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Brief paper

Adaptive iterative learning control for robot manipulators☆

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Abstract

In this paper, we propose some adaptive iterative learning control (ILC) schemes for trajectory tracking of rigid robot manipulators, with unknown parameters, performing repetitive tasks. The proposed control schemes are based upon the use of a proportional-derivative (PD) feedback structure, for which an iterative term is added to cope with the unknown parameters and disturbances. The control design is very simple in the sense that the only requirement on the PD and learning gains is the positive definiteness condition and the bounds of the robot parameters are not needed. In contrast to classical ILC schemes where the number of iterative variables is generally equal to the number of control inputs, the second controller proposed in this paper uses just two iterative variables, which is an interesting fact from a practical point of view since it contributes considerably to memory space saving in real-time implementations. We also show that it is possible to use a single iterative variable in the control scheme if some bounds of the system parameters are known. Furthermore, the resetting condition is relaxed to a certain extent for a certain class of reference trajectories. Finally, simulation results are provided to illustrate the effectiveness of the proposed controllers.
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Keywords: Iterative learning control; Adaptive control; Robot manipulators

1. Introduction

For the sake of implementation simplicity, the use of classical linear controllers such as PD and PID in robotics applications has attracted researchers and industrials for many decades. In fact, a PD controller with gravity compensation is able to globally asymptotically stabilize the joint positions of rigid robot manipulators at a given set-point as long as the gravity forces can be instantaneously evaluated or at least known at the desired final configuration (Takegaki & Arimoto, 1981; Spong & Vidyasagar, 1989; Tomei, 1991). This condition is not easy to satisfy in practical situations because the gravity terms generally depend on the unknown and possibly time-varying payloads manipulated by the robot during a given task. Without compensating for the gravity forces, the PD control scheme leads to a steady-state error, which can eventually be reduced by increasing the proportional and derivative gains (high-gain feedback) or by introducing an integral action. The drawback of the high-gain feedback solution is related to the fact that it may saturate the joint actuators or/and excite high-frequency modes. On the other hand, with the PID control scheme only local asymptotic stability was proven under some relatively complex conditions until the introduction of the passivity property for robot manipulators, which allowed to design globally asymptotically stabilizing PID controllers without gravity compensation (Arimoto, 1996). Again this result requires the control gains to satisfy some relatively complex relations involving particularly the robot manipulator dynamics. Besides that, owing to the physical property that the robot parameters appear linearly in the Lagrange equation, another interesting approach has been derived for trajectory tracking instead of set-point regulation (Slotine & Li, 1987; Slotine & Li, 1991). This approach consists basically of a PD controller with an additional term generated by an appropriate adaptive rule in order to cope with the unknown parameters assumed to be time-invariant.

Since robot manipulators are generally used in repetitive tasks, one should take advantage of the fact that the

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reference trajectory is repeated over a given operation time. In this context, ILC techniques can be applied in order to enhance the tracking performance from operation to operation. Since the early works of Arimoto, Kawamura, and Miyazaki (1984), Casalino and Bartolini (1984) and Craig (1984), several ILC schemes for robot manipulators have been proposed in the literature, see for instance Arimoto (1996), Bondi, Casalino, and Gambardella (1988), De Luca, Paesano, and Ulivi (1992), Horowitz (1993), Kavli (1992), Kawamura, Miyazaki, and Arimoto (1988) and Moon, Doh, and Chung (1997). These ILC algorithms, whether developed for the linearized model or the nonlinear model, are generally based upon the contraction mapping theory and require a certain a priori knowledge of the system dynamics.

On the other hand, another type of ILC algorithms has been developed using Lyapunov and Lyapunov-like methods. In fact, in French and Rogers (2000) a standard Lyapunov design is used to solve ILC problems. The idea consists of using a standard adaptive controller and to start the parameter estimates with their final values obtained at the preceding iteration. In the same spirit, Choi and Lee (2000) proposed an adaptive ILC for uncertain robot manipulators, where the uncertain parameters are estimated along the time horizon, whereas the repetitive disturbances are compensated along the iteration horizon. However, as in standard adaptive control design, this technique requires the unknown system parameters to be constant. In Kuc, Nam and Lee (1991), Ham, Qu, and Kaloust (1995), Ham, Qu, and Johnson (2000), Xu (2002), Xu, Badrinath, and Qu (2000), Xu and Tan (2001), several ILC algorithms have been proposed based upon the use of a positive definite Lyapunov-like sequence which is made monotonically decreasing along the iteration axis via a suitable choice of the control input. In contrast to the standard adaptive control, this technique is shown to be able to handle systems with time-varying parameters since the adaptation law in this case is nothing else but a discrete integration along the iteration axis. Based on this approach, Kuc et al. (1991) proposed an ILC scheme for the linearized robot manipulator model, while in Ham et al. (2000), Xu et al. (2000) nonlinear ILC schemes have been proposed for the nonlinear model. Again these control laws require a certain a priori knowledge of the system dynamics.

In this paper, we present three simple ILC schemes for the position tracking problem of rigid robot manipulators. The control schemes are built around a classical PD feedback structure, for which an iterative term is added in order to cope with the unknown parameters and disturbances. The convergence of the tracking error to zero, over the whole finite time-interval, is guaranteed when the iteration number tends to infinity. The proof of convergence is based upon the use of a Lyapunov-like positive definite sequence, which is made monotonically decreasing through an adequate choice of the control law and the iterative adaptation rule. In this framework, the acceleration measurements and the bounds of the robot parameters are not needed and the only requirement on the control gains is the positive definiteness condition. On the other hand, one of the proposed controllers uses just two iterative variables, which is an interesting advantage from a practical point of view. We also show that it is possible to bring down the number of iterative variables to one, at the expense of the knowledge of some bounds of the system parameters. Furthermore, the resetting condition is relaxed to a certain extent for a certain class of reference trajectories. Finally, simulation results are provided to illustrate the effectiveness of the proposed controllers.

