2012 American Control Conference
Fairmont Queen Elizabeth, Montréal, Canada
June 27-June 29, 2012

Rigid Body Attitude Synchronization With Communication Delays

Abdelkader Abdessameud, Abdelhamid Tayebi and Ilia G. Polushin

Abstract— **The paper addresses the cooperative attitude synchronization problem of multiple rigid bodies in the presence of communication delays and without angular velocity measurements. First, we present a solution to the leaderless and leader-follower problems in the case of time-varying communication delays and undirected communication topology. Next, we present an attitude synchronization scheme for the leaderless problem, under a directed graph topology, in the presence of constant communication delays. To demonstrate the effectiveness of the proposed control schemes, simulation results of a scenario of four rigid bodies are provided.**

I. Introduction

We consider the cooperative attitude synchronization problem of multiple rigid bodies. The objective is to drive a team of rigid bodies to synchronize their orientations (attitudes) to the same final orientation. This problem has recently gained an increased interest in applications related to spacecraft formations. Several attitude synchronization schemes have been reported in the literature, see for example [1]-[4] in the full state information case, and [5]-[7], in the case where the angular velocities are not available for feedback. In these papers, the interconnection topology plays a central role, however, communication delays that are inherently present in transmission systems have not been considered.

The effects of communication delays in linear multi-agent systems, described by second-order dynamics, have been extensively studied in [8]-[10] to cite a few, and sufficient stability conditions have been derived. The communication delays in nonlinear systems have also been considered in bilateral teleoperation [11]-[12], the synchronization of Euler-Lagrange systems [13]-[14] and the formation control of a class of unmanned aerial vehicles [15]. However, due to the complexity of the attitude dynamics, it is not straightforward to extend the results of the above papers to the attitude synchronization problem. This is the reason behind the existence of only few papers dealing with this problem in the available literature.

Using the Modified Rodriguez parameters (MRP) and the Lagrangian formulation for the attitude dynamics, the authors in [16] proposed a solution to the spacecraft attitude synchronization problem in the presence of constant communication delays. To avoid the inherent singularity of the MRP representation, the globally non-singular unit-quaternion representation has been considered in [17]-[18] and attitude synchronization schemes have been proposed in the presence of time-varying communication delays. The proposed control schemes in the above papers rely on some synchronization variables defined in terms of both attitude and angular velocity tracking errors. More recently, a different analysis method has been considered in [19], where the relative attitudes are defined using linear differences between individual attitudes given in terms of unit-quaternion. It is worth mentioning that in [16]-[19], only the cooperative attitude tracking problem has been considered, where a common desired attitude trajectory is required to be available to each spacecraft in the team. Moreover, the communication topology between spacecraft in the team is assumed to be undirected. In [20], the attitude kinematics of rigid bodies have been considered to design appropriate angular velocity inputs to solve the leaderless attitude synchronization problem with delayed communication and directed communication topology. However, the attitude dynamics have not been considered and the input torque that drives this type of systems has not been designed. In addition, all the aforementioned papers rely on the assumption that the angular velocities are available for feedback.

The main contribution of this paper is to propose new quaternion-based attitude synchronization schemes for a group of rigid bodies (or spacecraft) without angular velocity measurements and in the presence of communication delays. As mentioned earlier, some solutions to this problem exist in the case of no communication delays. However, it is generally difficult to study the effects of communication delays in the partial state feedback case, using Lyapunov-Krasovskii functionals for example. To overcome this difficulty, we propose an approach that handles, simultaneously, the communication delays and the lack of angular velocity measurements. Using this approach, we present first a unified scheme that solves the leaderless and leader follower attitude synchronization problems in the presence of time-varying communication delays. We derive sufficient conditions on the communication delays and the controller gains such that the control objectives are attained under a fixed and undirected communication topology. Then, we present a solution to the leaderless problem in the case of constant communication delays and directed communication topology. To the best of our knowledge, the above problems in the presence of communication delays have never been addressed in the partial state feedback case. In addition, only the leaderless attitude synchronization problem with delayed communication has been partly solved in the full state feedback case.

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

The authors are with the Department of Electrical and Computer Engineering, University of Western Ontario, London, Ontario, Canada. The second author is also with the Department of Electrical Engineering, Lakehead University, Thunder Bay, Ontario, Canada. aabdessa@uwo.ca, tayebi@ieee.org, ipolushi@uwo.ca

II. System model and notations

Consider a group of n-rigid bodies, where the equations of motion of the i^{th} rigid body are given by

$$\dot{\mathbf{Q}}_i = \frac{1}{2}\mathbf{T}(\mathbf{Q}_i)\boldsymbol{\omega}_i, \quad (1)$$

$$\mathbf{I}_{f_i}\dot{\boldsymbol{\omega}}_i = \boldsymbol{\Gamma}_i - \mathbf{S}(\boldsymbol{\omega}_i)\mathbf{I}_{f_i}\boldsymbol{\omega}_i, \quad (2)$$

for $i \in \mathcal{N} \triangleq \{1,...,n\}$, where $\boldsymbol{\omega}_i \in \mathbb{R}^3$ is the angular velocity of the i^{th} rigid body expressed in the body-fixed frame, $\mathcal{F}_i$, $\mathbf{I}_{f_i} \in \mathbb{R}^{3\times3}$ is a constant symmetric positive definite inertia matrix of the i^{th} rigid body with respect to $\mathcal{F}_i$, and the vector $\boldsymbol{\Gamma}_i$ is the external torque input expressed in $\mathcal{F}_i$. The unit-quaternion $\mathbf{Q}_i = (\mathbf{q}_i^\top, \eta_i)^\top$ is composed of a vector part $\mathbf{q}_i \in \mathbb{R}^3$ and a scalar part $\eta_i \in \mathbb{R}$, and represents the orientation of the i^{th} rigid body. The elements of the unit-quaternion $\mathbf{Q}_i$ satisfy the unity constraint: $\eta_i^2 + \mathbf{q}_i^\top\mathbf{q}_i = 1$. The matrix $\mathbf{T}(\mathbf{Q}_i) \in \mathbb{R}^{4\times3}$ satisfies: $\mathbf{T}(\mathbf{Q}_i)^\top\mathbf{T}(\mathbf{Q}_i) = \mathbf{I}_3$, and is given by

