Adaptive Position Tracking of VTOL UAVs

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***Abstract*—An adaptive position-tracking control scheme is proposed for vertical take-off and landing (VTOL) unmanned airborne vehicles (UAVs) for a set of bounded external disturbances. The control design is achieved in three main steps. The first step is devoted to the design of an *a priori* bounded linear acceleration driving the translational dynamics toward the desired trajectory. In the second step, we extract the required *a priori* bounded thrust and the desired attitude, in terms of unit quaternion, from the desired acceleration derived in the first step. In the last step, we design the required torque for the rotational dynamics, allowing the system's attitude to be driven toward the desired attitude obtained at the second step. Two control laws for the system control torque are rigorously designed. The first control law ensures that the position-tracking objective is satisfied for any initial conditions, whereas the second ensures that the tracking objective is satisfied for a set of initial conditions, which is dependant on the control gains. The latter case is included, since it is less complicated than the former control law and may be advantageous from a practical point of view. Finally, simulation results are provided to illustrate the effectiveness of the proposed control strategy.**

***Index Terms*—Adaptive control, unmanned airborne vehicle (UAV), vertical take-off and landing (VTOL).**

I. Introduction

IN the recent past, the use of vertical take-off and landing (VTOL) unmanned airborne vehicles (UAVs) has seen a significant increase in popularity. These types of vehicles are desired for a variety of applications, including visual inspection of structures (buildings, bridges, etc.), search and recovery, defence, etc. Most commonly these systems are piloted by a remote operator, however, a number of researchers have been working to develop systems that offer a higher degree of autonomy, thus reducing the complexity of the task presented to the operator. One example of a VTOL UAV is the ducted-fan aircraft, which is illustrated in Fig. 1. The system uses one or two main rotors/fans/propellers to generate vertical thrust, which provides the VTOL ability. This type of aircraft uses *vectored thrust*, which controls the movement of the system by directing the thrust using a set of ailerons/wings/control surfaces that are located at the underside of the system. In previous work, for example, [2], the goal was to achieve a tracking control law for the system attitude. This paper extends further to deal with the position-tracking problem of a ducted-fan VTOL UAV in the presence of external disturbances. Due to the nature of this underactuated system, a common practice is to use the system attitude as a means to direct the thrust in order to control the system position and velocity. This choice is intuitive and offers promising results when used with traditional backstepping approaches. Position control of VTOL UAVs has also been the focus of a number of researchers (for example, [3]–[7]), yet despite their tremendous efforts, there still exist some open problems in terms of handling external disturbances, coupling between system dynamics, singularities, as well as achieving global stability results. Previous work that address unknown disturbances include [3]–[5]. In most cases, the disturbance force is required to satisfy some assumptions in order to develop the control laws (for example, the disturbance is constant in the inertial frame of reference), which is also the case in this paper.

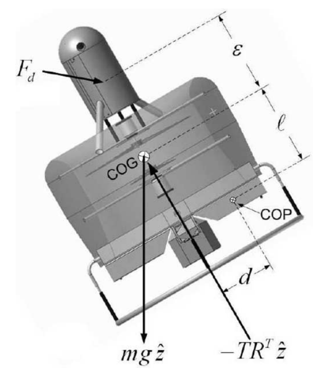

Fig. 1. Exogenous forces.

A second common problem is related to which system inputs are used to specify the control law. Usually, it is desired to obtain the control torque that is applied to the rotational dynamics of the system (that is generated by the vectored thrust). This goal can be challenging, especially in the presence of external disturbances, and as a result, the control law is often specified in terms of the desired system angular velocity (one integrator away from the control torque). The desired angular velocity must then be implemented using high-gain feedback. There are few examples of controllers in the literature that use the control torque as a system input while considering external disturbances. In [5], a result is achieved for the position regulation problem; however, this result only guarantees local stability and does not consider position tracking. In [8], a neural network adaptive controller has been proposed; however, it is not accompanied with proofs for stability, either locally or globally.

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For this type of aircraft, the vectored thrust action produces a translational force in addition to a moment. Due to this system characteristic, another well-known problem is due to this coupling between the rotational and translational dynamics. This coupling is usually in the form of a perturbing term (given as a function of the control torque or angular velocity) that affects the translational acceleration of the system. This problem is discussed in more detail in [9] and [10]. This coupling term is system dependant, and is not always present in certain VTOL systems, for example, the quadrotor aircraft. In [11], a position-tracking controller is proposed for the quadrotor aircraft achieving practical stability. In addition, an attitude controller for the quadrotor aircraft is proposed in [12]. As is the case with this paper, it is assumed that the coupling term is negligible and is thus omitted in the control design. However, as discussed in [9] and [10], depending on the strength of the coupling and the choice of control gains, this can lead to unexpected oscillations in the system states. There are some examples of controllers in the literature which address the coupling problem. For instance, in [10], a nice change of coordinates is presented; however, only for a planar system. In [5], a change of coordinates is also presented that removes the coupling due to the control torque. A consequence of this change of coordinates is that a new coupling is introduced in terms of the system angular velocity, which can only be neglected if the system yaw rate is assumed to be zero, which is the case in [5]. However, in practice, this would likely not be the case. Therefore, there still seems to be some potential for improvement regarding this coupling term in future work. Last, to the best of our knowledge, there are no results in the available literature to achieve almost global asymptotic stability for the position-tracking problem of UAVs in the presence of disturbance forces that use the control torque as the system input. In [6] and [11], controllers are proposed that achieve practical stability. Position-tracking control laws, that do not address disturbance forces, can be found in [7], [11], and [13]. In [4], a nice result is obtained to achieve almost global stability using a simple control law, which is given in terms of the system angular velocity.

A necessary step for this type of problem is to obtain a method to extract the magnitude and direction of the thrust from a given desired translational force. In the case where only one vector is used for attitude extraction, there exists an infinite number of solutions to this problem. However, similar to the work of [14], we utilize one solution to the attitude extraction problem in terms of the unit quaternion that has *almost* no restrictions on the demanded acceleration, except for a mild singularity that can be avoided.

Relying on this quaternion extraction method, we present two adaptive tracking controllers using the torque as a control input. Both controllers depend on an adaptive estimation method, which use a projection mechanism [15], [16]. The projection mechanism is required to avoid the singularity associated with the attitude extraction method. The first proposed controller achieves the position-tracking objective for any initial condition of the state, whereas the second controller achieves the position-tracking objective for a set of initial conditions, which are dependant on the control gains. The latter controller is included, since it is less complicated than the prior case and may be more suitable to use in practice. During the process of developing the two control laws, the disturbance forces are assumed to be constant in the inertial frame. In this case, both control laws are proven to achieve the position-tracking objective provided that an upper bound of the disturbance force is known *a priori* (although the actual magnitude of the disturbance force may be less than this limit). To evaluate the robustness of the proposed controller when the disturbance force is not constant, simulation results are provided, which considers a model of the aerodynamic forces that are exerted on the system in the presence of a uniform external wind, which is assumed to have a constant velocity.

II. Background

A. Attitude Representation

In this paper, we make use of two well-known forms of attitude representation, the rotation matrix (direct cosine matrix) and the unit quaternion [17]–[19]. The rotation matrix is the map from the inertial frame to the body frame, where $R \in \mathrm{SO}(3) := \{R \in \mathbb{R}^{3\times 3}; \det(R) = 1; RR^\mathsf{T} = R^\mathsf{T} R = I_{3\times 3}\}$. The dynamics of the rotation matrix are $\dot{R} = -S(\Omega)R$, where $S(\cdot)$ is the skew-symmetric matrix such that $S(u)v = u \times v$, where $\times$ denotes the vector cross product and $u, v \in \mathbb{R}^3$. The vector $\Omega \in \mathbb{R}^3$ is the body-referenced angular velocity of the rigid body. The system attitude can also be represented by the unit quaternion $Q = (q_0, q) \in \mathbb{Q} = \{Q \in S^3, \|Q\| = 1\}$. The rotation matrix corresponding to a unit quaternion $Q = (q_0, q)$ is given by

$$R(q_0, q) = I_{3\times 3} + 2S(q)^2 - 2q_0 S(q). \tag{1}$$

Other attitude representations and their transformations, including transformations from unit quaternion to rotation matrices can be found in [17]. Given two quaternion $Q, P \in \mathbb{Q}$, where $P = (p_0, p)$ we use the well-known noncommutative quaternion multiplication

$$Q \odot P = \left(p_0 q_0 - q^\mathsf{T} p, q_0 p + p_0 q + S(q)p\right). \tag{2}$$

The unit quaternion has the inverse $Q^{-1} = (q_0, -q)$, which has the property $Q \odot Q^{-1} = Q^{-1} \odot Q = (1, \mathbf{0})$, where $(1, \mathbf{0})$ is known as the *identity quaternion*, and $Q \odot (1, \mathbf{0}) = (1, \mathbf{0}) \odot Q = Q$. In general, the unit-quaternion dynamics is expressed using the *body-referenced angular velocity* Ω by

$$\dot{Q} = \frac{1}{2} Q \odot \begin{bmatrix} 0 \\ \Omega \end{bmatrix} = \frac{1}{2} A(q_0, q)\,\Omega \tag{3}$$

$$A(q_0, q) = \begin{bmatrix} -q^\mathsf{T} \\ q_0 I_{3\times 3} + S(q) \end{bmatrix}. \tag{4}$$

B. System Model

Using the representation given in Section II-A, the well-known rigid body model can be written as follows:

$$\dot{p} = v \tag{5}$$

$$\dot{v} = g\hat{z} - \frac{T}{m} R^\mathsf{T} \hat{z} - \frac{1}{m\ell} R^\mathsf{T} S(\hat{z}) u + \frac{1}{m} F_d \tag{6}$$

$$\dot{Q} = \frac{1}{2}\begin{bmatrix} -q^{\mathsf{T}} \\ q_0 I_{3\times 3} + S(q) \end{bmatrix}\Omega \tag{7}$$

$$I_b\dot{\Omega} = -S(\Omega)I_b\Omega + \epsilon_M S(\hat{z})RF_d + u \tag{8}$$

where $p, v, \in \mathbb{R}^3$ denote the inertial referenced system position and velocity, respectively, F_d denotes the inertial referenced disturbance force, I_b is the constant body-referenced inertia tensor, u is the control torque input, T is the system thrust, $\hat{z} = \mathrm{col}\,[0, 0, 1]$, m is the system mass, ℓ is the torque lever arm, g is the gravitational acceleration, and ϵ_M is the lever arm that creates a disturbance torque due to F_d. The model for the disturbance force and torque is similar to [5], since we assume that the disturbance force is applied to a point on the body-referenced z-axis at a distance ϵ_M away from the system center of gravity. The coupling between the translational and rotational dynamics appears in the equation of $\dot{v}$ in the form of the control torque u.

To simplify notation, we define two unknown parameters θ_a and θ_b, in addition to a scalar *thrust control input* u_t, which are given by

$$\theta_a = \frac{1}{m}F_d, \qquad \theta_b = \epsilon_M F_d, \qquad u_t = \frac{T}{m}. \tag{9}$$

C. Mathematical Preliminary

Throughout the paper, $\|u\|$ denotes the Euclidian norm of the vector u, and $\|M\|_F$ denotes the Frobenius norm of the matrix M. We also consider a bounded function $h : \mathbb{R}^3 \to \mathbb{R}^3$ such that $0 \leq \|h(u)\| < 1 \forall u$, whose first and second partial derivatives are denoted as follows:

$$\frac{\partial}{\partial u}h(u) := \phi_h(u), \quad \frac{\partial}{\partial u}\phi_h(u)v := f_{\phi_h}(u, v). \tag{10}$$

We choose the following bounded function:

$$h(u) = \left(1 + u^{\mathsf{T}}u\right)^{-1/2} u \tag{11}$$

from which we evaluate the partial derivatives as follows:

$$\phi_h(u) = \left(1 + u^{\mathsf{T}}u\right)^{-3/2}\left(I_{3\times 3} - S(u)^2\right) \tag{12}$$

$$f_{\phi_h}(u, v) = \left(1 + u^{\mathsf{T}}u\right)^{-5/2}\Big(3\left(S(u)^2 - I_{3\times 3}\right)vu^{\mathsf{T}} + \left(1 + u^{\mathsf{T}}u\right)\left(2S(u)S(v) - S(v)S(u)\right)\Big). \tag{13}$$

The eigenvalues of $\phi_h(u)$ are given by

$$\lambda\left(\phi_h(u)\right) = \begin{bmatrix} \left(1 + \|u\|^2\right)^{-3/2} \\ \left(1 + \|u\|^2\right)^{-1/2} \\ \left(1 + \|u\|^2\right)^{-1/2} \end{bmatrix} \tag{14}$$

and therefore, $0 < \|\phi_h(u)\| \leq 1 \forall u$.