2. Problem formulation

Using the Lagrangian formulation, the equations of motion of a n degrees-of-freedom rigid manipulator may be expressed by

$$M(q_k(t))\ddot{q}_k(t) + C(q_k(t),\dot{q}_k(t))\dot{q}_k(t) + G(q_k(t)) = \tau_k(t) + d_k(t), \quad (1)$$

where t denotes the time and the nonnegative integer $k \in \mathbb{Z}_+$ denotes the operation or iteration number. The signals $q_k \in \mathbb{R}^n$, $\dot{q}_k \in \mathbb{R}^n$ and $\ddot{q}_k \in \mathbb{R}^n$ are the joint position, joint velocity and joint acceleration vectors, respectively, at the iteration k. $M(q_k) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q_k,\dot{q}_k)\dot{q}_k \in \mathbb{R}^n$ is a vector resulting from Coriolis and centrifugal forces. $G(q_k) \in \mathbb{R}^n$ is the vector resulting from the gravitational forces. $\tau_k \in \mathbb{R}^n$ is the control input vector containing the torques and forces to be applied at each joint. $d_k(t) \in \mathbb{R}^n$ is the vector containing the unmodeled dynamics and other unknown external disturbances.

Assuming that the joint positions and the joint velocities are available for feedback, our objective is to design a control law $\tau_k(t)$ guaranteeing the boundedness of $q_k(t)$, $\forall t \in [0,T]$ and $\forall k \in \mathbb{Z}_+$, and the convergence of $q_k(t)$ to the desired reference trajectory $q_d(t)$ for all $t \in [0,T]$ when k tends to infinity.

Throughout this paper, we will use the $\mathscr{L}_{\rm pe}$ norm defined as follows:

$$\|x(t)\|_{\rm pe} \triangleq \begin{cases} \left(\int_0^t \|x(\tau)\|^p \,d\tau\right)^{1/p} & \text{if } p \in [0,\infty), \\ \sup_{0\leqslant\tau\leqslant t} \|x(\tau)\| & \text{if } p=\infty, \end{cases}$$

where $\|x\|$ denotes any norm of x, and t belongs to the finite interval $[0,T]$. We say that $x \in \mathscr{L}_{\rm pe}$ when $\|x\|_{\rm pe}$ exists (i.e., when $\|x\|_{\rm pe}$ is finite).

We assume that all the system parameters are unknown and we make the following reasonable assumptions:

(A1) The reference trajectory and its first and second time-derivative, namely $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$, as well as the disturbance $d_k(t)$ are bounded $\forall t \in [0, T]$ and $\forall k \in \mathbb{Z}_+$.

(A2) The resetting condition is satisfied, i.e., $\dot{q}_d(0) - \dot{q}_k(0) = q_d(0) - q_k(0) = 0$, $\forall k \in \mathbb{Z}_+$.

We will also need the following properties, which are common to robot manipulators

(P1) $M(q_k) \in \mathbb{R}^{n\times n}$ is symmetric, bounded, and positive definite.

(P2) The matrix $\dot{M}(q_k) - 2C(q_k, \dot{q}_k)$ is skew symmetric, hence $x^{\mathrm{T}}(\dot{M}(q_k) - 2C(q_k, \dot{q}_k))x = 0$, $\forall x \in \mathbb{R}^n$.

(P3) $G(q_k) + C(q_k, \dot{q}_k)\dot{q}_d(t) = \Psi(q_k, \dot{q}_k)\xi(t)$, where $\Psi(q_k, \dot{q}_k) \in \mathbb{R}^{n\times(m-1)}$ is a known matrix and $\xi(t) \in \mathbb{R}^{m-1}$ is an unknown continuous vector over $[0, T]$.

(P4) $\|C(q_k, \dot{q}_k)\| \leqslant k_c\|\dot{q}_k\|$ and $\|G(q_k)\| < k_g$, $\forall t \in [0, T]$ and $\forall k \in \mathbb{Z}_+$, where k_c and k_g are unknown positive parameters.

Note that (P1–P3) are needed to derive the result in Theorem 1 and (P1, P2, P4) are needed to derive the result in Theorems 2 and 3. Assumption (A2) will be relaxed to a certain extent in Theorem 4.

3. Adaptive ILC design

Now, one can state the following result

Theorem 1. *Consider system* (1) *with properties* (P1–P3) *under the following control law*:

$$\tau_k(t) = K_P\tilde{q}_k(t) + K_D\dot{\tilde{q}}_k(t) + \phi(q_k, \dot{q}_k, \dot{\tilde{q}}_k)\hat{\theta}_k(t), \tag{2}$$

with

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) + \Gamma\phi^{\mathrm{T}}(q_k, \dot{q}_k, \dot{\tilde{q}}_k)\dot{\tilde{q}}_k(t), \tag{3}$$

where $\hat{\theta}_{-1}(t) = 0$, $\tilde{q}_k(t) = q_d(t) - q_k(t)$ *and* $\dot{\tilde{q}}_k(t) = \dot{q}_d(t) - \dot{q}_k(t)$. *The matrix* $\phi(q_k, \dot{q}_k, \dot{\tilde{q}}_k) \in \mathbb{R}^{n\times m}$ *is defined as* $\phi(q_k, \dot{q}_k, \dot{\tilde{q}}_k) \triangleq [\Psi(q_k, \dot{q}_k)\, sgn(\dot{\tilde{q}}_k)]$, *where* $sgn(\dot{\tilde{q}}_k)$ *is the vector obtained by applying the signum function to all elements of* $\dot{\tilde{q}}_k$. *The matrices* $K_P \in \mathbb{R}^{n\times n}$, $K_D \in \mathbb{R}^{n\times n}$ *and* $\Gamma \in \mathbb{R}^{m\times m}$ *are symmetric positive definite. Let assumptions* (A1–A2) *be satisfied, then* $\tilde{q}_k(t) \in \mathscr{L}_{\infty e}$, $\dot{\tilde{q}}_k(t) \in \mathscr{L}_{\infty e}$, $\tau_k(t) \in \mathscr{L}_{2e}$ *for all* $k \in \mathbb{Z}_+$ *and* $\lim_{k\to\infty}\tilde{q}_k(t) = \lim_{k\to\infty}\dot{\tilde{q}}_k(t) = 0$, $\forall t \in [0, T]$.

The proof of this theorem is in three parts. The first part consists of taking a positive definite Lyapunov-like composite energy function (Xu, 2002; Xu & Tan, 2001), namely W_k, and show that this sequence is non-increasing with respect to k and hence bounded if W_0 is bounded. In the second part, we show that $W_0(t)$ is bounded for all $t \in [0, T]$. Finally, in the third part, we show that $\lim_{k\to\infty}\tilde{q}_k(t) = \lim_{k\to\infty}\dot{\tilde{q}}_k(t) = 0$, $\forall t \in [0, T]$.