$$\mathbf{T}(\mathbf{Q}_i) = \begin{pmatrix} \eta_i \ \mathbf{I}_3 + \mathbf{S}(\mathbf{q}_i) \\ -\mathbf{q}_i^\top \end{pmatrix}, \quad (3)$$

where the matrix $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $\mathbf{S}(\mathbf{x}_1)\mathbf{x}_2 = \mathbf{x}_1 \times \mathbf{x}_2$ for any vectors $\mathbf{x}_1 \in \mathbb{R}^3$ and $\mathbf{x}_2 \in \mathbb{R}^3$, where '$\times$' denotes the vector cross product. The orthogonal rotation matrix $\mathbf{R}(\mathbf{Q}_i) \in SO(3)$ related to the unit-quaternion $\mathbf{Q}_i$, that brings the inertial frame into the i^{th} body frame, can be obtained through the Rodriguez formula as: $\mathbf{R}(\mathbf{Q}_i) = (\eta_i^2 - \mathbf{q}_i^\top\mathbf{q}_i)\mathbf{I}_3 + 2\mathbf{q}_i\mathbf{q}_i^\top - 2\eta_i\mathbf{S}(\mathbf{q}_i)$, where $\mathbf{I}_3$ is the 3-by-3 identity matrix. The time-derivative of the rotation matrix $\mathbf{R}(\mathbf{Q}_i)$ is given as: $\dot{\mathbf{R}}(\mathbf{Q}_i) = -\mathbf{S}(\boldsymbol{\omega}_i)\mathbf{R}(\mathbf{Q}_i)$.

The multiplication between two unit-quaternion, $\mathbf{Q}_i = (\mathbf{q}_i^\top, \eta_i)^\top$ and $\mathbf{Q}_j = (\mathbf{q}_j^\top, \eta_j)^\top$, is defined by the following operation: $\mathbf{Q}_i \odot \mathbf{Q}_j = \left((\eta_i\mathbf{q}_j + \eta_j\mathbf{q}_i + S(\mathbf{q}_i)\mathbf{q}_j)^\top \ , \ \eta_i\eta_j - \mathbf{q}_i^\top\mathbf{q}_j\right)^\top$. The inverse or conjugate of the unit-quaternion $\mathbf{Q}_i$ is defined by, $\mathbf{Q}_i^{-1} = (-\mathbf{q}_i^\top, \eta_i)^\top$, with the quaternion identity given by $\mathbf{Q}_I := (\mathbf{0}_3^\top, 1)^\top$, where $\mathbf{0}_m \in \mathbb{R}^m$ is the vector of zero elements. Note that due to the redundancy in the unit-quaternion representation, $\pm\mathbf{Q}_I$ represents the same physical orientation. For more properties of the unit-quaternion representation of the attitude, the reader is referred to [21].

Notations: For the sake of the presentation clarity, we omit throughout the paper the arguments of time-dependent signals, (*e.g.,* $\mathbf{Q}_i \leftrightarrow \mathbf{Q}_i(t)$), except for those which are time-delayed (*e.g.,* $\mathbf{Q}_i(t-\tau_{ij})$). In addition, the argument of the signals inside the integrals is omitted, which is assumed to be equal to the variable on the differential, unless otherwise stated (*e.g.* $\int_0^t \dot{\mathbf{Q}}_i ds \leftrightarrow \int_0^t \dot{\mathbf{Q}}_i(s)ds$). Also, the limit of a signal at infinity is replaced by an arrow (*e.g.,* $\mathbf{Q}_i \to c \leftrightarrow \lim_{t\to\infty}\mathbf{Q}_i(t) = c$, for a constant c, and $\mathbf{Q}_i \to \mathbf{Q}_j \leftrightarrow \lim_{t\to\infty}\mathbf{Q}_i(t) = \lim_{t\to\infty}\mathbf{Q}_j(t)$).

III. Problem statement

To achieve attitude synchronization, rigid bodies in the team must exchange some of their states information. We assume that the information flow between members of the team is fixed and is represented by a weighted graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{K})$, where $\mathcal{N}$ is the set of nodes or vertices, describing the set of vehicles in the team, $\mathcal{E} \subseteq \mathcal{N}\times\mathcal{N}$ is the set of pairs of nodes, called edges, and $\mathcal{K} = [k_{ij}]$ is a weighted adjacency matrix. An edge $(i,j) \in \mathcal{E}$ indicates that the i^{th} rigid body receives information from the j^{th} rigid body, which is designated as its neighbor. The weighted adjacency matrix of a weighted graph is defined such that $k_{ij} > 0$ if and only if $(i,j) \in \mathcal{E}$ and $k_{ij} = 0$ if and only if $(i,j) \notin \mathcal{E}$. If the interconnection between rigid bodies is bidirectional, then $\mathcal{G}$ is undirected, the pairs of nodes in $\mathcal{E}$ are unordered, $(i,j) \in \mathcal{E} \Leftrightarrow (j,i) \in \mathcal{E}$, and $\mathcal{K}$ is symmetric, *i.e.,* $k_{ij} = k_{ji}$. In the case of unidirectional communication topology, $\mathcal{G}$ is a directed graph, $\mathcal{E}$ contains ordered pairs, and $\mathcal{K}$ is not necessarily symmetric. An undirected graph is said to be connected if there is an undirected path between any two distinct nodes of the graph. Similarly, a directed graph is said to be strongly connected if there exists a directed path between any two distinct nodes [22]. We also assume that each rigid body can sense its states with no delays, and the communication between the i^{th} and j^{th} rigid bodies, with $(i,j) \in \mathcal{E}$, is delayed by τ_{ij}, which is non-uniform, *i.e.,* τ_{ij} is not necessarily equal to τ_{ji}.