III. Main Problem Formulation

Using the model given by (5)–(8), our objective is to force the position p to track some continuous time-varying reference $r(t)$, given that it meets the following requirements.

Assumption 1: The second, third, and fourth derivatives (w.r.t. t) of the reference trajectory $r(t)$ are uniformly continuous. Furthermore, there exists positive constants δ_r and δ_{rz} such that the second derivative of the reference trajectory is bounded by $\|\ddot{r}\| \leq \delta_r$ and $\hat{z}^{\mathsf{T}}\ddot{r} < \delta_{rz} < g$.

Although, in general, the disturbance force exerted on the aircraft can be time-varying, for the purposes of developing the control laws we consider (for the time being) that the disturbance force is constant in the inertial frame of reference (for example, this may be valid if the system is moving with a constant velocity in the presence of a constant and uniform wind).

Assumption 2: The disturbances θ_a and θ_b are constant and there exists positive constants $\delta_a < g$ and δ_b such that the disturbances are contained in the set $\theta_a \in B_a := \{\theta_a \in \mathbb{R}^3; \|\theta_a\| < \delta_a < g\}$, $\theta_b \in B_b := \{\theta_b \in \mathbb{R}^3; \|\theta_b\| < \delta_b\}$.

Assumption 3: The control torque lever arm ℓ is sufficiently large such that $m\ell \gg 1$, and therefore, the coupling term $(m\ell)^{-1}R^{\mathsf{T}}S(\hat{z})u \approx 0$.

At this point, we define the error signals for the system states that will be used in the control design. Some of the error signals involve virtual control laws that will not be defined until later in the paper. Given the reference trajectory $r(t)$, we define the error signals as follows:

$$\tilde{p} = p - r(t), \quad \text{position error} \tag{15}$$

$$\tilde{v} = v - \dot{r}(t), \quad \text{velocity error.} \tag{16}$$

To use the system velocity v as a virtual control, we must use the system attitude R and thrust u_t as virtual inputs to the translational dynamics (5) and (6). Therefore, we define

$$\mu = g\hat{z} - u_t R^{\mathsf{T}}\hat{z} \tag{17}$$

which is the acceleration of the system due to gravity and the system thrust expressed in the inertial frame. We define μ_d as the *desired virtual acceleration*, which will be defined later as an intermediate step in the control design. The *acceleration error* can then defined as follows:

$$\tilde{\mu} = \mu - \mu_d, \quad \text{acceleration error.} \tag{18}$$

The use of μ as a virtual control requires the extraction of the thrust u_t and the *desired attitude* (or *desired orientation*) $Q_d = (q_{d0}, q_d)$ from μ_d. That is, we require a transformation $\chi_{u_t} : \mathbb{R}^3 \to \mathbb{R}$ and $\chi_Q : \mathbb{R}^3 \to S^3$, such that the signals $u_t = \chi_{u_t}(\mu_d)$ and $(q_{d0}, q_d) = \chi_Q(\mu_d) = (\chi_{q_0}(\mu_d), \chi_q(\mu_d))$ satisfy

$$\mu_d = \mathrm{col}\left[\mu_{d_1}, \mu_{d_2}, \mu_{d_3}\right] = g\hat{z} - u_t R_d^{\mathsf{T}}\hat{z} \tag{19}$$

where $R_d = R(q_{d0}, q_d)$ as defined by (1). The choice of χ_{u_t} and χ_Q is the focus of Section IV. Provided that the transformation χ_Q is differentiable, it is also possible to define the dynamics of the desired attitude $\dot{Q}_d$. The derivative of the desired attitude can then be used to obtain the *desired angular velocity* Ω_d, based on the following well-known relationship:

$$\dot{Q}_d = \frac{1}{2}A\left(q_{d0}, q_d\right)\Omega_d. \tag{20}$$

Since $A^{\mathsf{T}}A = I_{3\times3}$, one can obtain the desired angular velocity from

$$\Omega_d = 2A^{\mathsf{T}}\frac{d}{dt}\begin{bmatrix}\chi_{q_0}(\mu_d)\\ \chi_q(\mu_d)\end{bmatrix}. \tag{21}$$

Using the *desired orientation* Q_d, we also define the *attitude error* $\tilde{Q} = (\tilde{q}_0, \tilde{q})$ as the rotation from the actual attitude Q to the desired attitude Q_d, which is given by

$$\tilde{Q} = Q_d^{-1} \odot Q, \quad \text{attitude error.} \tag{22}$$

During the control design, it is necessary to obtain the dynamics of the attitude error $\tilde{Q}$. Differentiating (22) along the trajectories (7) and (20), we obtain

$$\dot{\tilde{Q}} = \frac{1}{2}\begin{bmatrix}\tilde{q}^{\mathsf{T}}(\Omega_d - \Omega)\\ \tilde{q}_0(\Omega - \Omega_d) + S(\tilde{q})(\Omega_d + \Omega)\end{bmatrix}. \tag{23}$$

Due to the use of the quaternion multiplication operator, the attitude error $\tilde{Q}$ preserves the properties of the unit quaternion and can also be considered as a rotation/orientation. Using this representation, one of the goals of the control design is to force the attitude error $\tilde{Q}$ to the identity quaternion $(1, \mathbf{0})$, or to the negated identity quaternion $(-1, \mathbf{0})$, since this represents a rotation of 2π from the desired attitude and is, therefore, the same physical orientation.

A. Error Dynamics

In this section, we define the dynamics of the error signals given in the previous section. In light of Assumption 3, for the purposes of the control design, we assume that the coupling term in the translational dynamics due to the control torque is negligible. As a result of the earlier formulation, the system error dynamics are given by

$$\dot{\tilde{p}} = \tilde{v} \tag{24}$$

$$\dot{\tilde{v}} = \mu_d + \tilde{\mu} + \theta_a - \ddot{r} \tag{25}$$

$$\dot{\tilde{Q}} = \frac{1}{2}\begin{bmatrix}\tilde{q}^{\mathsf{T}}(\Omega_d - \tilde{\Omega} - \beta)\\ \tilde{q}_0(\beta + \tilde{\Omega} - \Omega_d) + S(\tilde{q})(\beta + \tilde{\Omega} + \Omega_d)\end{bmatrix} \tag{26}$$

$$I_b\dot{\tilde{\Omega}} = -S(\Omega)I_b\Omega + S(\hat{z})R\theta_b - I_b\dot{\beta} + u \tag{27}$$

with $\tilde{\Omega} = \Omega - \beta$, where β is a virtual control law for the angular velocity that is defined later in the control design. More specifically, we find two virtual control laws $\beta = \beta_1$ for the first control law, and $\beta = \beta_2$ for the second control law. Based on this formulation, our control strategy can be separated into three tasks.

1) Specify the virtual control law for the desired virtual acceleration μ_d that satisfies the position- and velocity-tracking objectives.
2) Find continuous and differentiable transformations χ_{u_t} and χ_Q, which extract the desired thrust u_t and desired attitude Q_d, respectively, that forces the system to the desired virtual acceleration specified in step 1.
3) Specify the virtual control law for the angular velocity β that forces the system attitude Q to track the desired system attitude Q_d specified in step 2, which is used to specify the control law for the system control torque u to force the system angular velocity Ω to track the desired angular velocity β.

Section IV describes the proposed attitude extraction algorithm and defines the transformations χ_{u_t} and χ_Q. Section V-A briefly restates the projection mechanism from [16]. Section V-B and C describes the two proposed control laws. A step-by-step procedure describing the implementation of the two controllers is given in Section V-D. Finally, Section VI gives simulation results from the two proposed controllers.

IV. Attitude Extraction

In this section, we define the transformations χ_{u_t} and χ_Q, which extract the system thrust u_t, and desired attitude Q_d or R_d, from the virtual control law μ_d given by (19). In general, the attitude extraction problem for a single pair of vectors can be stated as follows: given two vectors u and v, where $\|u\| = \|v\|$, we wish to find an orientation $R_d \in \mathrm{SO}(3)$ that satisfies

$$R_d u = v. \tag{28}$$

Note that (28) can also be expressed using the unit quaternion as follows:

$$\begin{bmatrix}0\\ v\end{bmatrix} = Q_d^{-1} \odot \begin{bmatrix}0\\ u\end{bmatrix} \odot Q_d \tag{29}$$

where instead we seek an expression for the quaternion Q_d. In [14], an intuitive solution to this problem is described. A similar solution which yields a unit quaternion is stated in the following lemma.

Lemma 1: Given two vectors u and v, where $\|u\| = \|v\| \neq 0$, and where $u \neq -v$; then, a solution for the unit quaternion $Q_d = (q_{d0}, q_d)$ that satisfies (29) exists and is given by

$$q_{d0} = \frac{1}{\|u\|}\sqrt{\frac{\|u\|^2 + u^{\mathsf{T}}v}{2}} \tag{30}$$

$$q_d = \frac{1}{\|u\|}\sqrt{\frac{1}{2\left(\|u\|^2 + u^{\mathsf{T}}v\right)}}S(v)u. \tag{31}$$

Proof: The proof is given in Appendix A.

Remark 1: As discussed by [14], this extraction uses the vector which is orthogonal to both u and v as the axis of rotation. Due to this choice, this method extracts the solution to the problem that minimizes the angle of rotation. Given the solution of the unit quaternion by (30) and (31), a rotation matrix which satisfies (28) can also be found using (1).

We now apply the results of the quaternion extraction to our particular case. Recall (17)–(19), and let μ_d be the virtual control input that achieves the position-tracking objective for the translational dynamics. We need to extract the thrust u_t and the desired orientation R_d or Q_d for a given μ_d satisfying (19). A solution for the system thrust $u_t = \chi_{u_t}$ and desired attitude $Q_d = (\chi_{q_0}, \chi_q)$ that satisfies (19), where the rotation matrix R_d is found from (1) is given by the following lemma.

Lemma 2: Given μ_d and assuming that $\mu_d \notin L$

$$L = \{\mu_d \in \mathbb{R}^3; \mu_d = \mathrm{col}\,[0, 0, \mu_{d_3}]\,; \mu_{d_3} \in [g, \infty)\} \tag{32}$$

a solution for the transformations χ_{u_t} and χ_Q, which extracts the system thrust $u_t = \chi_{u_t}$ and desired attitude $Q_d = (q_{d0}, q_d) = (\chi_{q_0}, \chi_q)$, which satisfies (19), is given by

$$\chi_{u_t} = \|\mu_d - g\hat{z}\| \tag{33}$$

$$\chi_{q_0} = \left(\frac{1}{2}\left(1 + \frac{g - \mu_{d_3}}{\|\mu_d - g\hat{z}\|}\right)\right)^{1/2} \tag{34}$$

$$\chi_q = \frac{1}{2\|\mu_d - g\hat{z}\|\chi_{q_0}} \begin{bmatrix} \mu_{d_2} \\ -\mu_{d_1} \\ 0 \end{bmatrix}. \tag{35}$$

Proof: Note that (19) can also be written as $R_d^\top \hat{z} = u_t^{-1}(g\hat{z} - \mu_d)$. Choosing the system thrust input (33), then it is clear that this is similar to the problem described by Lemma 1 with $v = \hat{z}$ and $u = (g\hat{z} - \mu_d)/\|g\hat{z} - \mu_d\|$ and $\|u\| = \|v\| = 1$. Applying these values of u and v to (30) and (31) results in the expressions given by (34) and (35). □

Remark 2: The singularity (32) corresponds to the desired attitude being perfectly inverted such that the vertical acceleration of the vehicle is greater than or equal to the acceleration due to gravity. Note that the singularity only applies to the desired attitude Q_d, and the actual attitude of the system can take any value without encountering a singularity. Since the singularity corresponds to an undesirable operating mode of the aircraft, avoiding this singularity does not significantly limit the normal operating mode of the system. Avoiding this singularity can be achieved by using a bounded law for the desired virtual acceleration μ_d. This is the main purpose of using the function h described by (11) and the projection mechanism described by Section V-A.

During the process of the control design, it is necessary to obtain an expression for the angular velocity Ω_d, which can be found using (21) and by differentiating the expression for the desired attitude Q_d. The value of $\dot{Q}_d$ is obtained by straightforward differentiation of the extracted attitude given by (34) and (35). This procedure ultimately yields the following expression:

$$\Omega_d = M(\mu_d)\dot{\mu}_d \tag{36}$$

$$M(\mu_d) = \frac{1}{\|\mu_d - g\hat{z}\|^2 c_1} \cdot \begin{bmatrix} -\mu_{d_1}\mu_{d_2} & -\mu_{d_2}^2 + \|\mu_d - g\hat{z}\|c_1 & \mu_{d_2}c_1 \\ \mu_{d_1}^2 - \|\mu_d - g\hat{z}\|c_1 & \mu_{d_1}\mu_{d_2} & -\mu_{d_1}c_1 \\ \mu_{d_2}\|\mu_d - g\hat{z}\| & -\mu_{d_1}\|\mu_d - g\hat{z}\| & 0 \end{bmatrix} \tag{37}$$

where $c_1 = \|\mu_d - g\hat{z}\| + g - \mu_{d_3}$.