Proof. Part 1: Let us consider the following Lyapunov-like composite energy function [1] :

$$W_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t), \tilde{\theta}_k(t)) = V_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t)) + \frac{1}{2}\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau, \tag{4}$$

with $\tilde{\theta}_k(t) = \theta(t) - \hat{\theta}_k(t)$, where $\theta(t) = [\xi^{\mathrm{T}}(t)\ \ \beta]^{\mathrm{T}} \in \mathbb{R}^m$ and $\hat{\theta}_k(t) = [\hat{\xi}_k^{\mathrm{T}}(t)\ \ \hat{\beta}_k(t)]^{\mathrm{T}}$ is the estimated value of $\theta(t)$. The unknown vector $\xi(t)$ is defined in (P3) and the unknown parameter β is obtained according to (P1) and (A1) such that $\|M(q_k)\ddot{q}_d - d_k\| \leqslant \beta$, $\forall t \in [0, T]$ and $\forall k \in \mathbb{Z}_+$.

The term $V_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t))$ in (4) is chosen as follows

$$V_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t)) = \tfrac{1}{2}\dot{\tilde{q}}_k^{\mathrm{T}}M(q_k)\dot{\tilde{q}}_k + \tfrac{1}{2}\tilde{q}_k^{\mathrm{T}}K_P\tilde{q}_k. \tag{5}$$

The difference of W_k is given by

$$\begin{aligned}\Delta W_k &= W_k - W_{k-1} = V_k - V_{k-1} \\ &\quad + \frac{1}{2}\int_0^t (\tilde{\theta}_k^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_k - \tilde{\theta}_{k-1}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_{k-1})\,\mathrm{d}\tau \\ &= V_k - V_{k-1} - \frac{1}{2}\int_0^t (\bar{\theta}_k^{\mathrm{T}}\Gamma^{-1}\bar{\theta}_k + 2\bar{\theta}_k^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_k)\,\mathrm{d}\tau,\end{aligned} \tag{6}$$

where $\bar{\theta}_k = \hat{\theta}_k - \hat{\theta}_{k-1}$. On the other hand, one can rewrite V_k as follows

$$V_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t)) = V_k(\dot{\tilde{q}}_k(0), \tilde{q}_k(0)) + \int_0^t \left(\dot{\tilde{q}}_k^{\mathrm{T}}M\ddot{\tilde{q}}_k + \frac{1}{2}\dot{\tilde{q}}_k^{\mathrm{T}}\dot{M}\dot{\tilde{q}}_k + \dot{\tilde{q}}_k^{\mathrm{T}}K_P\tilde{q}_k\right)\mathrm{d}\tau. \tag{7}$$

Now, using (1) and (P2, P3), Eq. (7) leads to

$$\begin{aligned}V_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t)) &= V_k(\dot{\tilde{q}}_k(0), \tilde{q}_k(0)) \\ &\quad + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}[M(q_k)\ddot{q}_d - d_k + C(q_k, \dot{q}_k)\dot{q}_d \\ &\quad + G(q_k) + K_P\tilde{q}_k - \tau_k]\,\mathrm{d}\tau \\ &\leqslant V_k(\dot{\tilde{q}}_k(0), \tilde{q}_k(0)) \\ &\quad + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\Psi(q_k, \dot{q}_k)\xi + K_P\tilde{q}_k \\ &\quad + \beta\, sgn(\dot{\tilde{q}}_k) - \tau_k)\,\mathrm{d}\tau\end{aligned}$$

[1] Throughout this paper, we will use the abusive notation $W_k(t)$ and $V_k(t)$ instead of $W_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t), \tilde{\theta}_k(t))$ and $V_k(\dot{\tilde{q}}_k(t), \tilde{q}_k(t))$. We will also use W_k and V_k instead of $W_k(t)$ and $V_k(t)$ where this does not lead to any confusion.

$$\leqslant V_k(\dot{\tilde{q}}_k(0),\tilde{q}_k(0)) + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\phi(q_k,\dot{q}_k,\dot{\tilde{q}}_k)\theta + K_P\tilde{q}_k - \tau_k)\,\mathrm{d}\tau. \quad (8)$$

Now, substituting (2) in (8) we obtain

$$V_k(\dot{\tilde{q}}_k(t),\tilde{q}_k(t)) \leqslant V_k(\dot{\tilde{q}}_k(0),\tilde{q}_k(0)) + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\phi(q_k,\dot{q}_k,\dot{\tilde{q}}_k)\tilde{\theta}_k - K_D\dot{\tilde{q}}_k)\,\mathrm{d}\tau. \quad (9)$$

Using (3), (9) and (A2), Eq. (6) leads to

$$\Delta W_k \leqslant -V_{k-1} - \frac{1}{2}\int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\phi(q_k,\dot{q}_k,\dot{\tilde{q}}_k)\Gamma\phi^{\mathrm{T}}(q_k,\dot{q}_k,\dot{\tilde{q}}_k) + 2K_D)\dot{\tilde{q}}_k\,\mathrm{d}\tau \leqslant 0. \quad (10)$$

Hence, W_k is a non-increasing sequence. Thus if W_0 is bounded one can conclude that W_k is bounded. In Part 2 of the Proof we will show that $W_0(t)$ is bounded for all $t\in[0,T]$. Hence, $\tilde{q}_k(t)$, $\dot{\tilde{q}}_k(t)$ and $\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau$ are bounded for all $k\in\mathbb{Z}_+$ and all $t\in[0,T]$. Since $\theta(t)$ is continuous over $[0,T]$, the boundedness of $\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau$ implies the boundedness of $\int_0^t \hat{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\hat{\theta}_k(\tau)\,\mathrm{d}\tau$. Consequently, one can conclude that $\tau_k(t)\in\mathscr{L}_{2e}$ for all $k\in\mathbb{Z}_+$.