With the above assumptions, our objective is to design control laws for each rigid body, without angular velocity measurements and in the presence of communication delays, such that the following problems are solved:

- Leaderless problem (**LSP**). When no desired attitude is assigned to the team, all rigid bodies are required to synchronize their attitudes to the same attitude, such that $\boldsymbol{\omega}_i \to \mathbf{0}_3$, and $\mathbf{Q}_i \to \mathbf{Q}_j$, for all $i,j \in \mathcal{N}$.
- Leader-follower problem (**LFP**). Given a constant desired attitude, represented by the unit-quaternion $\mathbf{Q}_d := (\mathbf{q}_d^\top, \eta_d)^\top$, available to a single rigid body in the team acting as a leader. All rigid bodies are required to synchronize their attitudes to the desired attitude, *i.e.,* $\boldsymbol{\omega}_i \to \mathbf{0}_3$ and $\mathbf{Q}_i \to \mathbf{Q}_d$, for all $i \in \mathcal{N}$.

IV. Attitude synchronization without angular velocity measurements

In this section, we first present an approach to the attitude synchronization problem that removes the requirement of angular velocity measurements in the presence of communication delays. Thereafter, we propose two attitude synchronization schemes with different assumptions on the communication delays and interconnection topologies.

A. Intermediate reference trajectory

Let us associate to each rigid body the following dynamic system:

$$\dot{\mathbf{Q}}_{r_i} = \frac{1}{2}\mathbf{T}(\mathbf{Q}_{r_i})\boldsymbol{\omega}_{r_i}, \quad (4)$$

for $i \in \mathcal{N}$, where $\mathbf{Q}_{r_i} = (\mathbf{q}_{r_i}^\top, \eta_{r_i})^\top$ is the unit-quaternion representing the attitude of system (4), with $\mathbf{Q}_{r_i}(0)$ can be initialized arbitrarily, and $\boldsymbol{\omega}_{r_i}$ is the angular velocity input to be designed. The matrix $\mathbf{T}(\mathbf{Q}_{r_i})$ can be obtained similar to (3) as:

$$\mathbf{T}(\mathbf{Q}_{r_i}) = \begin{pmatrix} \eta_{r_i} \ \mathbf{I}_3 + \mathbf{S}(\mathbf{q}_{r_i}) \\ -\mathbf{q}_{r_i}^\top \end{pmatrix}.$$

The dynamic system (4) is introduced to generate an intermediate reference trajectory for each rigid body, which is given in terms of the time-varying reference attitude, $\mathbf{Q}_{r_i}$, with $\boldsymbol{\omega}_{r_i}$ and its time-derivative, $\dot{\boldsymbol{\omega}}_{r_i}$ being the reference angular velocity and acceleration respectively. In view of this definition, we let the discrepancy between the attitude of the i^{th} rigid body and its corresponding intermediate reference attitude be represented by the unit-quaternion $\mathbf{Q}_i^e := (\mathbf{q}_i^{e\top}, \eta_i^e)^\top$, and is defined as

$$\mathbf{Q}_i^e = \mathbf{Q}_{r_i}^{-1} \odot \mathbf{Q}_i, \tag{5}$$

and satisfies the unit-quaternion dynamics

$$\dot{\mathbf{Q}}_i^e = \frac{1}{2}\mathbf{T}(\mathbf{Q}_i^e)\boldsymbol{\omega}_i^e, \tag{6}$$

$$\boldsymbol{\omega}_i^e = \boldsymbol{\omega}_i - \mathbf{R}(\mathbf{Q}_i^e)\boldsymbol{\omega}_{r_i}, \tag{7}$$

where $\mathbf{T}(\mathbf{Q}_i^e)$ can be obtained similar to (3), and $\mathbf{R}(\mathbf{Q}_i^e)$ is the rotational matrix related to $\mathbf{Q}_i^e$ and is given as $\mathbf{R}(\mathbf{Q}_i^e) = \mathbf{R}(\mathbf{Q}_i)\mathbf{R}(\mathbf{Q}_{r_i})^\top$, [21].

The main idea in this approach is to design the input torque of each rigid body, $\boldsymbol{\Gamma}_i$, without angular velocity measurements such that each rigid body tracks asymptotically its corresponding intermediate reference trajectory. Then, attitude synchronization will be achieved if one determines an appropriate input of the dynamic system (4) such that the intermediate reference attitudes of all rigid bodies converge to the same final attitude in the presence of communication delays.