V. Position-Tracking Control

In this section, we propose two control laws in terms of the system torque control input u. The first control law guarantees the position-tracking objective is satisfied for any initial conditions. The second control law guarantees the position-tracking objective for a set of initial conditions that is dependant on the control gains. The latter controller is included, since it is somewhat less complicated and may be beneficial from a practical perspective.

The first step of the control design is to choose the desired virtual acceleration μ_d. Based on the earlier formulation, there are some requirements for this control law that must be considered.

1) To ensure a solution always exists for the desired orientation, Q_d, the desired virtual acceleration μ_d must be bounded such that it does not encounter the singularity (32).
2) In order to satisfy the tracking objective, the expression for the desired virtual acceleration μ_d must contain an (adaptive) estimate of the disturbance force.
3) Due to the use of backstepping, the expression for μ_d must be twice differentiable.

In order to satisfy the first two requirements, the adaptive estimate of the disturbance force must be guaranteed to be bounded *a priori*. To meet this criteria, we use a *projection-based* estimation algorithm. Furthermore, in order to satisfy the third requirement, the solution for the disturbance estimates obtained using the projection-based adaptation law must be twice differentiable. This motivates us to use the *sufficiently smooth projection* algorithm described by [16]. The use of other existing projection-based algorithms (for example, see [15] and [20]) is not directly applicable since they are not always differentiable.

As shown in [16], when utilizing projection, overparameterization is required when the system is of a sufficiently high order. For this reason, a second adaptive estimate $\hat{\theta}_2$ is used due to the unknown disturbance θ_a, and a third adaptive estimate $\hat{\theta}_3$ is used due to the unknown disturbance θ_b.

We begin by briefly describing the sufficiently smooth projection algorithm of [16] in Section V-A. This is followed by the two control laws in Section V-B and C respectively.

A. Adaptive Estimation Using Projection

In this section, we restate the projection algorithm described by [16], which yields adaptive estimates whose trajectories are sufficiently smooth. The use of the projection algorithm allows us to use an adaptive estimate of the disturbance force in the expression of the desired virtual acceleration μ_d, while ensuring that the desired attitude Q_d never encounters the singularity (32).

Consider a constant unknown parameter θ_p, which belongs to the set $B_p := \{\theta_p \in \mathbb{R}^3; \|\theta_p\| < \delta_p\}$, where the parameter δ_p is known *a priori*. Let $\hat{\theta}_p$ be the corresponding adaptive estimate of θ_p, and define the error $\tilde{\theta}_p = \theta_p - \hat{\theta}_p$. In general, the ideal adaptive estimation law is given by $\dot{\hat{\theta}}_p = \tau$, which does not necessarily guarantee that $\hat{\theta}_p \in B_p$. Based on the ideal adaptive estimation law, a projection-based adaptation law defined in [16], which guarantees the bound of $\hat{\theta}_p$ is given for our particular case by

$$\dot{\hat{\theta}}_p = \tau + \alpha(\hat{\theta}_p, \delta_p, \tau) \tag{38}$$

$$\alpha(\hat{\theta}_p, \delta_p, \tau) = -k_\alpha \eta_1 \eta_2 \hat{\theta}_p \tag{39}$$

$$k_\alpha = (2(\epsilon_\alpha^2 + 2\epsilon_\alpha \delta_p)^2 \delta_p^2)^{-1} \tag{40}$$

$$\eta_1 = \begin{Bmatrix} (\hat{\theta}_p^{\mathsf{T}}\hat{\theta}_p - \delta_p^2)^2, & \text{if } \hat{\theta}_p^{\mathsf{T}}\hat{\theta}_p > \delta_p^2 \\ 0, & \text{otherwise} \end{Bmatrix} \quad (41)$$

$$\eta_2 = \hat{\theta}_p^{\mathsf{T}}\tau + ((\hat{\theta}_p^{\mathsf{T}}\tau)^2 + \delta_\alpha^2)^{1/2} \quad (42)$$

where $\epsilon_\alpha > 0$, $\delta_\alpha > 0$ and has the properties

$$\|\hat{\theta}_p\| < \delta_p + \epsilon_\alpha, \quad \tilde{\theta}_p^{\mathsf{T}}\alpha \geq 0, \quad \dot{\hat{\theta}}_p \in C^1. \quad (43)$$

B. Controller 1

Let $\hat{\theta}_1$ denote the first adaptive estimate of the unknown parameter θ_a. Using the bounded function $h(u)$ defined by (11), we propose the following law for the desired virtual acceleration:

$$\mu_d = \ddot{r} - \hat{\theta}_1 - k_p\Gamma_v^{-1}h(\tilde{p}) - (k_v + k_\theta)\,h(\tilde{v}) \quad (44)$$

where $k_p, k_\theta, k_v > 0, \Gamma_v = \Gamma_v^{\mathsf{T}} > 0$, and $\hat{z} = \text{col}\,[0, 0, 1]$. Using the parameters δ_{rz} and δ_a defined in Assumptions 1 and 2, we place the restriction

$$k_p\|\hat{z}^{\mathsf{T}}\Gamma_v^{-1}\| + 2k_\theta + k_v + \epsilon_\alpha < g - \delta_{rz} - \delta_a \quad (45)$$

where $\epsilon_\alpha > 0$ is a control gain used in the projection algorithm. Applying the value of the desired virtual acceleration (44) to the thrust and attitude extraction algorithm specified by (33)–(35), we extract the system thrust and desired attitude

$$u_t = \chi_{u_t}(\mu_d), \quad q_{d0} = \chi_{q_0}(\mu_d), \quad q_d = \chi_q(\mu_d) \quad (46)$$

from which we obtain the attitude error $\tilde{Q} = (\tilde{q}_0, \tilde{q}) = Q_d^{-1} \odot Q = (q_{d0}, -q_d) \odot Q$. We propose the following estimation law for the first adaptive estimate:

$$\dot{\hat{\theta}}_1 = \gamma_{\theta_1}(\tau_2 + \alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2)) \quad (47)$$

$$\tau_2 = \Gamma_v\tilde{v} + \frac{k_\theta}{\gamma_{\theta_1}}k_p\phi_h(\tilde{v})\Gamma_v^{-1}h(\tilde{p}) + \frac{k_\theta k_v}{\gamma_{\theta_1}}\phi_h(\tilde{v})h(\tilde{v}) + \left(\gamma_q(k_\theta + k_v)\phi_h(\tilde{v})M(\mu_d)^{\mathsf{T}} - \frac{2u_t k_\theta}{\gamma_{\theta_1}}\phi_h(\tilde{v})R^{\mathsf{T}}S(\bar{q})\right)\tilde{q} \quad (48)$$

$$\bar{q} = \tilde{q}_0\hat{z} + S(\hat{z})\tilde{q} \quad (49)$$

where $\gamma_{\theta_1} > 0$, $\gamma_q > 0$, $\phi_h(u)$ is the partial derivative of $h(u)$ as defined by (12), and α is the projection function defined by (39). We propose the following virtual control law for the angular velocity:

$$\beta_1 = M(\mu_d)(r^{(3)} + w_\beta) - \frac{1}{\gamma_q}\Phi\tilde{v} - K_q\tilde{q} \quad (50)$$

$$w_\beta = k_p k_v \phi_h(\tilde{v})\Gamma_v^{-1}h(\tilde{p}) - \gamma_{\theta_1}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) \quad (51)$$

$$\Phi = (\gamma_{\theta_1}\gamma_q M(\mu_d) - 2u_t S(\bar{q})R)\Gamma_v + \gamma_q k_p M(\mu_d)\Gamma_v^{-1}\phi_h(\tilde{p}) \quad (52)$$

where $K_q = K_q^{\mathsf{T}} > 0$, and the matrix $M(\mu_d)$ is given by (37). In general, the derivative of (50) is given by

$$\dot{\beta}_1 = f_{\beta_1} + \bar{f}_{\beta_1}\theta_a \quad (53)$$

where the actual expressions for f_{β_1} and $\bar{f}_{\beta_1}$ are given in Appendix B. Let $\hat{\theta}_2$ denote the second estimate of θ_a, $\hat{\theta}_3$ denote the estimate of θ_b, and let $\tilde{\Omega} = \Omega - \beta_1$ denote the angular velocity error. We propose the following control law for the system torque control input:

$$u = -\gamma_q\tilde{q} + S(\Omega)I_b\Omega - S(\hat{z})R\hat{\theta}_3 + I_b f_{\beta_1} + I_b\bar{f}_{\beta_1}\hat{\theta}_2 - K_\omega\tilde{\Omega} \quad (54)$$

where $K_\omega = K_\omega^{\mathsf{T}} > 0$, in addition to the following adaptation laws:

$$\dot{\hat{\theta}}_2 = \gamma_{\theta_2}(-\bar{f}_{\beta_1}^{\mathsf{T}}I_b\tilde{\Omega} + \alpha(\hat{\theta}_2, \delta_a, -\bar{f}_{\beta_1}^{\mathsf{T}}I_b\tilde{\Omega})) \quad (55)$$

$$\dot{\hat{\theta}}_3 = \gamma_{\theta_3}(-R^{\mathsf{T}}S(\hat{z})\tilde{\Omega} + \alpha(\hat{\theta}_3, \delta_b, -R^{\mathsf{T}}S(\hat{z})\tilde{\Omega})) \quad (56)$$

where $\gamma_{\theta_{2,3}} > 0$, and $\delta_b > 0$ is given based on Assumption 2.

Theorem 1: Consider the system described (24)–(27), where we apply the control and estimation laws (47) and (54)–(56). Provided that the following inequalities are satisfied:

$$\lambda_{\min}(K_q) > \frac{2\sqrt{2}k_v\bar{u}_t\delta_{\mu_d} + 2\gamma_{\theta_1}\gamma_q(k_v + k_\theta)}{\delta_{\mu_d}^2} + \frac{k_v^2}{2\epsilon_1}$$

$$\lambda_{\min}(\Gamma_v) > \frac{\gamma_q k_v \epsilon_1}{\delta_{\mu_d}^2} \quad (57)$$

where $\epsilon_1 > 0$, and

$$\bar{u}_t = g + \delta_r + \delta_a + k_p\|\Gamma_v^{-1}\| + k_v + 2k_\theta + \epsilon_\alpha \quad (58)$$

$$\delta_{\mu_d} = g - \delta_{rz} - \delta_a - k_p\|\hat{z}^{\mathsf{T}}\Gamma_v^{-1}\| - k_v - 2k_\theta - \epsilon_\alpha \quad (59)$$

then the system thrust input u_t is bounded and nonvanishing such that $0 < \delta_{\mu_d} < u_t < \bar{u}_t$, the system states (p, v, Ω) are bounded, and

$$\lim_{t\to\infty}[p(t) - r(t), v(t) - \dot{r}(t), \tilde{q}(t), \tilde{\Omega}(t)] = 0 \quad (60)$$

for any initial condition.[1]

Furthermore, the adaptive estimates $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$ are bounded, and in particular, the estimate $\hat{\theta}_1$ converges asymptotically to the constant unknown disturbance θ_a.

Proof: The proof is given in Appendix B. ■

C. Controller 2

In this section, we propose a similar albeit simpler version of the control law described in Section V-B. The motivation to simplify the previous result is largely due to the complexity of the virtual control law for the angular velocity β_1 from (50), and it is derivative of (53). It is possible to simplify this virtual control law, if an additional constraint is satisfied that is based on the initial conditions of the state and the control gains. This simplified version, which may be more suitable from a practical perspective, is described in the following.

Using the previous law for the desired virtual acceleration from (44) under the restriction (45), and the extraction of the

[1] There are two equilibrium solutions (physically identical), which satisfy the position-tracking objective, which are defined by $(\tilde{p}, \tilde{v}, \tilde{q}, \tilde{\Omega}) = 0$, $\tilde{q}_0 = \pm 1$. The equilibrium solution characterized by $\tilde{q}_0 = 1$ is stable, while the one characterized by $\tilde{q}_0 = -1$ is unstable (repeller equilibrium). This is due to the well-known topological obstruction on SO(3) under strictly continuous feedback. For more details regarding this topological obstruction (see [21]). One solution to this problem consists of using discontinuous or hybrid feedback control laws (see, for instance, [22]).

thrust control input u_t and desired attitude Q_d from (46), we obtain the attitude error $\tilde{Q} = (\tilde{q}_0, \tilde{q}) = Q_d^{-1} \odot Q$. Let $\hat{\theta}_1$ denote the first estimate of θ_a, where we use the estimation law (47). We propose the following virtual control law for the angular velocity:

$$\beta_2 = M(\mu_d)(r^{(3)} + w_\beta) - K_q \tilde{q} \tag{61}$$

where $K_q = K_q^\top > 0$, and w_β is given by (51). The derivative of β_2 can be written as follows:

$$\dot{\beta}_2 = f_{\beta_2} + \bar{f}_{\beta_2} \theta_a \tag{62}$$

where the actual expressions for f_{β_2} and $\bar{f}_{\beta_2}$ are given in Appendix D. Using the angular velocity error $\tilde{\Omega} = \Omega - \beta_2$, we propose the following control law for the system control torque input:

$$u = -\gamma_q \tilde{q} + S(\Omega) I_b \Omega - S(\hat{z}) R \hat{\theta}_3 + I_b f_{\beta_2} + I_b \bar{f}_{\beta_2} \hat{\theta}_2 - K_\omega \tilde{\Omega} \tag{63}$$

where $K_\omega = K_\omega^\top > 0$. Using the new expression for angular velocity error $\tilde{\Omega}$, we apply the adaptive estimation laws (55) and (56).