Part 2: Now, we will show that $W_0(t)$ is bounded over the time interval $[0,T]$. In fact, considering (4) with $k=0$, the time-derivative of W_0 can be bounded as follows

$$\dot{W}_0 \leqslant \dot{\tilde{q}}_0^{\mathrm{T}}(\phi(q_0,\dot{q}_0,\dot{\tilde{q}}_0)\tilde{\theta}_0 - K_D\dot{\tilde{q}}_0) + \frac{1}{2}\tilde{\theta}_0^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_0. \quad (11)$$

Since $\hat{\theta}_{-1}(t)=0$, one has $\hat{\theta}_0(t)=\Gamma\phi^{\mathrm{T}}(q_0,\dot{q}_0,\dot{\tilde{q}}_0)\dot{\tilde{q}}_0(t)$. Hence

$$\dot{W}_0 \leqslant -\dot{\tilde{q}}_0^{\mathrm{T}}K_D\dot{\tilde{q}}_0 + \left(\hat{\theta}_0^{\mathrm{T}} + \frac{1}{2}\tilde{\theta}_0^{\mathrm{T}}\right)\Gamma^{-1}\tilde{\theta}_0$$
$$\leqslant -\dot{\tilde{q}}_0^{\mathrm{T}}K_D\dot{\tilde{q}}_0 - \frac{1}{2}\tilde{\theta}_0^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_0 + \theta^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_0. \quad (12)$$

Using Young's inequality, we have

$$\theta^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}_0 \leqslant \mathscr{K}\|\Gamma^{-1}\tilde{\theta}_0\|^2 + \frac{1}{4\mathscr{K}}\|\theta\|^2$$

for any $\mathscr{K}>0$. Hence,

$$\dot{W}_0 \leqslant -\rho_1\|\dot{\tilde{q}}_0\|^2 - \rho_2\|\tilde{\theta}_0\|^2 + \frac{1}{4\mathscr{K}}\|\theta\|^2, \quad (13)$$

with $\rho_1=\lambda_{\min}(K_D)$, $\rho_2=\frac{1}{2}\lambda_{\min}(\Gamma^{-1})-\mathscr{K}\lambda_{\max}^2(\Gamma^{-1})$ and $0<\mathscr{K}\leqslant\lambda_{\min}(\Gamma^{-1})/2\lambda_{\max}^2(\Gamma^{-1})$, where $\lambda_{\min}(*)(\lambda_{\max}(*))$ denotes the minimal (maximal) eigenvalue of $(*)$. Since θ is continuous over $[0,T]$, it is clear that it is bounded over $[0,T]$, i.e., $\|\theta\|_{\infty e}\leqslant\theta_{\max}$. Hence, one can conclude from (13) that $\dot{W}_0(t)\leqslant\theta_{\max}^2/(4\mathscr{K})$, which implies that $W_0(t)$ is uniformly continuous and thus bounded over $[0,T]$.

Part 3: Note that W_k can be written as follows $W_k=W_0+\sum_{j=1}^{k}\Delta W_j$. Hence, using (10), one has

$$W_k \leqslant W_0 - \sum_{j=1}^{k} V_{j-1}$$
$$\leqslant W_0 - \frac{1}{2}\sum_{j=1}^{k}\tilde{q}_{j-1}^{\mathrm{T}}K_P\tilde{q}_{j-1} - \frac{1}{2}\sum_{j=1}^{k}\dot{\tilde{q}}_{j-1}^{\mathrm{T}}M(q_{j-1})\dot{\tilde{q}}_{j-1}, \quad (14)$$

which implies that

$$\sum_{j=1}^{k}\tilde{q}_{j-1}^{\mathrm{T}}K_P\tilde{q}_{j-1} + \sum_{j=1}^{k}\dot{\tilde{q}}_{j-1}^{\mathrm{T}}M(q_{j-1})\dot{\tilde{q}}_{j-1} \leqslant 2(W_0 - W_k) \leqslant 2W_0.$$

Hence, $\lim_{k\to\infty}\tilde{q}_k(t)=\lim_{k\to\infty}\dot{\tilde{q}}_k(t)=0$, $\forall t\in[0,T]$, since $W_k(t)$ is bounded $\forall k\in\mathbb{Z}_+$, $\forall t\in[0,T]$. □

Note that under properties (P1–P3) the control law (2)–(3) involves m iterative parameters, where m is generally larger than the number of degrees-of-freedom n. It also requires the knowledge of the matrix $\Psi(q_k,\dot{q}_k)$. However, by using (P4) instead of (P3), the knowledge of the matrix $\Psi(q_k,\dot{q}_k)$ is not required anymore and the number of iterative parameters is reduced to two as stated in the following theorem.

Theorem 2. *Consider system* (1) *with properties* (P1, P2, P4) *under the following control law*:

$$\tau_k(t) = K_P\tilde{q}_k(t) + K_D\dot{\tilde{q}}_k(t) + \eta(\dot{\tilde{q}}_k)\hat{\theta}_k(t) \quad (15)$$

with

$$\hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) + \Gamma\eta^{\mathrm{T}}(\dot{\tilde{q}}_k)\dot{\tilde{q}}_k(t), \quad (16)$$

where $\hat{\theta}_{-1}(t)=0$. *The matrices* $K_P\in\mathbb{R}^{n\times n}$, $K_D\in\mathbb{R}^{n\times n}$ *and* $\Gamma\in\mathbb{R}^{2\times 2}$ *are symmetric positive definite. The matrix* $\eta(\dot{\tilde{q}}_k)$ *is defined as* $\eta(\dot{\tilde{q}}_k)\triangleq[\dot{\tilde{q}}_k\ \ sgn(\dot{\tilde{q}}_k)]$. *Let assumptions* (A1–A2) *be satisfied, then* $\tilde{q}_k(t)\in\mathscr{L}_{\infty e}$, $\dot{\tilde{q}}_k(t)\in\mathscr{L}_{\infty e}$, $\tau_k(t)\in\mathscr{L}_{2e}$ *for all* $k\in\mathbb{Z}_+$ *and* $\lim_{k\to\infty}\tilde{q}_k(t)=\lim_{k\to\infty}\dot{\tilde{q}}_k(t)=0$, $\forall t\in[0,T]$.