B. Case of time-varying communication delays and undirected graph topology

Consider the case where the interconnection graph between neighboring rigid bodies is bidirectional, represented by the undirected graph $\mathcal{G}$, and is subject to time-varying communication delays. To achieve attitude synchronization based on the above approach, communicating rigid bodies need to transmit their intermediate reference attitudes. Since the communication between rigid bodies is delayed, we let the error between the intermediate reference attitudes of the i^{th} and j^{th} rigid bodies be represented by the unit-quaternion $\bar{\mathbf{Q}}_{r_{ij}} := (\bar{\mathbf{q}}_{r_{ij}}^\top, \bar{\eta}_{r_{ij}})^\top$, defined as

$$\bar{\mathbf{Q}}_{r_{ij}} = \mathbf{Q}_{r_j}^{-1}(t - \tau_{ij}) \odot \mathbf{Q}_{r_i}. \tag{8}$$

In addition, in the case where the constant desired attitude, represented by $\mathbf{Q}_d$, is available to a rigid body in the team, case of the **LFP**, the error between the desired attitude and the intermediate reference attitude of the leader rigid body is represented by the unit-quaternion $\tilde{\mathbf{Q}}_{r_l} := (\tilde{\mathbf{q}}_{r_l}^\top, \tilde{\eta}_{r_l})^\top$, defined as:

$$\tilde{\mathbf{Q}}_{r_l} = \mathbf{Q}_d^{-1} \odot \mathbf{Q}_{r_l}, \tag{9}$$

and satisfies the unit-quaternion dynamics

$$\dot{\tilde{\mathbf{Q}}}_{r_l} = \frac{1}{2}\mathbf{T}(\tilde{\mathbf{Q}}_{r_l})\boldsymbol{\omega}_{r_l}, \tag{10}$$

with $\mathbf{T}(\tilde{\mathbf{Q}}_{r_l})$ being defined similar to (3), and the subscript "l" is used to designate the leader.

With these definitions, we propose the following input torque in (2)

$$\begin{aligned}\boldsymbol{\Gamma}_i = \mathbf{I}_{f_i}\mathbf{R}(\mathbf{Q}_i^e)\dot{\boldsymbol{\omega}}_{r_i} + \mathbf{S}(\mathbf{R}(\mathbf{Q}_i^e)\boldsymbol{\omega}_{r_i})\mathbf{I}_{f_i}\mathbf{R}(\mathbf{Q}_i^e)\boldsymbol{\omega}_{r_i} \\ - k_i^p\mathbf{q}_i^e - k_i^d\tilde{\mathbf{q}}_i^e,\end{aligned} \tag{11}$$

with

$$\dot{\boldsymbol{\omega}}_{r_i} = -k_i^\omega\boldsymbol{\omega}_{r_i} - \alpha\bar{\mathbf{u}}_i - \sum_{j=1}^{n} k_{ij}\bar{\mathbf{q}}_{r_{ij}}, \tag{12}$$

$$\bar{\mathbf{u}}_i = \begin{cases} k_l^q\tilde{\mathbf{q}}_{r_l} & \text{for} \quad i = l \\ 0 & \text{for} \quad i \neq l, \end{cases}$$

for $i \in \mathcal{N}$, where k_i^p, k_i^d, k_i^ω and k_l^q are strictly positive scalar gains, $k_{ij} \geq 0$ is the $(i,j)^{th}$ entry of the adjacency matrix of the weighted undirected graph $\mathcal{G}$, $\mathbf{q}_i^e$ is the vector part of the unit-quaternion $\mathbf{Q}_i^e$ defined in (5), $\tilde{\mathbf{q}}_{r_l}$ is the vector part of the unit-quaternion $\tilde{\mathbf{Q}}_{r_l}$ defined in (9), with the subscript "l" being used to designate the leader, $\bar{\mathbf{q}}_{r_{ij}}$ is the vector part of the unit-quaternion $\bar{\mathbf{Q}}_{r_{ij}}$ defined in (8), and the scalar α is selected as: $\alpha = 0$ for the **LSP** and $\alpha = 1$ for the **LFP**. The reference angular velocity $\boldsymbol{\omega}_{r_i}$ is the solution of (12), and can take arbitrary initial values. The vector $\tilde{\mathbf{q}}_i^e$ in (11) is the vector part of the unit-quaternion $\tilde{\mathbf{Q}}_i^e := (\tilde{\mathbf{q}}_i^{e\top}, \tilde{\eta}_i^e)^\top$ defined as

$$\tilde{\mathbf{Q}}_i^e = \mathbf{P}_i^{-1} \odot \mathbf{Q}_i^e, \tag{13}$$

$$\dot{\mathbf{P}}_i = \frac{1}{2}\mathbf{T}(\mathbf{P}_i)\boldsymbol{\beta}_i, \tag{14}$$

where $\mathbf{P}_i$ is a unit-quaternion that can be initialized arbitrarily, $\mathbf{T}(\mathbf{P}_i)$ is given similar to (3), and $\boldsymbol{\beta}_i \in \mathbb{R}^3$ is an input to be determined. We can verify that $\tilde{\mathbf{Q}}_i^e$ satisfies the following unit-quaternion dynamics:

$$\dot{\tilde{\mathbf{Q}}}_i^e = \frac{1}{2}\mathbf{T}(\tilde{\mathbf{Q}}_i^e)\tilde{\boldsymbol{\omega}}_i^e, \tag{15}$$

$$\tilde{\boldsymbol{\omega}}_i^e = \boldsymbol{\omega}_i^e - \mathbf{R}(\tilde{\mathbf{Q}}_i^e)\boldsymbol{\beta}_i, \tag{16}$$

with $\mathbf{T}(\tilde{\mathbf{Q}}_i^e)$ being defined similar to (3).

The role of the above input torque is to drive each rigid body to track its corresponding intermediate reference trajectory, *i.e.,* $\boldsymbol{\omega}_i^e \to \mathbf{0}_3$ and $\mathbf{Q}_i^e \to \pm\mathbf{Q}_I$ for $i \in \mathcal{N}$. It should be noted that the torque input (11) is given in terms of the intermediate reference trajectory, *i.e.,* $\mathbf{Q}_{r_i}$, $\boldsymbol{\omega}_{r_i}$ and $\dot{\boldsymbol{\omega}}_{r_i}$, and the absolute attitudes of the rigid bodies, and does not depend on the angular velocity of the rigid bodies. The auxiliary system (14) is used to achieve tracking of the intermediate reference trajectory without angular velocity measurements [23], and the input of the dynamic system (4) is designed in (12) such that all rigid bodies synchronize their intermediate reference trajectories in the presence of time-varying communication delays.