Theorem 2: Consider the system described (24)–(27), where we apply the control and estimation laws (47) and (63). Using the angular velocity error $\tilde{\Omega} = \Omega - \beta_2$, where β_2 is obtained using (61), we apply the estimation laws (55) and (56). Provided that the following inequalities are satisfied:

$$\lambda_{\min}(K_q) > \frac{2\sqrt{2} k_v \bar{u}_t \delta_{\mu_d} + 2\gamma_{\theta_1} \gamma_q (k_v + k_\theta)}{\delta_{\mu_d}^2} + \frac{k_v^2}{2\epsilon_1} + \frac{\delta_1^2}{2\gamma_q \epsilon_2} \tag{64}$$

$$\lambda_{\min}(\Gamma_v) > \frac{\gamma_q k_v \epsilon_1}{\delta_{\mu_d}^2} \tag{65}$$

$$\delta_1 = \left(2\bar{u}_t + \frac{\sqrt{2}\gamma_q \gamma_{\theta_1}}{\delta_{\mu_d}}\right) \|\Gamma_v\| + \frac{\sqrt{2}\gamma_q k_p}{\delta_{\mu_d}} \|\Gamma_v^{-1}\| \tag{66}$$

where $\epsilon_2 > 0$, then the system thrust input u_t is bounded and nonvanishing such that $0 < \delta_{\mu_d} < u_t < \bar{u}_t$, the system states (p, v, Ω) are bounded, and

$$\lim_{t\to\infty} [p(t) - r(t), v(t) - \dot{r}(t), \tilde{q}(t), \tilde{\Omega}(t)] = 0 \tag{67}$$

for all system initial conditions that satisfy

$$k_p \left(\sqrt{1 + \tilde{p}(0)^\top \tilde{p}(0)} - 1\right) + \frac{1}{2} X(0)^\top \bar{C} X(0) < \lambda_{\min}(\Gamma_v) \left(\frac{2\|\Delta_v\|}{\epsilon_2} - \frac{1}{2}\right) \tag{68}$$

$$\Delta_v = k_v \left(\Gamma_v - \frac{\gamma_q k_v \epsilon_1}{\delta_{\mu_d}^2} I_{3\times3}\right) \tag{69}$$

$$X = \text{col}[\tilde{v}, 1 - \tilde{q}_0, \tilde{q}, \tilde{\Omega}, \theta_a - k_\theta h(\tilde{v}) - \hat{\theta}_1, \theta_a - \hat{\theta}_2, \theta_b - \hat{\theta}_3] \tag{70}$$

$$\bar{C} = \text{diag}[\|\Gamma_v\| I_{3\times3}, 4\gamma_q I_{4\times4}, \|I_b\| I_{3\times3}, \gamma_{\theta_1}^{-1} I_{3\times3}, \gamma_{\theta_2}^{-1} I_{3\times3}, \gamma_{\theta_3}^{-1} I_{3\times3}]. \tag{71}$$

Furthermore, the adaptive estimates $\hat{\theta}_1, \hat{\theta}_2,$ and $\hat{\theta}_3$ are bounded, and in particular, the estimate $\hat{\theta}_1$ converges asymptotically to the constant unknown disturbance θ_a.

Proof: The proof is given in Appendix C. ■

D. Implementation

To implement the controllers given in Section V-B and C, consider the following iterative procedure.

1) Obtain the signals p, v, Q, Ω, and the desired reference trajectory $r(t)$ and $d^i/dt^i r(t)$, $i = 2, 3, 4$, and calculate the error signals $\tilde{p}$, $\tilde{v}$.
2) Calculate the virtual control law μ_d using (44), which is used to obtain the system thrust input u_t from (33), desired attitude Q_d using (34) and (35), and the matrix $M(\mu_d)$ from (37).
3) Calculate the error signals $\tilde{Q}$ and $\tilde{\mu}$ from (22) and (18), respectively, which is used to obtain τ_2 from (48) and $\dot{\hat{\theta}}_1$ from (47).
4) Using the virtual control law for the desired angular velocity (50) (or (61) for controller 2), calculate the angular velocity error $\tilde{\Omega} = \Omega - \beta_1$ ($\tilde{\Omega} = \Omega - \beta_2$ for controller 2).
5) Using the expression for the derivative $\dot{\beta}_1$ ($\dot{\beta}_2$ for the second control law) given by (103) and (104) [(101) and (102) for controller 2], apply the control law u from (54) (or (63) for controller 2), and the estimation laws $\dot{\hat{\theta}}_2$ and $\dot{\hat{\theta}}_3$ from (55) and (56), respectively.

VI. Simulations

To aid in the development of the control laws, some assumptions were required regarding the disturbance force F_d and the control torque lever arm ℓ. Assumption 2 states that the disturbance forces were assumed to be constant in the inertial frame, and Assumption 3 requires that the control torque lever arm is sufficiently large enough that an undesired coupling term can be omitted. However, these may be unlikely and/or unrealistic assumptions, since the disturbance forces are dependant on system aerodynamic forces caused by wind and the motion of the system, and the control torque lever arm may not be sufficiently large. To address these shortcomings, we demonstrate the robustness of the proposed controller by including in the simulations a time-varying disturbance force (which is based on an aerodynamic model) and the coupling term that was previously omitted (using reasonable values for the system mass m and the control torque lever arm ℓ). The disturbance model used is defined by

$$F_d = F_{\text{drag}} + F_{\text{ram}}$$
$$F_{\text{drag}} = \|v_w - v\| R^\top C_d R (v_w - v)$$
$$F_{\text{ram}} = \sqrt{\frac{T\rho A}{2}} R^\top I_{xy} R (v_w - v)$$

where F_{drag} are frictional drag forces that are proportional to the square of the external airflow, F_{ram} is the *ram-drag*

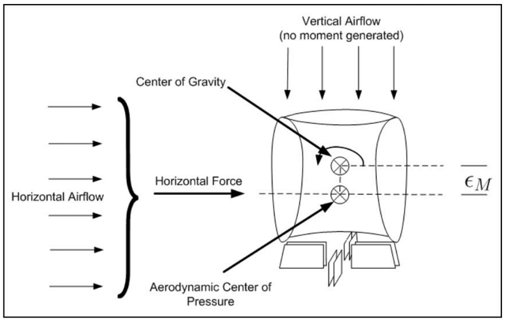

Fig. 2. Torque-generating aerodynamic forces as a result of airflow in body-fixed frame.

force,[2] $I_{xy} = \text{diag}\,(1,1,0)$, v_w is an inertially referenced unknown wind velocity (which is assumed to be uniform and constant in the inertial frame), ρ is the air density, A is the duct cross-sectional area, and $C_d \in \mathbb{R}^3$ is a matrix that consists of system-dependant aerodynamic constants expressed in the body-fixed frame. For the purposes of simulation, the aerodynamic-drag matrix and wind velocity was chosen to be $C_d = \text{diag}\,[0.1, 0.1, 0.05]$ kg/m and $v_w = [-1,-1,0]$ m/s. Assuming that the external airflow is uniform, due to the cylindrical symmetry of the system, the net aerodynamic force is assumed to be applied at a point on the body-referenced z-axis, located at a constant distance of $\epsilon_M = 0.1$ m from the system center of gravity, which is often referred to as the aerodynamic center-of-pressure [see Fig. 2]. In addition, to simulate uncertainty in the system inertia tensor, the controllers were implemented using an expected value of $I_b = \text{diag}\,[0.5, 0.5, 0.25]\ \text{kg}{\cdot}\text{m}^2$, where the actual value was specified as $I_b = \text{diag}\,[0.6, 0.6, 0.3]\ \text{kg}{\cdot}\text{m}^2$. For both simulations, the desired reference trajectory was specified as $r(t) = [10t, 30\sin(0.1t + 3.48), 20\sin(0.1t + 4.71)]^{\mathsf{T}}$.

1) *Other system parameters used in simulations:* system mass $m = 5$ kg, gravitational acceleration $g = 9.81\,\text{m/s}^2$, control torque lever arm $\ell = 0.5$ m, upper bound for θ_a, $\delta_a = 5\,\text{m}{\cdot}\text{s}^{-2}$, upper bound for θ_b, $\delta_b = 3\,\text{N}{\cdot}\text{m}$, air density $\rho = 1.2\,\text{kg/m}^3$, and duct cross-sectional area $A = 0.114\,\text{m}^2$ (corresponds to 15 in duct inner diameter).
2) *Initial conditions used in simulations:* $p(0) = \text{col}[50, 10, 0]$ m, $v(0) = \text{col}\,[5, 0, 0.5]$ m/s, $Q(0) = (q_{d0}(0), q_d(0)) = (0, 1, 0, 0)$, and $\Omega(0) = \text{col}\,[0, 0, 0]$.
3) *Control/adaptation gains for controllers 1 and 2:* $k_p = 1$, $k_v = 0.1$, $\Gamma_v = \text{diag}\,[0.2, 0.2, 0.8]$, $k_\theta = 1$, $\gamma_q = 10$, $K_q = 20 I_{3\times 3}$, $K_\omega = 20 I_{3\times 3}$, $\gamma_{\theta_1} = 0.2$, and $\gamma_{\theta_2} = \gamma_{\theta_3} = 1$.

The simulation results are given by Figs. 3–11. Although the second controller is easier to implement, in situations where the system initial conditions are sufficiently far from the desired trajectory, some control gains are required to have extremely large values [as specified by requirements (64)–(68)]. These requirements are likely conservative, and simulations suggest that the domain of attraction is actually larger than the region specified by (68).

[2] In addition to generating the thrust T along the body-referenced vertical axis $\hat{z}$, the change in momentum of the airflow (due to the system rotors/propellers) can cause an additional force when the external duct airflow velocity has a component, which is orthogonal to the thrust vector $T\hat{z}$. This force (which is also orthogonal to the thrust vector) is caused due to the deceleration of the horizontal component of the airflow, and is known as the *ram drag*. For further information, see [23].

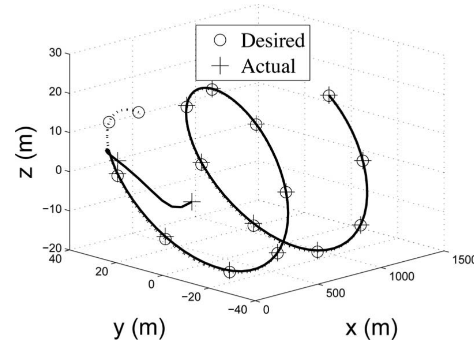

Fig. 3. Three-dimensional plot of desired versus actual system trajectory for controller 1.

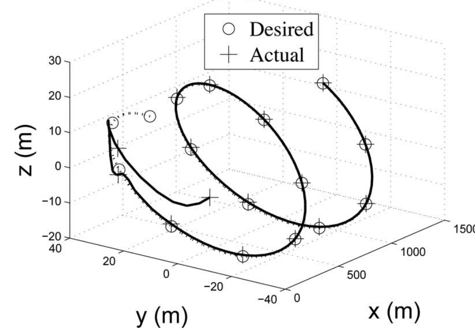

Fig. 4. Three-dimensional plot of desired versus actual system trajectory for controller 2.

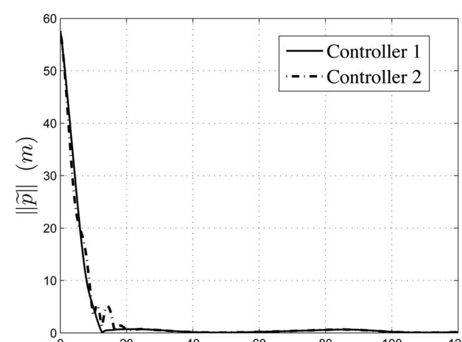

Fig. 5. Norm of position error $\|\tilde{p}\|$.