Proof. Here we use the same Lyapunov-like composite energy function (4), with $\Gamma\in\mathbb{R}^{2\times 2}$ and $\tilde{\theta}\in\mathbb{R}^2$. The vector θ is defined as $\theta=[\alpha,\ \delta]^{\mathrm{T}}\in\mathbb{R}^2$. The unknown parameters α and δ are defined as follows:

$$\dot{\tilde{q}}_k^{\mathrm{T}}(M(q_k)\ddot{q}_d + C(q_k,\dot{q}_k)\dot{q}_d + G(q_k) - d_k)$$
$$\leqslant \|\dot{\tilde{q}}_k\|(\beta + k_g + k_c\|\dot{q}_d\|\|\dot{q}_k\|)$$
$$\leqslant \|\dot{\tilde{q}}_k\|(\beta + k_g + k_c\|\dot{q}_d\|^2 + k_c\|\dot{q}_d\|\|\dot{\tilde{q}}_k\|), \quad (17)$$

where k_c and k_g are defined in (P4), and β is obtained according to (A1) and (P1). Since $\dot{q}_d$ is bounded, inequality (17) leads to

$$\dot{\tilde{q}}_k^{\mathrm{T}}(M(q_k)\ddot{q}_d + C(q_k,\dot{q}_k)\dot{q}_d + G(q_k) - d_k) \leqslant \dot{\tilde{q}}_k^{\mathrm{T}}(\alpha\dot{\tilde{q}}_k + \delta\, sgn(\dot{\tilde{q}}_k)) \leqslant \dot{\tilde{q}}_k^{\mathrm{T}}\eta(\dot{\tilde{q}}_k)\theta, \tag{18}$$

where $\alpha = k_c\ Sup_{t\in[0,T]}\|\dot{q}_d\|$ and $\delta = \beta + k_g + k_c\ Sup_{t\in[0,T]}\|\dot{q}_d(t)\|^2$.

Following the steps of the Proof of Theorem 1, and using (18), we obtain

$$V_k(\dot{\tilde{q}}_k(t),\tilde{q}_k(t)) \leqslant V_k(\dot{\tilde{q}}_k(0),\tilde{q}_k(0)) + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\eta(\dot{\tilde{q}}_k)\theta + K_P\tilde{q}_k - \tau_k)\,\mathrm{d}\tau. \tag{19}$$

Substituting (15) in (19) we obtain

$$V_k(\dot{\tilde{q}}_k(t),\tilde{q}_k(t)) \leqslant V_k(\dot{\tilde{q}}_k(0),\tilde{q}_k(0)) + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\eta(\dot{\tilde{q}}_k)\tilde{\theta}_k - K_D\dot{\tilde{q}}_k)\,\mathrm{d}\tau. \tag{20}$$

Using (16) and (20) in (6), in view of (A2), we obtain

$$\Delta W_k \leqslant -V_{k-1} - \frac{1}{2}\int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\eta(\dot{\tilde{q}}_k)\Gamma\eta^{\mathrm{T}}(\dot{\tilde{q}}_k) + 2K_D)\dot{\tilde{q}}_k\,\mathrm{d}\tau \leqslant 0. \tag{21}$$

The remaining of the proof is omitted since it is similar to the proof of Theorem 1. □

Remark 1. Generally, in contraction-mapping-based ILC schemes, the number of iterative parameters is equal to the number of the control inputs which is equal to the number of degrees of freedom n. In our approach (Theorem 2), we use only two iterative parameters $\hat{\theta}_k \in \mathbb{R}^2$, which is an interesting fact from a practical point of view since it contributes considerably to memory space saving. Now, the question is: Can we bring the number of iterative parameters from two to one and at what expense? The answer is yes, but to the expense of a certain knowledge of the system dynamics. In fact, in contrast with the result in Theorem 2, the knowledge of the parameter k_c defined in (P4) is needed to guarantee the convergence of the tracking error to zero as stated bellow in Theorem 3.

Theorem 3. *Consider system* (1) *with properties* (P1, P2, P4) *under the following control law*:

$$\tau_k(t) = K_P\tilde{q}_k(t) + K_D\dot{\tilde{q}}_k(t) + \hat{\delta}_k(t)\,sgn(\dot{\tilde{q}}_k(t)) \tag{22}$$

with

$$\hat{\delta}_k(t) = \hat{\delta}_{k-1}(t) + \gamma\dot{\tilde{q}}_k^{\mathrm{T}}(t)\,sgn(\dot{\tilde{q}}_k(t)), \tag{23}$$

where $\hat{\delta}_{-1}(t) = 0$. *The matrices* $K_P \in \mathbb{R}^{n\times n}$ *and* $K_D \in \mathbb{R}^{n\times n}$ *are symmetric positive definite, and* γ *is a positive scalar. Let assumptions* (A1–A2) *be satisfied. If* $(K_D - \alpha I)$ *is positive semi-definite, with* $\alpha = k_c\ Sup_{t\in[0,T]}\|\dot{q}_d(t)\|$, *then* $\tilde{q}_k(t) \in \mathscr{L}_{\infty e}$, $\dot{\tilde{q}}_k(t) \in \mathscr{L}_{\infty e}$, $\tau_k(t) \in \mathscr{L}_{2e}$ *for all* $k \in \mathbb{Z}_+$ *and* $\lim_{k\to\infty}\tilde{q}_k(t) = \lim_{k\to\infty}\dot{\tilde{q}}_k(t) = 0$, $\forall t \in [0,T]$.

Proof. Here we use the following Lyapunov-like composite energy function

$$W_k = V_k(\dot{\tilde{q}}_k(t),\tilde{q}_k(t)) + \frac{1}{2}\int_0^t \gamma^{-1}\tilde{\delta}_k^2(\tau)\,\mathrm{d}\tau, \tag{24}$$

with $\tilde{\delta}_k(t) = \delta - \hat{\delta}_k(t)$, where the unknown parameter δ is defined in (18). The term V_k is defined in (5) and can be written as (19), by virtue of (18).