Under the assumption that neighboring rigid bodies can communicate their intermediate reference attitudes, *i.e.,* $\mathbf{Q}_{r_i}$, the following result holds:

Theorem 1: Consider system (1)-(2) with the torque input law (11) with (4) and (12)-(14). Let the time-varying communication delays be bounded such that $\tau_{ij} \leq \tau$ for $(i,j) \in \mathcal{E}$, where τ is a positive constant, and let the controller gains satisfy:

$$k_i^{\omega} > \sum_{j=1}^{n} \frac{k_{ij}}{4}\left(\epsilon + \frac{\tau^2}{\epsilon}\right), \qquad (17)$$

for some $\epsilon > 0$. Let the vector $\boldsymbol{\beta}_i$ in (14) be given as: $\boldsymbol{\beta}_i = \lambda_i \tilde{\mathbf{q}}_i^e$, with λ_i a strictly positive scalar gain. If the undirected communication graph is a tree[1], then all the signals are globally bounded and the **LSP** and the **LFP** are solved by setting $\alpha = 0$, 1 respectively. Moreover, if there exists a time $t_0 > 0$ such that $\eta_{r_i}(t) > 0$ (or $\eta_{r_i}(t) < 0$) for $t \geq t_0$, then the above results hold for any connected undirected communication graph.

Proof: The proof is omitted due to space limitations and can be found in [24]. ■

Remark 1: Note that the above attitude synchronization scheme can be extended to the case where the angular velocities are available for feedback. In this case, the intermediate reference trajectories are not required and we can show that the **LSP** and the **LFP** will be solved under the same conditions reported in Theorem 1 if the following control input is implemented:

$$\boldsymbol{\Gamma}_i = -\alpha\bar{\mathbf{u}}_i - k_i^{\omega}\boldsymbol{\omega}_i - \sum_{i=1}^{n} k_{ij}\bar{\mathbf{q}}_{ij}, \qquad (18)$$

where the control gains are defined as in Theorem 1, $\bar{\mathbf{q}}_{ij}$ is the vector part of the unit-quaternion $\bar{\mathbf{Q}}_{ij} = \mathbf{Q}_j^{-1}(t-\tau_{ij}) \odot \mathbf{Q}_i$, $\bar{\mathbf{u}}_i = k_l^q \tilde{\mathbf{q}}_l$, for $i = l$, and $\bar{\mathbf{u}}_i = 0$, for $i \neq l$, with $\tilde{\mathbf{q}}_l$ being the vector part of the unit-quaternion $\tilde{\mathbf{Q}}_l = \mathbf{Q}_d^{-1} \odot \mathbf{Q}_l$.

Theorem 1 provides a sufficient condition that ensures the stability of the closed loop system. This condition relates the controller gains and the upper bound of the communication delays, which can be satisfied with the reasonable assumption that this upper bound is known. In addition, the above result restricts the undirected communication graph to be a tree. It is clear that this restriction is due to the nonlinear expression of the relative attitudes between neighboring rigid bodies, which makes the convergence analysis in this case difficult. In fact, a connected and acyclic communication graph is often considered in unit-quaternion based solutions to the **LSP** and **LFP** in the case of no communication delays, as can be seen in [1]-[2] and [6]-[7]. To relax this condition on the communication graph, we present in the next subsection a different design of the auxiliary system 4 that achieves attitude synchronization under a directed communication topology.

[1] An undirected graph is a tree if it is connected and contains no cycles, [22]

C. Case of constant communication delays and directed graph topology

We assume that the communication delays are constant and the interconnection between rigid bodies is unidirectional and is represented by the directed graph $\mathcal{G}$. To solve the **LSP** in this case, we propose the following design of the intermediate reference angular velocity in (4) and (11):

$$\boldsymbol{\omega}_{r_i} = -\sum_{j=1}^{n} k_{ij}(\mathbf{q}_{r_i} - \mathbf{q}_{r_j}(t-\tau_{ij})), \qquad (19)$$

for $i \in \mathcal{N}$, where $k_{ij} \geq 0$ is the $(i,j)^{th}$ entry of the adjacency matrix of the directed communication graph $\mathcal{G}$. It is easy to verify that

$$\dot{\boldsymbol{\omega}}_{r_i} = -\sum_{j=1}^{n} k_{ij}(\dot{\mathbf{q}}_{r_i} - \dot{\mathbf{q}}_{r_j}(t-\tau_{ij})), \qquad (20)$$

with

$$\dot{\mathbf{q}}_{r_i} = \frac{1}{2}(\eta_{r_i}\mathbf{I}_3 + \mathbf{S}(\mathbf{q}_{r_i}))\boldsymbol{\omega}_{r_i}. \qquad (21)$$

Under the assumption that neighboring rigid bodies can communicate their intermediate reference attitudes and reference angular velocities, *i.e.,* $\mathbf{Q}_{r_i}$ and $\boldsymbol{\omega}_{r_i}$, the following result holds:

Theorem 2: Consider system (1)-(2) with the control law (11) with (4), (13)-(14) and (19)-(21). Let the vector β_i in (14) be given as in Theorem 1. If the directed communication graph is strongly connected, then all the signals are globally bounded and the **LSP** is solved in the presence of arbitrary constant communication delays.