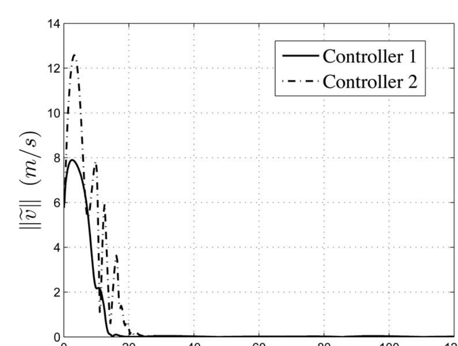

Fig. 6. Norm of velocity error $\|\tilde{v}\|$.

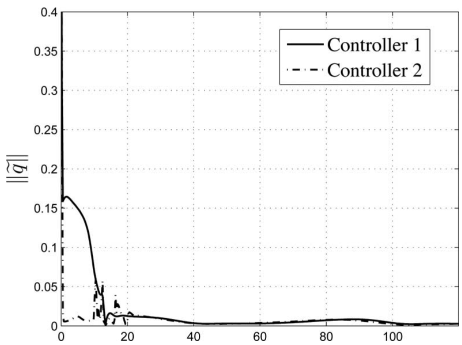

Fig. 7. Norm of attitude error $\|\tilde{q}\|$.

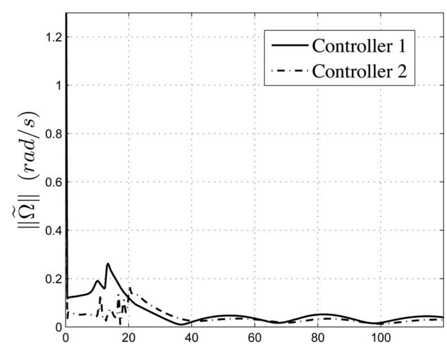

Fig. 8. Norm of angular velocity error $\|\tilde{\Omega}\|$.

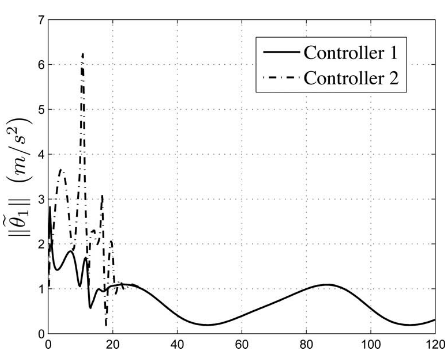

Fig. 9. Norm of estimation error $\|\tilde{\theta}_1\|$.

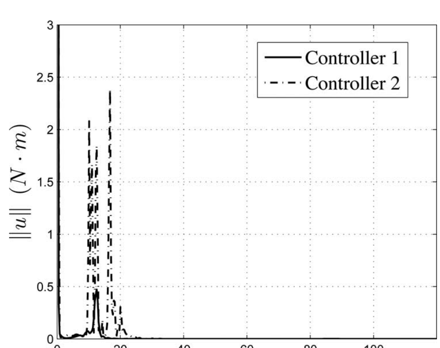

Fig. 10. Norm of control torque $\|u\|$.

The simulations show that both controllers are successful in forcing the system to the desired trajectory. For each case, the system attitude was initialized at $Q = (0, 1, 0, 0)$, which corresponds to the system being completely inverted. This is done to demonstrate the effectiveness of the control laws for extreme deviations in the system attitude. The control laws were effective, despite the time-varying disturbance due to aerodynamic drag, the coupling term, which was omitted during the control design, and uncertainty in the inertia tensor.

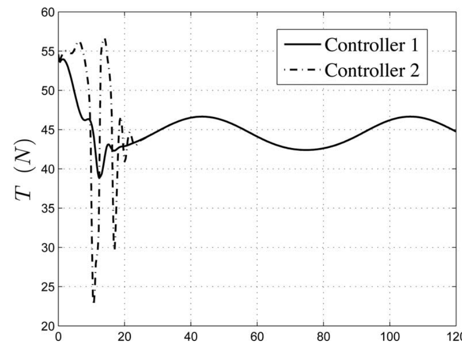

Fig. 11. System thrust T.

VII. Conclusion

Two adaptive position-tracking controllers have been proposed for a VTOL UAV in the presence of external disturbances. The second control law is included since it is somewhat less complicated to implement than the first controller, which may be desirable from a practical point of view. The two control laws are dependant on a quaternion extraction algorithm, which allows the extraction of the system attitude and thrust from the desired virtual acceleration that is required to achieve the tracking objective. This quaternion extraction method provides almost global results, with the exception of a mild singularity, which is easily avoided by using a bounded control law for the desired virtual acceleration required to meet the tracking objective. To compensate for the external disturbances while ensuring the bound of the required acceleration, projection is used in the adaptive estimation algorithms. Although this improves the robustness of the proposed controllers, this method requires the use of an overparameterization of the parameters estimates.

Simulations were performed for the two proposed controllers. In both cases, the controllers successfully achieved the tracking objective even in the presence of the coupling term that is omitted in the control design, time-varying disturbance due to aerodynamic drag and uncertainty in the inertia tensor of the system. Due to the time-varying nature of the disturbances, a slight error was observed in the simulations. The implementation of the second controller was less complicated; however, in general, high gains were required to guarantee stability, although simulations suggest that the stability requirements are quite conservative. Still, even when using reasonable gains, the second controller was shown to be effective.

There is also potential for improvement due to the open problem related to the coupling term that is neglected in the control design. Another improvement would be to remove the overparameterization that is required due to the projection algorithm.

APPENDIX A
PROOF OF ATTITUDE EXTRACTION

One definition of a unit quaternion as it pertains to a rotational transformation is given by $Q_d = (\cos(\delta/2), \sin(\delta/2)\hat{k})$, where δ is an angle of rotation about the normalized axis of rotation $\hat{k}$. One possible solution for the angle and axis of rotation can be found by using the scalar and vector products, i.e., $u^\mathsf{T} v = \|u\|\|v\|\cos(\delta)$, and $S(v)u = \|u\|\|v\|\sin(\delta)\hat{k}$. From the definition of the scalar product, we find $\cos(\delta) = u^\mathsf{T} v/\|u\|^2$, where we assume $\|u\| = \|v\|$. The result $\sin(\delta) = 1/\|u\|^2\sqrt{(\|u\|^2 + u^\mathsf{T} v)(\|u\|^2 - u^\mathsf{T} v)}$ follows from the fact that $\sin^2(\delta) = 1 - \cos^2(\delta)$. Applying the double angle formula $\cos(\delta) = 1 - 2\sin^2(\delta/2)$, we obtain $\sin(\delta/2) = 1/\|u\|\sqrt{\frac{\|u\|^2 - u^\mathsf{T} v}{2}}$. Subsequently, the normalized axis of rotation is given by $\hat{k} = ((\|u\|^2 + u^\mathsf{T} v)(\|u\|^2 - u^\mathsf{T} v))^{-1/2} S(v)u$. Therefore, the solution for the vector and scalar parts of the quaternion are given by

$$\hat{k}\sin(\delta/2) = \frac{1}{\|u\|}\sqrt{\frac{1}{2(\|u\|^2 + u^\mathsf{T} v)}}S(v)u$$

$$\cos(\delta/2) = \frac{\sin(\delta)}{2\sin(\delta/2)} = \frac{1}{\|u\|}\sqrt{\frac{\|u\|^2 + u^\mathsf{T} v}{2}}.$$

■

APPENDIX B
PROOF OF THEOREM 1

In the following sections, we present the proof of the control law proposed in Section V-B. The proof is completed in a number of stages. In subsection 1, we focus on the upper and lower bounds for the system thrust as a result of the proposed control law. In subsection 2, we analyze the system translational dynamics, the dynamics of the estimation error, and the dynamics of the angular velocity associated with the quaternion Q_d. The parts of the proof contained in Appendix B-1 and 2 are the same for both proposed control laws. Appendix B-3 finalizes the proof for the first proposed control law, where the proof for the second control law can be found in Appendix C. Appendix D provides derivatives of a number of functions that are necessary to implement the controller.

1) Bounded Control: The proposed control laws are dependent on an attitude extraction algorithm that encounters a singularity defined by (32). Fortunately, the singularity can be avoided if the third component of the virtual control law μ_d is bounded such that $\mu_{d_3} = \hat{z}^\mathsf{T}\mu_d < g$. Recall the expression for the signal μ_d given by (44). Due to Assumption 1, the acceleration of the reference trajectory is bounded such that $\hat{z}^\mathsf{T}\ddot{r} < \delta_{rz} < g$ and $\|\ddot{r}\| < \delta_r$, where the parameters δ_r and δ_{rz} are known *a priori*. Given the projection-based estimation law (47) and the property (43), the disturbance estimate $\hat{\theta}_1$ is bounded such that $\|\hat{\theta}_1\| < \delta_a + k_\theta + \epsilon_\alpha$. In addition, the function $h(\cdot)$, defined by (11), is bounded such that $0 \le \|h(\cdot)\| < 1$. Consequently, the signal μ_d is also bounded by

$$\|\mu_d\| < \bar{\mu}_d = k_p\|\Gamma_v^{-1}\| + \delta_r + \delta_a + 2k_\theta + k_v + \epsilon_\alpha \quad (72)$$

$$|\hat{z}^\mathsf{T}\mu_d| < \bar{\mu}_{d_3} = k_p\|\hat{z}^\mathsf{T}\Gamma_v^{-1}\| + \delta_{rz} + \delta_a + 2k_\theta + k_v + \epsilon_\alpha \quad (73)$$

where due to (45), the bound on the third component of μ_d is limited to $|\hat{z}^\mathsf{T}\mu_d| = |\mu_{d_3}| < \bar{\mu}_{d_3} < g$. As a result, we define

$$\delta_{\mu_d} = g - \bar{\mu}_{d_3} > 0. \quad (74)$$

As a result of the bounds (72)–(74), the system thrust, given by (33), is also bounded such that $\delta_{\mu_d} < u_t < \bar{u}_t$, where $\bar{u}_t = \bar{\mu}_d + g$. Therefore, the system thrust never vanishes and the singularity in the attitude and thrust extraction (32) is avoided.

2) Translational and Quaternion Dynamics: Recall the expression for the velocity error dynamics defined by (25), and let $\tilde{\theta}_1$ denote the following estimation error function:

$$\tilde{\theta}_1 = \theta_a - \hat{\theta}_1 - k_\theta h(\tilde{v}). \quad (75)$$

Given the virtual control law μ_d defined by (44) and the estimation error $\tilde{\theta}_1$, the time derivative of the velocity error can now be written as follows:

$$\begin{aligned}\dot{\tilde{v}} &= \tilde{\mu} - K_1 h(\tilde{p}) - K_2 h(\tilde{v}) - \hat{\theta}_1 + \theta_a \\ &= \tilde{\mu} - K_1 h(\tilde{p}) - k_v h(\tilde{v}) + \tilde{\theta}\end{aligned} \quad (76)$$

where $K_1 = k_p\Gamma_v^{-1}$, and $K_2 = (k_v + k_\theta) I_{3\times3}$. Furthermore, in light of Assumption 2 and the velocity error (76), the time derivative of (75) is given by

$$\dot{\tilde{\theta}}_1 = -k_\theta\phi_h(\tilde{v})\tilde{\theta}_1 + \tau_1 - \dot{\hat{\theta}}_1 \quad (77)$$

$$\tau_1 = -k_\theta\phi_h(\tilde{v})(\tilde{\mu} - K_1 h(\tilde{p}) - k_v h(\tilde{v})). \quad (78)$$

The expressions for $\dot{\tilde{v}}$ and $\dot{\tilde{\theta}}_1$ are dependant on the error function $\tilde{\mu} = \mu - \mu_d$. A more convenient notation is to express the error function $\tilde{\mu}$ in terms of the attitude error $\tilde{Q} = (\tilde{q}_0, \tilde{q}) = Q_d^{-1} \odot Q$, where Q_d is the desired attitude defined by (34) and (35). This can be achieved if we consider the rotation matrix $\tilde{R} = RR_d^\mathsf{T}$ (which corresponds to the unit quaternion $\tilde{Q}$) and the fact that $\tilde{R} = I_{3\times3} + 2S(\tilde{q})^2 - 2\tilde{q}_0 S(\tilde{q})$ to obtain

$$\tilde{\mu} = W_1^\mathsf{T}\tilde{q} \quad W_1 = -2u_t S(\bar{q})R \quad \bar{q} = S(\hat{z})\tilde{q} + \tilde{q}_0\hat{z}. \quad (79)$$

Consequently, the expressions for $\dot{\tilde{v}}$ and $\dot{\tilde{\theta}}_1$ can be written as functions of the attitude error $\tilde{q}$.