Now, substituting (22) in (19) we obtain

$$V_k(\dot{\tilde{q}}_k(t),\tilde{q}_k(t)) \leqslant V_k(\dot{\tilde{q}}_k(0),\tilde{q}_k(0)) + \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\tilde{\delta}_k\,sgn(\dot{\tilde{q}}_k) + \alpha\dot{\tilde{q}}_k - K_D\dot{\tilde{q}}_k)\,\mathrm{d}\tau. \tag{25}$$

In the same way as in the Proof of the Theorem 2, using (23) and (25), in view of (A2), we obtain

$$\Delta W_k \leqslant -V_{k-1} - \frac{1}{2}\int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\gamma\,sgn(\dot{\tilde{q}}_k)\,sgn(\dot{\tilde{q}}_k)^{\mathrm{T}} + 2(K_D - \alpha I))\dot{\tilde{q}}_k\,\mathrm{d}\tau, \tag{26}$$

which is non-increasing if the matrix $(K_D - \alpha I)$ is positive semi-definite. The remaining of the proof is omitted since it is similar to the proof of Theorem 1. □

Note that to carry out the results in Theorems 1–3, we assumed that the resetting condition is satisfied. In fact, this condition, i.e., (A2), can be relaxed to a certain extent if the alignment condition is satisfied (Xu et al., 2000; Xu and Tan, 2001), i.e., $\dot{\tilde{q}}_k(0)=\dot{\tilde{q}}_{k-1}(T)$ and $\tilde{q}_k(0)=\tilde{q}_{k-1}(T)$ that is $V_k(0) = V_{k-1}(T)$. In other words, if the reference trajectory satisfies $q_d(0) = q_d(T)$ and $\dot{q}_d(0) = \dot{q}_d(T)$, we can start the system from where it was stopped at the last operation instead of bringing the system to the same initial position at each operation. According to this discussion one can state the following result.

Theorem 4. *If the alignment condition, i.e.,* $V_k(0) = V_{k-1}(T)$, *is used in theorems* 1–3 *instead of* (A2), *then*

- $\tilde{q}_k(t) \in \mathscr{L}_{\infty e}$, $\dot{\tilde{q}}_k(t) \in \mathscr{L}_{\infty e}$, $\tau_k(t) \in \mathscr{L}_{2e}$ *for all* $k \in \mathbb{Z}_+$.
- $\lim_{k\to\infty}\tilde{q}_k(t) = \varepsilon$, $\forall t \in [0,T]$, *where* ε *is finite and tends to zero when* $\lambda_{\min}(K_P)$ *tends to infinity*.

Proof. Since the proof is similar for Theorems 1–3, we will prove this result just for Theorem 2. In fact, using (20) in (6), and using the alignment condition instead of (A2), for

$t=T$, we obtain

$$\Delta W_k(T) \leqslant V_k(0) - V_{k-1}(T) - \frac{1}{2}\int_0^T \dot{\tilde{q}}_k^{\mathrm{T}}(\eta(\dot{\tilde{q}}_k)\Gamma\eta^{\mathrm{T}}(\dot{\tilde{q}}_k) + 2K_D)\dot{\tilde{q}}_k\,\mathrm{d}\tau \leqslant -\frac{1}{2}\int_0^T \dot{\tilde{q}}_k^{\mathrm{T}}(\eta(\dot{\tilde{q}}_k)\Gamma\eta^{\mathrm{T}}(\dot{\tilde{q}}_k) + 2K_D)\dot{\tilde{q}}_k\,\mathrm{d}\tau \leqslant 0. \quad (27)$$

In the same way as in the Proof of Theorem 1, one can show that $W_k(T)$ is bounded for all $k\in\mathbb{Z}_+$ and

$$W_k(T) = W_0(T) + \sum_{j=1}^{k}\Delta W_j(T) \leqslant W_0(T) - \sum_{j=1}^{k}\left\{\int_0^T \dot{\tilde{q}}_j^{\mathrm{T}}K_D\dot{\tilde{q}}_j\,\mathrm{d}\tau\right\}, \quad (28)$$

which leads to

$$\sum_{j=1}^{k}\left\{\int_0^T \dot{\tilde{q}}_j^{\mathrm{T}}K_D\dot{\tilde{q}}_j\,\mathrm{d}\tau\right\} \leqslant W_0(T) - W_k(T) \leqslant W_0(T). \quad (29)$$

Hence,

$$\lim_{k\to\infty}\int_0^T \dot{\tilde{q}}_k^{\mathrm{T}}K_D\dot{\tilde{q}}_k\,\mathrm{d}\tau = 0. \quad (30)$$

Since $W_k(T)$ is bounded for all $k\in\mathbb{Z}_+$, one can conclude that $\frac{1}{2}\int_0^T \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau \triangleq \varpi_k(T) \leqslant \varpi < \infty$. Since $\varpi_k(t) \leqslant \varpi_k(T) \leqslant \varpi$, $\forall k\in\mathbb{Z}_+$, $\forall t\in[0,T]$, one has $W_k(t) = V_k(t) + \varpi_k(t) \leqslant V_k(t) + \varpi$. Thus,

$$W_{k-1}(t) \leqslant V_{k-1}(t) + \varpi. \quad (31)$$

On the other hand, one has

$$\Delta W_k(t) = W_k(t) - W_{k-1}(t) \leqslant V_k(0) - V_{k-1}(t). \quad (32)$$

From (31) and (32), one can conclude that

$$W_k(t) \leqslant V_k(0) + \varpi = V_{k-1}(T) + \varpi. \quad (33)$$

Since $W_k(T)$ is bounded $\forall k\in\mathbb{Z}_+$, it is clear that $V_k(T)$ is bounded $\forall k\in\mathbb{Z}_+$. Hence, from (33), one can conclude that $W_k(t)$ is bounded $\forall k\in\mathbb{Z}_+$, $\forall t\in[0,T]$. Consequently, $\tilde{q}_k(t)$, $\dot{\tilde{q}}_k(t)$ and $\int_0^t \tilde{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\tilde{\theta}_k(\tau)\,\mathrm{d}\tau$ are bounded for all $t\in[0,T]$ and all $k\in\mathbb{Z}_+$. Since θ is continuous, the boundedness of $\int_0^t \hat{\theta}_k^{\mathrm{T}}(\tau)\Gamma^{-1}\hat{\theta}_k(\tau)\,\mathrm{d}\tau$ is guaranteed. Therefore, $\tau_k(t)\in\mathscr{L}_{2e}$ for all $k\in\mathbb{Z}_+$ and so is $\ddot{q}_k(t)$.