Proof: The proof is omitted due to space limitations and can be found in [24]. ■

Remark 2: It should be noted that the above control scheme can be extended in an obvious way to the case where the angular velocity vectors are available for feedback. Moreover, note that the input torque (11) with (19)-(21) consists of pure unit-quaternion terms and the inertia matrix of the rigid body. As a result, a natural saturation is achieved for the control effort as follows:

$$\|\boldsymbol{\Gamma}_i\| \leq \|\mathbf{I}_{f_i}\|(\varrho_i + \rho_i^2) + k_i^p + k_i^d,$$

with ρ_i and ϱ_i can be obtained respectively from (19) and (20) as: $\|\boldsymbol{\omega}_{r_i}\| \leq \rho_i := 2\sum_{j=1}^{n} k_{ij}$, and $\|\dot{\boldsymbol{\omega}}_{r_i}\| \leq \varrho_i := \frac{1}{2}\rho_i^2$.

V. Simulation results

In this section, we provide simulation results showing the effectiveness of the proposed control schemes. We consider a team of four rigid bodies with the inertia matrices: $\mathbf{I}_{f_1} = \text{diag}(20,20,30)$ kg.m^2, $\mathbf{I}_{f_2} = \text{diag}(10,5,15)$ kg.m^2, $\mathbf{I}_{f_3} = \text{diag}(10,3,8)$ kg.m^2 and $\mathbf{I}_{f_4} = \text{diag}(5,8,15)$ kg.m^2, and the following initial conditions: $\mathbf{Q}_1(0) = (0,0,sin(-\pi/4),\cos(-\pi/4))^{\top}$, $\mathbf{Q}_2(0) = (1,0,0,0)^{\top}$, $\mathbf{Q}_3(0) = (0,1,0,0)^{\top}$, $\mathbf{Q}_4(0) = (0,0,1,0)^{\top}$, $\boldsymbol{\omega}_1(0) = (-0.1,0.09,0.1)^{\top}$ rad/sec, $\boldsymbol{\omega}_2(0) = (0.2,-0.05,0.1)^{\top}$ rad/sec, $\boldsymbol{\omega}_3(0) = (-0.2,0.1,-0.05)^{\top}$ rad/sec, $\boldsymbol{\omega}_4(0) = (0.1,0.1,-0.25)^{\top}$ rad/sec. The initial states of the dynamic systems (4) and the auxiliary systems (14) are selected as $\mathbf{Q}_{r_i}(0) = \mathbf{Q}_i(0)$ and $\mathbf{P}_i(0) = (0, 0, 1, 0)^\top$, for $i \in \mathcal{N}$.

We consider first the control scheme in Theorem 1 with the control gains $k_i^\omega = 15$, $k_i^p = 5$, $k_i^d = 30$, $\lambda_i = 3$, for $i \in \mathcal{N}$, and $k_l^q = 25$ for $l = 1$, with $\boldsymbol{\omega}_{r_i}(0) = \mathbf{0}_3$ rad/sec. The information flow between rigid bodies is represented by the undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{K})$, given in Fig. 1, where $\mathcal{K} = [k_{ij}]$, with $k_{ij} = k_{ji} = 15$ for all $(i, j) \in \mathcal{E} := \{(1, 2), (1, 4), (2, 3)\}$ and $k_{ij} = 0$ otherwise. Also, the communication delays are selected as $\tau_{ij} = \bar{\tau}_{ij}|\sin(0.2\ t)|$ sec, for $i, j \in \mathcal{E}$, with $\bar{\tau}_{1i} = 0.1$, $\bar{\tau}_{2i} = 0.15$, $\bar{\tau}_{3i} = \bar{\tau}_{4i} = 0.2$, for $i \in \mathcal{N}$. Note that the control gains satisfy (17) with $\tau = 0.3$ and $\epsilon = 1$.

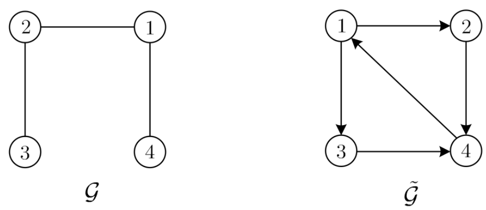

Fig. 1: Interconnection graphs.

Fig. 2 shows the attitudes of the systems, with $\mathbf{Q}_i = (\mathbf{q}_i^1, \mathbf{q}_i^2, \mathbf{q}_i^3, \boldsymbol{\eta}_i)^\top$, in the case where a desired attitude, represented by $\mathbf{Q}_d = (0, 0, 0, 1)^\top$, is available to the first rigid body designated as the leader, *i.e.,* case of the **LFP** with $l = 1$ and $\alpha = 1$. It can be observed that all systems synchronize their attitudes to the desired attitude in the presence of time-varying communication delays. Also, Fig. 3 depicts the obtained results in the case of the **LSP**, with $\alpha = 0$, where it is clear that all rigid bodies synchronize their attitudes to the same constant final attitude.

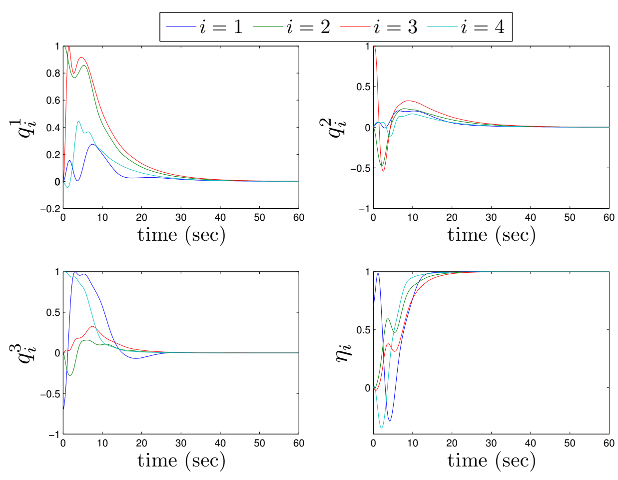

Fig. 2: Attitudes of rigid bodies, Theorem 1, case of the **LFP**.