At this point, we focus our attention on the dynamics of the attitude error in terms of the quaternion scalar $\tilde{q}_0$. From (26), we note that the time derivative of $\tilde{q}_0$ is given by $\dot{\tilde{q}}_0 = 1/2\tilde{q}^\mathsf{T}(\Omega_d - \Omega)$, where from (36), we recall $\Omega_d = M(\mu_d)\dot{\mu}_d$. Differentiating (44) in light of (47) and (76), we find the derivative $\dot{\mu}_d$ to be given by

$$\begin{aligned}\dot{\mu}_d = {} & r^{(3)} + w_\beta + W_2 h(\tilde{v}) + W_3\tilde{q} + W_4\tilde{v} \\ & - (k_\theta + k_v)\phi_h(\tilde{v})\tilde{\theta}_1\end{aligned} \quad (80)$$

$$\begin{aligned}W_2 &= k_v^2\phi_h(\tilde{v}) \\ W_3 &= -\gamma_{\theta_1}\gamma_q(k_\theta + k_v)\phi_h(\tilde{v})M(\mu_d)^\mathsf{T} - k_v\phi_h(\tilde{v})W_1^\mathsf{T} \\ W_4 &= -\gamma_{\theta_1}\Gamma_v - k_p\Gamma_v^{-1}\phi_h(\tilde{p}) \\ w_\beta &= k_v k_p\phi_h(\tilde{v})\Gamma_v^{-1}h(\tilde{p}) - \gamma_{\theta_1}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2)\end{aligned} \quad (81)$$

from which we obtain the desired attitude dynamics

$$\Omega_d = M(\mu_d)(r^{(3)} + w_\beta + W_2 h(\tilde{v}) + W_3\tilde{q} + W_4\tilde{v}) - (k_\theta + k_v)M(\mu_d)\phi_h(\tilde{v})\tilde{\theta}_1. \quad (82)$$

Since Ω_d is not entirely known (due to the presence of the signal $\tilde{\theta}_1$), it is necessary to study the upper bound of the undesired terms in (82). For the most part, this analysis is straightforward except for the matrix $M(\mu_d)$. To determine an upper bound for this matrix, we apply the Frobenius norm to the expression for $M(\mu_d)$ given by (37). For convenience, we let $\xi = \mathrm{col}\,[\xi_1, \xi_2, \xi_3] = \mu_d - g\hat{z}$ and find the value of the norm to be given by $\|M(\mu_d)\|_F = \sqrt{2/(\|\xi\|^2 + \|\xi\||\xi_3|) + 1/\|\xi\|^2}$.

Since $\inf\{\|\xi\|\} = \inf\{|\xi_3|\} = \delta_{\mu_d}$, we obtain

$$\|M(\mu_d)\|_F \leq \frac{\sqrt{2}}{\delta_{\mu_d}}. \quad (83)$$

We now propose the following Lyapunov function candidate:

$$V_1 = k_p(\sqrt{1 + \tilde{p}^\mathsf{T}\tilde{p}} - 1) + \frac{1}{2}\tilde{v}^\mathsf{T}\Gamma_v\tilde{v} + 2\gamma_q(1 - \tilde{q}_0) + \frac{1}{2\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\tilde{\theta}_1. \quad (84)$$

Given (24), (26), (76)–(78), (82), and the adaptive estimation law (48), we differentiate V_1 to obtain

$$\dot{V}_1 = -k_v\tilde{v}^\mathsf{T}\Gamma_v h(\tilde{v}) - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) + \tilde{q}^\mathsf{T}(\Phi\tilde{v} + \gamma_q(\Omega - M(\mu_d)r^{(3)} - M(\mu_d)(w_\beta + W_2h(\tilde{v}) + W_3\tilde{q}))) \quad (85)$$

$$\Phi = W_1\Gamma_v - \gamma_q M(\mu_d)W_4. \quad (86)$$

3) Angular Velocity Error Dynamics—Controller 1: Recall the expression for the angular velocity error is given by $\tilde{\Omega} = \Omega - \beta_1$. Applying the virtual control law β_1, which is given by (50), to $\dot{V}_1$ defined by (85), we obtain

$$\dot{V}_1 = -k_v\tilde{v}^\mathsf{T}\Gamma_v h(\tilde{v}) - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 + \gamma_q\tilde{q}^\mathsf{T}\tilde{\Omega} - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) - \gamma_q\tilde{q}^\mathsf{T}M(\mu_d)W_2h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}(K_q + M(\mu_d)W_3)\tilde{q}. \quad (87)$$

To further simplify this result, using (14) and (83), we apply Young's inequality to obtain the following upper bound:

$$|\gamma_q\tilde{q}^\mathsf{T}M(\mu_d)W_2h(\tilde{v})| \leq \frac{\gamma_q k_v^2}{2\epsilon_1}\tilde{q}^\mathsf{T}\tilde{q} + \frac{\gamma_q k_v^2\epsilon_1}{\delta_{\mu_d}^2}\tilde{v}^\mathsf{T}h(\tilde{v}) \quad (88)$$

where $\epsilon_1 > 0$. Furthermore, due to $\|S(\bar{q})\| \leq 1$ and $u_t < \bar{u}_t = g + \bar{\mu}_d$, we also find

$$\|M(\mu_d)W_3\| \leq \frac{2\sqrt{2}k_v\bar{u}_t}{\delta_{\mu_d}} + \frac{2\gamma_{\theta_1}\gamma_q(k_\theta + k_v)}{\delta_{\mu_d}^2}. \quad (89)$$

Due to the bounds (88) and (89), the Lyapunov function derivative (87) is bounded by

$$\dot{V}_1 \leq -\tilde{v}^\mathsf{T}\Delta_v h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}\Delta_q\tilde{q} - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) + \gamma_q\tilde{q}^\mathsf{T}\tilde{\Omega}, \quad (90)$$

$$\Delta_v = k_v\left(\Gamma_v - \frac{\gamma_q k_v\epsilon_1}{\delta_{\mu_d}^2}I_{3\times3}\right) \quad (91)$$

$$\Delta_q = K_q - \left(\frac{2\sqrt{2}k_v\bar{u}_t\delta_{\mu_d} + 2\gamma_{\theta_1}\gamma_q(k_\theta + k_v)}{\delta_{\mu_d}^2} + \frac{k_v^2}{2\epsilon_1}\right)I_{3\times3}. \quad (92)$$

Provided that (57) is satisfied, then Δ_v and Δ_q are positive definite matrices. Introducing the error signals $\tilde{\theta}_2 = \theta_a - \hat{\theta}_2$ and $\tilde{\theta}_3 = \theta_b - \hat{\theta}_3$, we introduce the second Lyapunov function

$$V_2 = V_1 + \frac{1}{2}\tilde{\Omega}^\mathsf{T}I_b\tilde{\Omega} + \frac{1}{2\gamma_{\theta_2}}\tilde{\theta}_2^\mathsf{T}\tilde{\theta}_2 + \frac{1}{2\gamma_{\theta_3}}\tilde{\theta}_3^\mathsf{T}\tilde{\theta}_3 = k_p(\sqrt{1 + \tilde{p}^\mathsf{T}\tilde{p}} - 1) + \frac{1}{2}X^\mathsf{T}CX \quad (93)$$

$$X = \mathrm{col}[\tilde{v}, 1 - \tilde{q}_0, \tilde{q}, \tilde{\Omega}, \tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3] \quad (94)$$

$$C = \mathrm{diag}[\Gamma_v, 4\gamma_q I_{4\times4}, I_b, \gamma_{\theta_1}I_{3\times3}, \gamma_{\theta_2}I_{3\times3}, \gamma_{\theta_3}I_{3\times3}]. \quad (95)$$

The time derivative of (93) is subsequently found using (90), in addition to the control and estimation laws defined by (54)–(56), to obtain the following result:

$$\dot{V}_2 \leq -\tilde{v}^\mathsf{T}\Delta_v h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}\Delta_q\tilde{q} - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\Omega}^\mathsf{T}K_\omega\tilde{\Omega} - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) - \tilde{\theta}_2^\mathsf{T}\alpha(\hat{\theta}_2, \delta_a, -\bar{f}_{\beta_1}^\mathsf{T}I_b\tilde{\Omega}) - \tilde{\theta}_3^\mathsf{T}\alpha(\hat{\theta}_3, \delta_b, -R^\mathsf{T}S(\hat{z})\tilde{\Omega}).$$

Due to the property of the projection law given by (43), $\tilde{\theta}_i^\mathsf{T}\alpha > 0$, and the Lyapunov function derivative can be simplified as follows:

$$\dot{V}_2 \leq -\tilde{v}^\mathsf{T}\Delta_v h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}\Delta_q\tilde{q} - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\Omega}^\mathsf{T}K_\omega\tilde{\Omega}.$$

Therefore, $\dot{V}_2 \leq 0$ and the states $(\tilde{p}, \tilde{v}, \tilde{\Omega})$ are bounded. The attitude error $\tilde{Q}$ is bounded by definition, and the adaptive estimation error $\tilde{\theta}_{1,2,3}$ are bounded due to Assumption 2 and due to the property of the projection mechanism (43). Applying Barbalat's Lemma, $\ddot{V}_2$ is bounded due to Assumption 1, which shows that $(\tilde{p}, \tilde{v}, \tilde{q}, \tilde{\Omega}, \tilde{\theta}_1) \to 0$ as $t \to \infty$. Since $\tilde{\theta}_1 \to 0$ and $\tilde{v} \to 0$, then $\hat{\theta}_1 \to \theta_a$. In addition, since $\dot{\tilde{v}} \to 0$, and $\dot{\tilde{v}} = W_1^\mathsf{T}\tilde{q} - K_1h(\tilde{p}) - (k_v + k_\theta)h(\tilde{v}) - \hat{\theta}_1 + \theta_a = -K_1h(\tilde{p}) = 0$, then $\tilde{p} \to 0$, which satisfies the tracking objective.

APPENDIX C
PROOF OF THEOREM 2

In this section, we present the proof of the control law proposed in Section V-C. The two proposed controllers differ due to the choice of the virtual control law for the angular velocity. Consequently, the sections pertaining to the bounded control and translational and quaternion dynamics are similar to both proofs. Therefore, before proceeding further see Appendix B-1 and 2. Based on the framework outlined in Appendix B-1 and 2, the following section finalizes the proof of Theorem 2. Appendix D provides derivatives of a number of functions that are necessary to implement the controller.

Using the second control scheme, the angular velocity error is now defined as $\tilde{\Omega} = \Omega - \beta_2$, where the virtual control law β_2 is given by (61). To study the stability of the system using the second controller, we use the same Lyapunov function candidate V_1 given by (84). Using the expression for $\dot{V}_1$ given by (85), in addition to the virtual control law β_2, the bounds defined by (88) and (89), and the matrices (91) and (92), the upper bound of the $\dot{V}_1$ is now given by

$$\dot{V}_1 \leq -\frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) + \tilde{q}^\mathsf{T}\Phi\tilde{v} - \tilde{v}^\mathsf{T}\Delta_v h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}\Delta_q\tilde{q} + \gamma_q\tilde{q}^\mathsf{T}\tilde{\Omega}.$$

Using (86) in addition to (79) and (81), the expression for Φ can also be written as follows:

$$\Phi = (\gamma_{\theta_1}\gamma_q M(\mu_d) - 2u_t S(\bar{q})R)\,\Gamma_v + \gamma_q k_p M(\mu_d)\Gamma_v^{-1}\phi_h(\tilde{p}).$$

Due to bound of the matrix $M(\mu_d)$ given by (83) and the fact $u_t < \bar{u}_t = g + \bar{\mu}_d$, we find the upper bound of the matrix Φ given by

$$\|\Phi\| \leq \left(\frac{\sqrt{2}\gamma_q\gamma_{\theta_1}}{\delta_{\mu_d}} + 2\bar{u}_t\right)\|\Gamma_v\| + \frac{\sqrt{2}\gamma_q k_p}{\delta_{\mu_d}}\|\Gamma_v^{-1}\|.$$

From the definition of δ_1 given by (66), we see that $\|\Phi\| \leq \delta_1$, and therefore, using Young's inequality, we find

$$|\tilde{q}^\mathsf{T}\Phi\tilde{v}| \leq \frac{\delta_1^2}{2\epsilon_2}\tilde{q}^\mathsf{T}\tilde{q} + \frac{\epsilon_2}{2}\tilde{v}^\mathsf{T}\tilde{v}$$

for any $\epsilon_2 > 0$. Therefore, the time derivative of the Lyapunov function $\dot{V}_1$ is modified as follows:

$$\dot{V}_1 \leq -\frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a, \tau_2) + \gamma_q\tilde{q}^\mathsf{T}\tilde{\Omega} - \tilde{q}^\mathsf{T}\bar{\Delta}_q\tilde{q} - \tilde{v}^\mathsf{T}\bar{\Delta}_v\tilde{v} \quad (96)$$

where we define the matrices

$$\bar{\Delta}_q = \gamma_q\Delta_q - \frac{\delta_1^2}{2\epsilon_2}I_{3\times3}$$
$$\bar{\Delta}_v = \frac{1}{(1+\tilde{v}^\mathsf{T}\tilde{v})^{1/2}}\Delta_v - \frac{\epsilon_2}{2}I_{3\times3}. \quad (97)$$