Now, using the fact that

$$\|\tilde{q}_k(t) - \tilde{q}_k(0)\|^2 = \left\|\int_0^t \dot{\tilde{q}}_k(\tau)\,\mathrm{d}\tau\right\|^2, \quad (34)$$

and using Schwartz inequality, we obtain

$$\|\tilde{q}_k(t) - \tilde{q}_k(0)\|^2 \leqslant \int_0^t \dot{\tilde{q}}_k^{\mathrm{T}}(\tau)\dot{\tilde{q}}_k(\tau)\,\mathrm{d}\tau\int_0^t 1^2\,\mathrm{d}\tau \leqslant T\int_0^T \dot{\tilde{q}}_k^{\mathrm{T}}(\tau)\dot{\tilde{q}}_k(\tau)\,\mathrm{d}\tau \leqslant \frac{T}{\lambda_{\min}(K_D)}\int_0^T \dot{\tilde{q}}_k^{\mathrm{T}}(\tau)K_D\dot{\tilde{q}}_k(\tau)\,\mathrm{d}\tau. \quad (35)$$

From (30) and (35), one can conclude that

$$\lim_{k\to\infty}\|\tilde{q}_k(t) - \tilde{q}_k(0)\| = \lim_{k\to\infty}\|\tilde{q}_k(t) - \tilde{q}_{k-1}(T)\| = 0, \quad (36)$$

for all $t\in[0,T]$. Hence $\lim_{k\to\infty}\tilde{q}_k(t) = \varepsilon$, $\forall t\in[0,T]$, where ε is a finite value. Finally, using (1) and (15), one can show that ε tends to zero when $\lambda_{\min}(K_P)$ tends to infinity. □

Remark 2. In all theorems proposed in this paper, one can show that the tracking error and its time derivative, at the first iteration, can be made arbitrarily small, over the finite time interval $[0,T]$, by increasing the minimal eigenvalues of the control and learning gains K_P, K_D and (Γ or γ).

Remark 3. It is worth noting that the proposed control strategy can be used in a straightforward manner for industrial robot manipulators already functioning under a PD controller by just adding the iterative term to the control input in order to enhance the tracking performance from operation to operation.

Remark 4. Note that the problem of set-point regulation is included in Theorem 4 since a constant reference trajectory satisfies the alignment condition. Basically, in this particular case, our algorithms will ensure a step-by-step convergence to the position set-point. In fact, after one operation we arrive to a certain position q_1, from which we start the next operation to arrive to q_2 and so one, we arrive to the position q_k after k iterations. The tracking error is reduced progressively and converges to the finite value ε as stated in Theorem 4.

Remark 5. Note that, in all theorems proposed in this paper, one can show that the control input $\tau_k(t)\in\mathscr{L}_{\infty e}$ for any finite k. This follows from the fact that $\tilde{q}_k(t), \dot{\tilde{q}}_k(t)\in\mathscr{L}_{\infty e}$, $\forall k\in\mathbb{Z}_+$ which implies that the parameters estimates are bounded for any finite k.

Remark 6. In practical applications, the manipulator joints are equipped with incremental encoders to measure the joint positions. In general, the joint velocities are not measured but estimated from the joint positions using a filtered derivative (*e.g.*, $s\tilde{q}_k/(1+T_c s)$). However, the measurement noise amplification, due to the derivative action, will accumulate through the iterative process. A potential solution to this problem is to design a P-type iterative parametric updating law that does not require the joint velocities measurements. In this case, the noise effect will be reduced considerably, but will not be totally eliminated since the measurement noise from the joint positions will accumulate from iteration to iteration. Consequently, in practical applications, it is important to stop the learning process after a certain number of iterations once the tracking error reaches a certain acceptable level. This crucial point will be investigated in our future research.

Remark 7. It is worth noting that the sign function used in the proposed control laws might lead to the chattering phenomenon. In practical applications, the sign function can be replaced by a continuous approximation (*e.g.*, saturation) in order to smooth out the control input and reduce the chattering. In fact, if we substitute in our algorithms the sign function by the following function: $f(\dot{\tilde{q}}_k) = \dot{\tilde{q}}_k / \max\{\varepsilon, \|\dot{\tilde{q}}_k\|\}$, $\varepsilon > 0$, one can show that the tracking error converges to a certain domain around zero, which can be made arbitrarily small by decreasing ε. It might also be possible to show the convergence to zero of the time-weighted norm (also called λ-norm) of the tracking error, i.e., $e^{-\lambda t}\|\tilde{q}_k\|$, for a sufficiently large λ, under the saturation function. This point needs further investigation and will be part of our future research. In fact, in Cao and Xu (2002), a saturation-type learning variable structure control, guaranteeing the convergence of the λ-norm of the tracking error to zero over a given finite-time interval, was proposed for a class of uncertain nonlinear systems. However, it is well known that the λ-norm leads generally to low convergence rates.

4. Simulation results

Let us consider a two degrees-of-freedom planar manipulator with revolute joints described by (1). The matrix $M = [m_{ij}]_{2\times 2}$ is given by $m_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2}\cos q_2) + I_1 + I_2$, $m_{12} = m_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos q_2) + I_2$, and $m_{22} = m_2 l_{c2}^2 + I_2$. The matrix $C = [c_{ij}]_{2\times 2}$ is given by $c_{11} = h\dot{q}_2$, $c_{12} = h\dot{q}_1 + h\dot{q}_2$, $c_{21} = -h\dot{q}_1$ and $c_{22} = 0$, where $h = -m_2 l_1 l_{c2}\sin q_2$. The vector $G = [G_1, G_2]^{\mathrm{T}}$ is given by $G_1 = (m_1 l_{c1} + m_2 l_1) g\cos q_1 + m_2 l_{c2} g\cos(q_1 + q_2)$ and, $G_2 = m_2 l_{c2} g\cos(q_1 + q_2)$. The robot parameters are given by $m_1 = m_2 = 1\,Kg$, $l_1 = l_2 = 0.5\,m$, $l_{c1} = l_{c2} = 0.25\,m$, $I_1 = I_2 = 0.1\,Kg.m^2$, $g = 9.81\,m/s^2$. The disturbances are assumed to be time-varying and also varying from iteration to iteration as follows $d_1 = d_2 = rand(k)\sin(t)$, where $rand(k)$ is a random function taking its values between 0 and 1. The matrix $\phi(q, \dot{q}, \dot{\tilde{q}}_k) = [\phi_{ij}]_{2\times 5}$ is given by $\phi_{11} = \dot{q}_2\sin q_2$, $\phi_{21} = -\dot{q}_1\sin q_2$, $\phi_{12} = \phi_{11} - \phi_{21}$, $\phi_{22} = \phi_{23} = 0$, $\phi_{13} = \cos q_1$, $\phi_{14} = \phi_{24} = \cos(q_1 + q_2)$, $\phi_{15} = sgn(\dot{\tilde{q}}_1)$ and $\phi_{25} = sgn(\dot{\tilde{q}}_2)$.