The control scheme in Theorem 2 is considered next, with the strongly connected directed communication graph $\tilde{\mathcal{G}}$ in Fig. 1. The control gains are selected as: $k_j^p = 8$, $k_i^d = 45$, $\lambda_i = 5$, $k_{ij} = 1$, for $(i, j) \in \tilde{\mathcal{E}} := \{(1, 4), (2, 1), (3, 1), (4, 2), (4, 3)\}$, and $k_{ij} = 0$ otherwise.

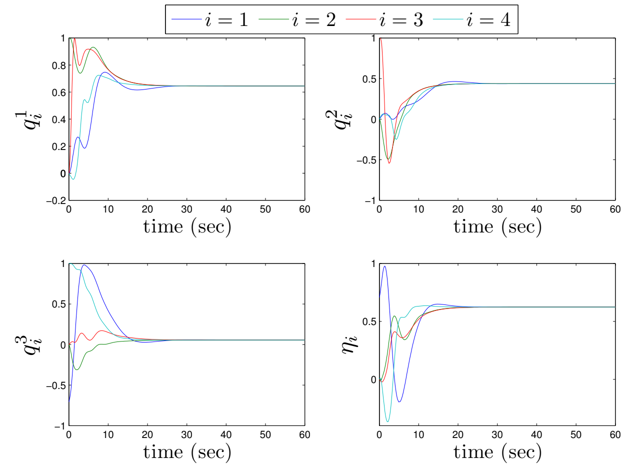

Fig. 3: Attitudes of rigid bodies, Theorem 1, case of the **LSP**.

The constant communication delays are considered as: $\tau_{1i} = 0.1$ sec, $\tau_{2i} = 0.15$ sec, $\tau_{3i} = \tau_{4i} = 0.2$ sec, for $i \in \mathcal{N}$. Fig. 4 illustrates the obtained results in this case, the **LSP**, where it is clear that all rigid bodies align their attitudes to the same constant final attitude despite the constant communication delays.

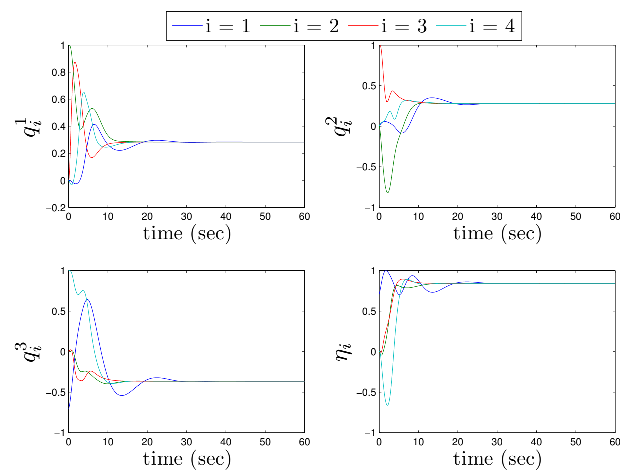

Fig. 4: Attitudes of rigid bodies, Theorem 2.

VI. Concluding Remarks

We addressed the attitude synchronization problem for a group of rigid bodies without angular velocity measurements and in the presence of communication delays. In Theorem 1, we proposed a solution to the **LSP** and the **LFP** in the presence of time-varying communication delays and undirected information exchange between members of the team. It is shown that attitude synchronization is achieved under a sufficient condition that can be satisfied with an appropriate choice of the control gains. Also, the communication graph is restricted to an undirected tree. Theorem 2 presents a solution to the **LSP** under a strongly connected directed communication graph in the presence of arbitrary constant communication delays. This removes the restrictions obtained in Theorem 1 in this case, and considers a more general communication topology between rigid bodies. The extension of this result to the case of the **LFP**, as well as the case of time-varying communication delays is not straightforward and will be examined in our future work.

The proposed approach in this paper is based on the introduction of dynamic systems that generate an intermediate reference trajectory for each rigid body, and all rigid bodies negotiate to reach an agreement on a common final reference trajectory in the presence of communication delays. This way, the attitude synchronization design problem is reduced to a separate design of a tracking control law, without angular velocity measurements, and an attitude synchronization with communication delays using the internally synthesized, and hence available, intermediate reference vectors.

As mentioned earlier, very few papers have considered the attitude synchronization problem with delayed communication in the full state feedback case. The authors in [20] have addressed the **LSP** with constant communication delays in the full state feedback case. In this reference, only the attitude kinematics have been considered to design a desired angular velocity that achieves attitude synchronization under strongly connected directed graphs. In addition, the result of [20] relies on the assumption that the rotation matrix of each rigid body is always positive definite. Besides the non-requirement of angular velocity measurements, the result of Theorem 2 provides an input torque design that solves this problem without conditions on the initial attitudes of the rigid bodies.

In [16]-[19], the attitude dynamics have been considered to design attitude synchronization schemes with delayed communication in the full state feedback case. In these papers, only the cooperative attitude tracking problem under undirected communication topologies has been addressed. Due to the definition of the error variables in the aforementioned papers, it is not trivial to extend their results to solve the **LSP** and the **LFP** in the full state feedback case with delayed communications. Theorem 1 provides solutions to these problems with time-varying communication delays and removes the requirements of angular velocity measurements. Note that the proposed control schemes in Theorem 1 and Theorem 2 can be extended to solve the cooperative attitude tracking problem in the presence of communication delays without angular velocity measurements. Furthermore, our results can be extended in a straightforward manner to the full state information case, which constitutes on its own right a new contribution in view of the existing literature.