Using the two estimation error functions $\tilde{\theta}_2 = \theta_a - \hat{\theta}_2$ and $\tilde{\theta}_3 = \theta_b - \hat{\theta}_3$, we introduce the following Lyapunov function:

$$V_2 = V_1 + \frac{1}{2}\tilde{\Omega}^\mathsf{T}I_b\tilde{\Omega} + \frac{1}{2\gamma_{\theta_2}}\tilde{\theta}_2^\mathsf{T}\tilde{\theta}_2 + \frac{1}{2\gamma_{\theta_3}}\tilde{\theta}_3^\mathsf{T}\tilde{\theta}_3 = k_p(\sqrt{1+\tilde{p}^\mathsf{T}\tilde{p}} - 1) + \frac{1}{2}X^\mathsf{T}CX \quad (98)$$

where X and C are given by (94) and (95), respectively. In light of (96), the estimation laws (55) and (56) and the control law (63), we find the following upper bound for $\dot{V}_2$:

$$\dot{V}_2 \leq -\tilde{v}^\mathsf{T}\bar{\Delta}_v h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}\bar{\Delta}_q\tilde{q} - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\Omega}^\mathsf{T}K_\omega\tilde{\Omega} - \tilde{\theta}_1^\mathsf{T}\alpha(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) - \tilde{\theta}_2^\mathsf{T}\alpha(\hat{\theta}_2, \delta_a, -\bar{f}_{\beta_2}^\mathsf{T}I_b\tilde{\Omega}) - \tilde{\theta}_3^\mathsf{T}\alpha(\hat{\theta}_3, \delta_b, -R^\mathsf{T}S(\hat{z})\tilde{\Omega}).$$

Due to the property of the projection law (43), $\tilde{\theta}_i^\mathsf{T}\alpha > 0$, and

$$\dot{V}_2 \leq -\tilde{v}^\mathsf{T}\bar{\Delta}_v h(\tilde{v}) - \gamma_q\tilde{q}^\mathsf{T}\bar{\Delta}_q\tilde{q} - \frac{k_\theta}{\gamma_{\theta_1}}\tilde{\theta}_1^\mathsf{T}\phi_h(\tilde{v})\tilde{\theta}_1 - \tilde{\Omega}^\mathsf{T}K_\omega\tilde{\Omega}.$$

Given the requirements (64) and (65) are satisfied, then $\bar{\Delta}_q > 0$ and $\Delta_v > 0$. However, in light of (97), to ensure that $\bar{\Delta}_v > 0$, we must also satisfy the following inequality:

$$\|\tilde{v}\|^2 < \frac{4}{\epsilon_2^2}\|\Delta_v\|^2 - 1.$$

Due to the definition of the Lyapunov function (98), the following inequality is always satisfied:

$$\frac{1}{2}\lambda_{\min}(\Gamma_v)\|\tilde{v}\|^2 \leq V_2 \leq k_p(\sqrt{1+\tilde{p}^\mathsf{T}\tilde{p}} - 1) + X^\mathsf{T}\bar{C}X$$

where $\bar{C}$ is given by (71), which we can further simplify to obtain

$$\|\tilde{v}\|^2 \leq 2\lambda_{\min}(\Gamma_v)^{-1}(k_p(\sqrt{1+\tilde{p}^\mathsf{T}\tilde{p}} - 1) + X^\mathsf{T}\bar{C}X).$$

Therefore, to ensure $\dot{V}_2 \leq 0$, it is sufficient to have

$$2\lambda_{\min}(\Gamma_v)^{-1}(k_p(\sqrt{1+\tilde{p}(0)^\mathsf{T}\tilde{p}(0)} - 1) + X(0)^\mathsf{T}\bar{C}X(0)) < \frac{4}{\epsilon_2^2}\|\Delta_v\|^2 - 1$$

which is satisfied due to (68), and consequently, $\bar{\Delta}_v$ is positive definite. Therefore, $\dot{V}_2 \leq 0$ and the states $(\tilde{p}, \tilde{v}, \tilde{\Omega})$ are bounded. The attitude error $\tilde{Q}$ is bounded by definition, and the adaptive estimation error $\tilde{\theta}_{1,2,3}$ are bounded due to Assumption 2 and due to the property of the projection mechanism (43). Applying Barbalat's lemma, $\ddot{V}_2$ is bounded due to Assumption 1, which shows that $(\tilde{p}, \tilde{v}, \tilde{q}, \tilde{\Omega}, \tilde{\theta}_1) \to 0$ as $t \to \infty$. Since $\tilde{\theta}_1 \to 0$ and $\tilde{v} \to 0$, then $\hat{\theta}_1 \to \theta_a$. In addition, since $\dot{\tilde{v}} \to 0$, and $\dot{\tilde{v}} = W_1^\mathsf{T}\tilde{q} - K_1 h(\tilde{p}) - K_2 h(\tilde{v}) - \hat{\theta}_1 + \theta_a = -K_1 h(\tilde{p}) = 0$, then $\tilde{p} \to 0$, which satisfies the tracking objective.

Appendix D

Derivatives of Angular Velocity Virtual Control Laws β_2 and β_1

In this section, we obtain the derivatives of the two virtual control laws for the angular velocity β_1 and β_2, which are given by (50) and (61), respectively. Due to the complexity of the virtual control laws, we begin by evaluating the derivatives of several signals before continuing to the derivatives of the virtual control laws. We first focus on the partial derivative of the matrix $M(\mu_d)$ and its transpose. Let $\mu_d = \mathrm{col}\,[\mu_{d_1}, \mu_{d_2}, \mu_{d_3}]$ and

$v = \mathrm{col}[v_1, v_2, v_3]$ denote two arbitrary vectors, and define the functions $Z_1, Z_2 : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}^3$ such that

$$Z_1(\mu_d, v) := \frac{\partial}{\partial \mu_d} M(\mu_d) v \quad Z_2(\mu_d, v) := \frac{\partial}{\partial \mu_d} M(\mu_d)^\mathsf{T} v.$$

From the definition of $M(\mu_d)$ given by (37), after some straightforward albeit tedious calculations, we evaluate these functions to be

$$Z_1(\mu_d, v) = \gamma_M^{-1} M(\mu_d) v f_\gamma + \gamma_M \Lambda_1(\mu_d, v)$$
$$Z_2(\mu_d, v) = \gamma_M^{-1} M(\mu_d)^\mathsf{T} v f_\gamma + \gamma_M \Lambda_2(\mu_d, v)$$
$$f_\gamma = \gamma_M^2 (g\hat{z} - \mu_d)^\mathsf{T} (3c_1 I_{3\times3} + S(\hat{z}) S(g\hat{z} - \mu_d))$$

where $\gamma_M = \|\mu_d - g\hat{z}\|^{-2} c_1^{-1}$, $c_1 = \|\mu_d - g\hat{z}\| + g - \mu_{d_3}$, and

$$\Lambda_1 = \begin{pmatrix} c_1 v_2 \\ -c_1 v_1 \\ \mu_{d_2} v_1 - \mu_{d_1} v_2 \end{pmatrix} \alpha_1(\mu_d)^\mathsf{T} + \begin{pmatrix} -\mu_{d_2} v_1 & -\mu_{d_1} v_1 - 2v_2 \mu_{d_2} + c_1 v_3 & 0 \\ 2v_1 \mu_{d_1} + v_2 \mu_{d_2} - c_1 v_3 & \mu_{d_1} v_2 & 0 \\ -u_t v_2 & u_t v_1 & 0 \end{pmatrix}$$
$$+ \begin{pmatrix} u_t v_2 + \mu_{d_2} v_3 & -u_t v_1 - \mu_{d_1} v_3 & 0 \end{pmatrix}^\mathsf{T} \alpha_2(\mu_d)^\mathsf{T},$$
$$\Lambda_2 = \begin{pmatrix} 2v_2 \mu_{d_1} - \mu_{d_2} v_1 & u_t v_3 - \mu_{d_1} v_1 & 0 \\ \mu_{d_2} v_2 - u_t v_3 & \mu_{d_1} v_2 - 2\mu_{d_2} v_1 & 0 \\ -c_1 v_2 & c_1 v_2 & 0 \end{pmatrix} + \begin{pmatrix} \mu_{d_2} v_3 - c_1 v_2 \\ c_1 v_1 - \mu_{d_1} v_3 \\ 0 \end{pmatrix} \alpha_1(\mu_d)^\mathsf{T}$$
$$+ \begin{pmatrix} -u_t v_2 & u_t v_1 & \mu_{d_2} v_1 - \mu_{d_1} v_2 \end{pmatrix}^\mathsf{T} \alpha_2(\mu_d)^\mathsf{T}$$

with $\alpha_1 = (\mu_d - g\hat{z}) / \|\mu_d - g\hat{z}\|$ and $\alpha_2 = \alpha_1 - \hat{z}$.

In order to obtain the derivative of the projection law α, we first focus on obtaining the derivative of the signal τ_2. Leading up to this goal, we first differentiate several signals. Due to the unknown parameter θ_a, in general, we group the derivative of an arbitrary signal x into known and unknown components as $\dot{x} = f_x + \bar{f}_x \theta_a$.

Recall the expression for the signal $\dot{\mu}_d$ given by (80). This result can also be written as $\dot{\mu}_d = f_{\mu_d} + \bar{f}_{\mu_d} \theta_a$, where the functions f_{μ_d} and $\bar{f}_{\mu_d}$ are given by

$$f_{\mu_d} = r^{(3)} + w_\beta + (W_2 + k_\theta K_2 \phi_h(\tilde{v})) h(\tilde{v}) + W_3 \tilde{q} + W_4 \tilde{v} + K_2 \phi_h(\tilde{v}) \hat{\theta}_1$$
$$\bar{f}_{\mu_d} = -K_2 \phi_h(\tilde{v})$$

where $K_2 = (k_\theta + k_v) I_{3\times3}$. Similarly, in light of (26), the derivative of $\tilde{q}$ can be written as $\dot{\tilde{q}} = f_{\tilde{q}} + \bar{f}_{\tilde{q}} \theta_a$ using the following expressions:

$$f_{\tilde{q}} = \frac{1}{2} (\tilde{q}_0 I + S(\tilde{q})) \Omega + \frac{1}{2} (S(\tilde{q}) - \tilde{q}_0 I) (M(\mu_d) f_{\mu_d})$$
$$\bar{f}_{\tilde{q}} = \frac{1}{2} (S(\tilde{q}) - \tilde{q}_0 I) \left(M(\mu_d) \bar{f}_{\mu_d} \right).$$

From the definition of $\tilde{\mu}$ given by (18), the derivative $\dot{\tilde{\mu}} = f_{\tilde{\mu}} + \bar{f}_{\tilde{\mu}} \theta_a$ is obtained, where

$$f_{\tilde{\mu}} = -(I + u_t^{-1} R^\mathsf{T} \hat{z} (\mu_d - g\hat{z})^\mathsf{T}) f_{\mu_d} - u_t R^\mathsf{T} S(\Omega) \hat{z}$$
$$\bar{f}_{\tilde{\mu}} = -(I + u_t^{-1} R^\mathsf{T} \hat{z} (\mu_d - g\hat{z})^\mathsf{T}) \bar{f}_{\mu_d}.$$

The expression for $\dot{\tilde{v}}$, previously given by (76), can also be given by $\dot{\tilde{v}} = f_{\tilde{v}_1} + \tilde{\theta}_1 = f_{\tilde{v}_2} + \theta_a$, where the functions $f_{\tilde{v}_1}$ and $f_{\tilde{v}_2}$ are given by

$$f_{\tilde{v}_1} = -K_1 h(\tilde{p}) - k_v h(\tilde{v}) + \tilde{\mu}$$
$$f_{\tilde{v}_2} = -K_1 h(\tilde{p}) - K_2 h(\tilde{v}) + \tilde{\mu} - \hat{\theta}_1.$$

Since we require the derivative of the signal $f_{\tilde{v}_1}$, we also find $\dot{f}_{\tilde{v}_1} = f_{f_{\tilde{v}}} + \bar{f}_{f_{\tilde{v}}}$, where

$$f_{f_{\tilde{v}}} = -K_1 \phi_h(\tilde{p}) \tilde{v} - k_v \phi_h(\tilde{v}) f_{\tilde{v}_2} + f_{\tilde{\mu}}$$
$$\bar{f}_{f_{\tilde{v}}} = -k_v \phi_h(\tilde{v}) + \bar{f}_{\tilde{\mu}}.$$

