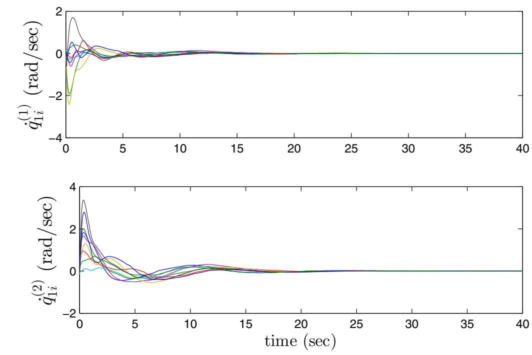

Fig. 3. Relative velocity vectors, $\dot{\mathbf{q}}_{1i} = (\dot{q}_{1i}^{(1)}, \dot{q}_{1i}^{(2)})^\top$.

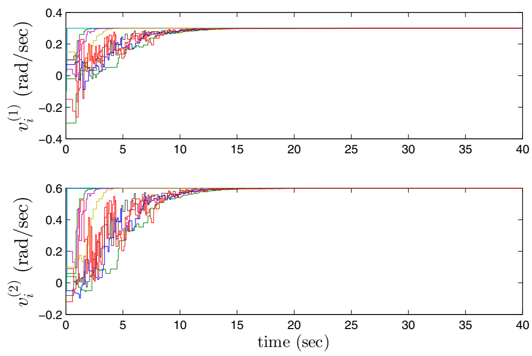

Fig. 4. Desired velocity estimates, $\hat{\mathbf{v}}_i = (\hat{v}_i^{(1)}, \hat{v}_i^{(2)})^\top$.

6. CONCLUSION

We addressed the synchronization problem of uncertain Euler-Lagrange systems interconnected under directed graphs in the presence of communication constraints. Using the IOS small-gain framework, we proposed a distributed control algorithm that achieves position synchronization, and all agents velocities match a desired velocity available to only some leaders. In contrast to the available relevant literature, the proposed control scheme allows agents to exchange information at irregular discrete time-intervals with irregular time-delays and possible packet loss. In fact, we prove that synchronization is still achieved even if each agent in the team runs its control algorithm without receiving any information from its neighbors during some allowable interval of time. The conditions for synchronization derived in this paper can be satisfied by an appropriate choice of the control gains. Future research will consider the extension of this work to the case of variable desired velocity.

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