Moreover, we believe that the result of Theorem 1 carries an additional feature, which consists in the fact that the time-varying communication delays are only assumed to be bounded. Also, the control scheme in Theorem 2 is guaranteed to be *a priori* bounded. This enables the designer to select appropriate control gains to account for input saturations. The extension of this work to the case of dynamically switching topologies is a challenging problem and will be the focus of our future work.

REFERENCES

[1] W. Ren, "Distributed attitude alignment in spacecraft formation flying," *International Journal of Adaptive Control and Signal Processing*, vol. 21, pp. 95-113, 2007.

[2] H. Bai, M. Arcak and J. T. Wen, "Rigid body attitude coordination without inertial frame information," *Automatica*, vol. 44, no. 12, pp. 3170-3175, 2008.

[3] D. V. Dimarogonas, P. Tsiotras and K. J. Kyriakopoulos, "Leader follower cooperative attitude control of multiple rigid bodies," *Systems and Control Letters*, vol. 58, pp. 429-435, 2009.

[4] W. Ren, "Distributed cooperative attitude synchronization and tracking for multiple rigid bodies," *IEEE Transactions on Control Systems Technologies*, vol. 18, no. 2, pp. 383-392, 2010.

[5] J. Lawton and R. W. Beard, "Synchronized multiple spacecraft rotations," *Automatica*, vol. 38, no. 8, pp. 1359-1364, 2002.

[6] A. Abdessameud and A. Tayebi, "Attitude synchronization of a group of spacecraft without velocity measurement," *IEEE Transactions on Automatic Control*, vol. 54, no. 11, pp. 2642-2648, 2009.

[7] A. Abdessameud and A. Tayebi, "On the coordinated attitude alignment of a group of spacecraft without velocity measurements," in *Proceedings of the 48^{th} IEEE Conference on Decision and Control*, 2009, pp. 1476-1481.

[8] U. Münz, A. Papachristodoulou and F. Allgöwer. "Delay-dependent rendezvous and flocking of large scale multi-agent systems with communication delays," in *Proceddings of the 47^{th} Conference on Decision and Control*, 2008, pp. 2038-2043.

[9] Y. Hong-Yong, Z. Xun-Lin and Z. Si-Ying. "Consensus of second-order delayed multi-agent systems with leader-following," *European Journal of Control*, vol. 15, pp. 1-15, 2010.

[10] Z. Meng, W. Yu, and W. Ren. "Discussion on: "Consensus of second-order delayed multi-agent systems with leader-following"," *European Journal of Control*, vol. 2, pp. 200-205, 2010.

[11] I. G. Polushin, A. Tayebi, and H. J. Marquez, "Control schemes for stable teleoperation with communication delay based on IOS small gain theorem," *Automatica*, vol. 42, pp. 905-915, 2006.

[12] N. Chopra, M. W. Spong and R. Lozano, "Synchronization of bilateral teleoperators with time delay," *Automatica*, vol. 44, no. 8, pp. 2142-2148, 2008.

[13] S.-J. Chung and J. J. E. Slotine, "Cooperative robot control and concurrent synchronization of Lagrangian systems," *IEEE Transactions on Robotics*, vol. 25, no. 3, pp. 686-700, 2009.

[14] E. Nuño, R. Ortega, L. Basañez & D. Hill. "Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays," *IEEE Transactions on Automatic Control*, vol. 56, no. 4, pp. 935-941, 2011.

[15] A. Abdessameud and A. Tayebi, "Formation control of VTOL Unmanned Aerial Vehicles with communication delays," *Automatica*, vol. 47, pp. 2383–2394, 2011.

[16] S.-J. Chung, U. Ahsun and J. J. E. Slotine, "Application of synchronization to formation flying spacecraft: Lagrangian approach," *Journal of Guidance, Control and Dynamics*, vol. 32, no. 2, pp. 512-526, 2009.

[17] J. Erdong, J. Xiaoleib and S. Zhaoweia, "Robust decentralized attitude coordination control of spacecraft formation," *Systems and Control Letters*, vol. 57, pp. 567-577, 2008.

[18] J. Erdong and S. Zhaowei, "Robust attitude synchronization controllers design for spacecraft formation," *IET Control Theory and Applications*, vol. 3, no.3, pp. 325-339, 2009.

[19] Z. Meng, Z. You, G. Li and C. Fan, "Cooperative attitude control of multiple rigid bodies with multiple time-varying delays and dynamically changing topologies," Hindawi Publishing Corporation, Mathematical Problems in Engineering vol. 2010.

[20] Y. Igarashi, T. Hatanaka, M. Fujita & M. W. Spong. "Passivity-based attitude synchronization in SE(3)," *IEEE Transaction on Control Systems Technology*, vol. 17, no. 5, pp. 1119-1134, 2009.

[21] M. D. Shuster, "A survey of attitude representations," *Journal of Astronautical Sciences*, vol. 41, no. 4, pp. 439-517, 1993.

[22] D. Jungnickel, *Graphs, Networks and Algorithms*, Algorithms and Computation in Mathematics, volume 5, 2005, second edition, Springer.

[23] A. Tayebi, "Unit quaternion based output feedback for the attitude tracking problem," *IEEE Transactions on Automatic Control*, vol. 53, no. 6, pp. 1516-1520, 2008.

[24] A. Abdessameud, A. Tayebi and I. G. Polushin, "Attitude synchronization of multiple rigid bodies with communication delays", *IEEE Transactions on Automatic Control*, In press.