At this point we require the derivative of the signal τ_2, given by (48). Using the partial derivative of $\phi(u)$ given by (13), we obtain $\dot{\tau}_2 = f_{\tau_2} + \bar{f}_{\tau_2} \theta_a$, where

$$\begin{aligned} f_{\tau_2} = {} & \Gamma_v f_{\tilde{v}_2} - \frac{k_\theta}{\gamma_{\theta_1}} (f_{\phi_h}(\tilde{v}, f_{\tilde{v}_1}) f_{\tilde{v}_2} - \phi_h(\tilde{v}) K_1 \phi_h(\tilde{p}) \tilde{v} \\ & - k_v \phi_h(\tilde{v})^2 f_{\tilde{v}_2} + \phi_h(\tilde{v}) f_{\tilde{\mu}}) \\ & + \gamma_q (f_{\phi_h}(\tilde{v}, K_2 M(\mu_d)^\mathsf{T} \tilde{q}) f_{\tilde{v}_2} + \phi_h(\tilde{v}) K_2 Z_2(\mu_d, \tilde{q}) f_{\mu_d} \\ & + \phi_h(\tilde{v}) K_2 M(\mu_d)^\mathsf{T} f_{\tilde{q}}) \end{aligned} \tag{99}$$

$$\begin{aligned} \bar{f}_{\tau_2} = {} & \Gamma_v - \frac{k_\theta}{\gamma_{\theta_1}} (f_{\phi_h}(\tilde{v}, f_{\tilde{v}_1}) - k_v^2 \phi_h(\tilde{v})^2 + \phi_h(\tilde{v}) \bar{f}_{\tilde{\mu}}) \\ & + \gamma_q (f_{\phi_h}(\tilde{v}, K_2 M(\mu_d)^\mathsf{T} \tilde{q}) + \phi_h(\tilde{v}) K_2 Z_2(\mu_d, \tilde{q}) \bar{f}_{\mu_d} \\ & + \phi_h(\tilde{v}) K_2 M(\mu_d)^\mathsf{T} \bar{f}_{\tilde{q}}). \end{aligned} \tag{100}$$

In light of the work presented in [16] and using the aforementioned derivatives, we now differentiate the projection law α. Using the projection algorithm defined by (39)–(42), in addition to the derivative of τ_2 as defined by (99) and (100), we obtain $\dot{\alpha}(\hat{\theta}_1, \delta_a + k_\theta, \tau_2) = f_\alpha + \bar{f}_\alpha \theta_a$, where the functions f_α and $\bar{f}_\alpha$ are given by

$$\begin{aligned} f_\alpha = {} & -k_\alpha \dot{\eta}_1 \eta_2 \hat{\theta}_1 - k_\alpha \eta_1 \eta_2 \dot{\hat{\theta}}_1 \\ & - k_\alpha \eta_1 \frac{\eta_2}{\eta_2 - \hat{\theta}_1 \tau_2} (\tau_2^\mathsf{T} \dot{\hat{\theta}}_1 + \hat{\theta}_1^\mathsf{T} f_{\tau_2}) \hat{\theta}_1 \end{aligned}$$
$$\bar{f}_\alpha = -k_\alpha \eta_1 \frac{\eta_2}{\eta_2 - \hat{\theta}_1^\mathsf{T} \tau_2} \hat{\theta}_1 \hat{\theta}_1^\mathsf{T} \bar{f}_{\tau_2}$$
$$\dot{\eta}_1 = \begin{cases} 4(\hat{\theta}_1^\mathsf{T} \hat{\theta}_1 - \theta_0^2) \hat{\theta}_1^\mathsf{T} \dot{\hat{\theta}}_1, & \text{if } \|\hat{\theta}_1\|^2 > \theta_0^2 \\ 0, & \text{otherwise.} \end{cases}$$

Having obtained the derivative of α, we differentiate the signal w_β given by (51) to obtain $\dot{w}_\beta = f_{w_\beta} + \bar{f}_{w_\beta} \theta_a$ with

$$\begin{aligned} f_{w_\beta} = {} & k_p k_v f_{\phi_h} \left(\tilde{v}, \Gamma_v^{-1} h(\tilde{p}) \right) f_{\tilde{v}_2} + k_p k_v \phi_h(\tilde{v}) \Gamma_v^{-1} \phi_h(\tilde{p}) \tilde{v} \\ & - \gamma_{\theta_1} f_\alpha \end{aligned}$$
$$\bar{f}_{w_\beta} = k_p k_v f_{\phi_h} \left(\tilde{v}, \Gamma_v^{-1} h(\tilde{p}) \right) - \gamma_{\theta_1} \bar{f}_\alpha.$$

Using the expression for $\bar{q}$ given by (49), we also find $\dot{\bar{q}} = f_{\bar{q}} + \bar{f}_{\bar{q}}\theta_a$, where

$$f_{\bar{q}} = S(\hat{z})f_{\tilde{q}} + \frac{1}{2}\hat{z}\,\tilde{q}^{\mathsf{T}}M(\mu_d)f_{\mu_d} - \frac{1}{2}\hat{z}\,\tilde{q}^{\mathsf{T}}\Omega$$

$$\bar{f}_{\bar{q}} = S(\hat{z})\bar{f}_{\tilde{q}} + \frac{1}{2}\hat{z}\,\tilde{q}^{\mathsf{T}}M(\mu_d)\bar{f}_{\mu_d}.$$

In light of the earlier results, we finally obtain the derivative of the virtual control law for the second controller $\dot{\beta}_2 = f_{\beta_2} + \bar{f}_{\beta_2}\theta_a$, where

$$f_{\beta_2} = Z_1(\mu_d, r^{(3)} + w_\beta)f_{\mu_d} + M(\mu_d)(r^{(4)} + f_{w_\beta}) - K_q f_{\tilde{q}} \tag{101}$$

$$\bar{f}_{\beta_2} = Z_1(\mu_d, r^{(3)} + w_\beta)\bar{f}_{\mu_d} + M(\mu_d)\bar{f}_{w_\beta} - K_q\bar{f}_{\tilde{q}} \tag{102}$$

which is used to specify the derivative of the virtual control law for the first controller $\dot{\beta}_1 = f_{\beta_1} + \bar{f}_{\beta_1}\theta_a$, where

$$\begin{aligned} f_{\beta_1} &= f_{\beta_2} - \gamma_{\theta_1} Z_1(\mu_d, \Gamma_v\tilde{v})\,f_{\mu_d} - \gamma_{\theta_1} M(\mu_d)\Gamma_v f_{\tilde{v}_2} \\ &\quad - k_p Z_1(\mu_d, \Gamma_v^{-1}\phi_h(\tilde{p})\tilde{v})f_{\mu_d} - k_p M(\mu_d)\Gamma_v^{-1} f_{\phi_h}(\tilde{p},\tilde{v})\tilde{v} \\ &\quad - k_p M(\mu_d)\Gamma_v^{-1}\phi_h(\tilde{p})f_{\tilde{v}_2} \\ &\quad + \frac{2}{\gamma_q u_t}S(\bar{q})R\Gamma_v\tilde{v}\,(\mu_d - g\hat{z})^{\mathsf{T}} f_{\mu_d} - \frac{2u_t}{\gamma_q}S(R\Gamma_v\tilde{v})f_{\bar{q}} \\ &\quad - \frac{2u_t}{\gamma_q}S(\bar{q})S(\Omega)R\Gamma_v\tilde{v} + \frac{2u_t}{\gamma_q}S(\bar{q})R\Gamma_v f_{\tilde{v}_2} \end{aligned} \tag{103}$$

$$\begin{aligned} \bar{f}_{\beta_1} &= \bar{f}_{\beta_2} - \gamma_{\theta_1} Z_1(\mu_d, \Gamma_v\tilde{v})\,\bar{f}_{\mu_d} - \gamma_{\theta_1} M(\mu_d)\Gamma_v \\ &\quad - k_p Z_1\left(\mu_d, \Gamma_v^{-1}\phi_h(\tilde{p})\tilde{v}\right)\bar{f}_{\mu_d} - k_p M(\mu_d)\Gamma_v^{-1}\phi_h(\tilde{p}) \\ &\quad + \frac{2}{\gamma_q u_t}S(\bar{q})R\Gamma_v\tilde{v}\,(\mu_d - g\hat{z})^{\mathsf{T}}\bar{f}_{\mu_d} - \frac{2u_t}{\gamma_q}S(R\Gamma_v\tilde{v})\bar{f}_{\bar{q}} \\ &\quad + \frac{2u_t}{\gamma_q}S(\bar{q})R\Gamma_v. \end{aligned} \tag{104}$$

References

[1] A. Roberts and A. Tayebi, "Adaptive position tracking of vtol uavs," in *Proc. 48th IEEE Conf. Decision Control and 28th Chin. Control Conf.,*, Shanghai, China, 2009.

[2] A. Tayebi, "Unit quaternion-based output feedback for the attitude tracking problem," *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1516–1520, Jul. 2008.

[3] M. D. Hua, P. Morin, and C. Samson, "Balanced-force-control of underactuated thrust-propelled vehicles," in *Proc. 46th IEEE Conf. Decision Control*, Dec.12–14, 2007, pp. 6435–6441.

[4] M. D. Hua, T. Hamel, P. Morin, and C. Samson, "Control of thrust-propelled underactuated vehicles," INRIA, Paris, France, Tech. Rep. 6453, Feb. 2008.

[5] J. Pflimlin, P. Soueres, and T. Hamel, "Position control of a ducted fan vtol uav in crosswind," *Int. J. Control*, vol. 80, no. 5, pp. 666–683, 2007.

[6] A. P. Aguiar and J. P. Hespanha, "Position tracking of underactuated vehicles," in *Proc. Amer. Control Conf.*, Jun. 4–6, 2003, vol. 3, pp. 1988–1993.

[7] E. Frazzoli, M. A. Dahleh, and E. Feron, "Trajectory tracking control design for autonomous helicopters using a backstepping algorithm," in *Proc. Amer. Control Conf.*, Jun. 28–30, 2000, vol. 6, pp. 4102–4107.

[8] E. N. Johnson and M. A. Turbe, "Modeling, control, and flight testing of a small ducted fan aircraft," *AIAA J. Guidance, Control, Dynamics*, vol. 29, no. 4, pp. 769-779, Jul./Aug. 2006.

[9] J. Hauser, S. Sastry, and G. Meyer, "Nonlinear control design for slightly non-minimum phase systems: Application to v/stol aircraft," *Automatica*, vol. 28, no. 4, pp. 665–679, 1992.

[10] R. Olfati-Saber, "Global configuration stabilization for the vtol aircraft with strong input coupling," *IEEE Trans. Autom. Control*, vol. 47, no. 11, pp. 1949–1952, Nov. 2002.

[11] T. Hamel, R. Mahony, R. Lozano, and J. Ostrowski, "Dynamic modelling and configuration stabilization for an X4-flyer," in *Proc. 15th Triennial World Cong.*, Barcelona, Spain, 2002.

[12] A. Tayebi and S. McGilvray, "Attitude stabilization of a vtol quadrotor aircraft," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 3, pp. 562–571, May 2006.

[13] A. Abdessameud and A. Tayebi, "Global trajectory tracking control of vtol-uavs without linear velocity measurements," *Automatica*, vol. 46, pp. 1053–1059, 2010.

[14] R. Reynolds, "Quaternion parameterization and a simple algorithm for global attitude estimation," *AIAA J. Guid., Control, Dyn.*, vol. 21, no. 4, 1998.

[15] I. P., J. Sun, *Robust Adaptive Control*. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[16] Z. Cai, M. S. de Queiroz, and D. M. Dawson, "A sufficiently smooth projection operator," *IEEE Trans. Autom. Control*, vol. 51, no. 1, pp. 135–139, Jan. 2006.

[17] M. Shuster, "A survey of attitude representations," *J. Astronautical Sci.*, vol. 41, no. 4, pp. 439–517, 1993.

[18] R. Murray, Z. Li, and S. Sastry, *A Mathematical Introduction to Robotic Manipulation*. Boca Raton, FL: CRC, 1994.

[19] P. Hughes, *Spacecraft Attitude Dynamics*. Hoboken, NJ: Wiley, 1986.

[20] R. Marino and P. Tomei, "Robust adaptive state-feedback tracking for nonlinear systems," *IEEE Trans. Autom. Control*, vol. 43, no. 1, pp. 84–89, Jan. 1998.

[21] S. Bhat and D. Bernstein, "A topological obstruction to continuous global stabilization of rotational motion and the unwinding phenomenon," *Syst. Control Lett.*, vol. 39, no. 1, pp. 63–70, 2000.

[22] C. Mayhew, R. Sanfelice, and A. Teel, "Robust global asymptotic attitude stabilization of a rigid body by quaternion-based hybrid feedback," in *Proc. 48th IEEE Conf. Decision Control 28th Chin. Control Conf.*, 2009, pp. 2522–2527.

[23] A. Ko, O. Ohanian, and P. Gelhausen, "Ducted fan UAV modeling and simulation in preliminary design," in *Proc. AIAA Model. Simulation Technol. Conf. Exhibit*, 2007, AIAA Paper 6375.

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