The desired reference trajectories for q_1 and q_2 are chosen as $q_{1,d}(t) = \sin(2\pi t)$ and $q_{2,d}(t) = \cos(2\pi t)$ over the time interval $[0, 1s]$. Applying the control law (2)–(3), with the resetting condition, with $K_P = K_D = 10 I_{2\times 2}$ and $\Gamma = 10 I_{5\times 5}$, where $I_{i\times i}$ is an $i \times i$ identity matrix, we obtain the result shown in Fig. 1. Applying the control law (15)–(16), with the resetting condition, with $K_P = K_D = 10 I_{2\times 2}$ and $\Gamma = 10 I_{2\times 2}$ we obtain the result shown in Fig. 2. Applying the control law (22)–(23), with the resetting condition, with $K_P = K_D = 10 I_{2\times 2}$ and $\gamma = 10$ we obtain the result shown in Fig. 3. Applying the control law (15)–(16), with the alignment condition, with $K_P = 500 I_{2\times 2}$, $K_D = \Gamma = 10 I_{2\times 2}$, and

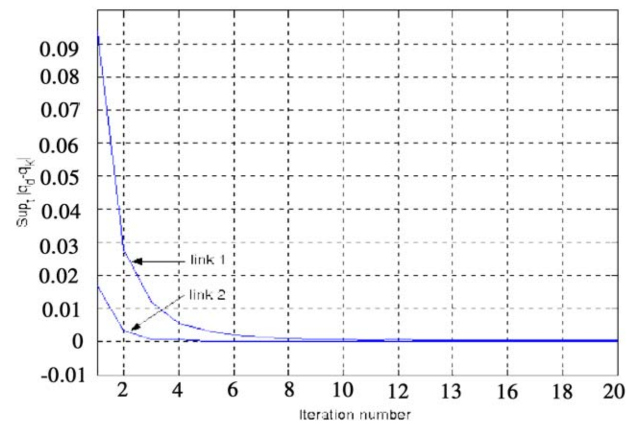

Fig. 1. Sup-norm of the tracking error versus the number of iterations for links 1 and 2 with the control law (2)–(3), with the resetting condition.

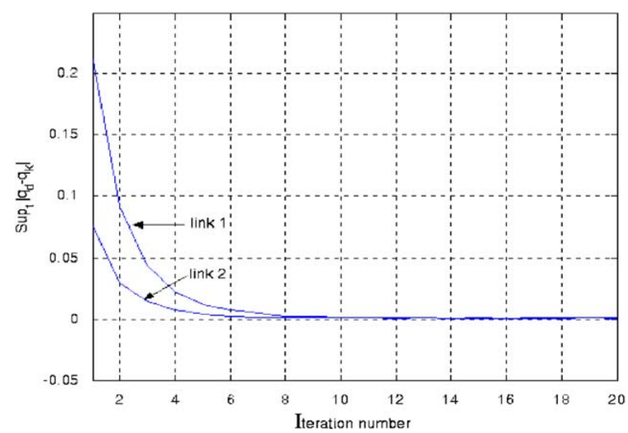

Fig. 2. Sup-norm of the tracking error versus the number of iterations for links 1 and 2 with the control law (15)–(16), with the resetting condition.

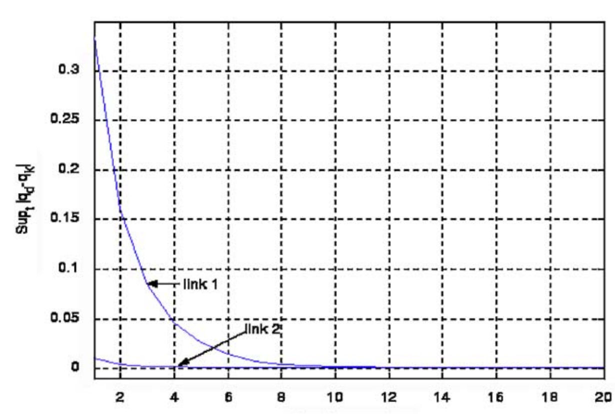

Fig. 3. Sup-norm of the tracking error versus the number of iterations for links 1 and 2 with the control law (22)–(23), with the resetting condition.

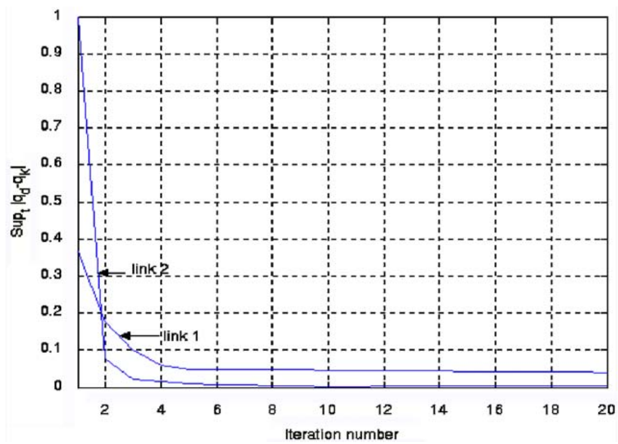

Fig. 4. Sup-norm of the tracking error versus the number of iterations for links 1 and 2 with the control law (15)–(16), with the alignment condition.

$q_{1,0}(0)=q_{2,0}(0)=\dot{q}_{1,0}(0)=\dot{q}_{2,0}(0)=0$, we obtain the result shown in Fig. 4.

5. Conclusion

Three adaptive ILC schemes have been proposed for the position tracking problem of rigid robot manipulators with unknown parameters and subject to external disturbances. The proposed controllers are based upon a PD feedback structure plus an iterative term designed to cope with the unknown parameters and disturbances. The proof of convergence is based upon the use of a Lyapunov-like positive definite sequence, which is shown to be monotonically decreasing under the proposed control schemes. The controllers are very simple to design and to implement since the only requirement on the PD and learning gains is the positive definiteness condition. The result in Theorem 2 is particularly interesting since it does not require any a priori knowledge of the system dynamics, and the number of iterative parameters used in this scheme is just two. Furthermore, in Theorem 3, we propose a solution to the problem using a single iterative variable at the expense of the knowledge of some bounds of the system parameters. Finally, the relaxation of the resetting condition for a certain class of reference trajectories has been discussed and formulated in Theorem 4.

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