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Control schemes for stable teleoperation with communication delay based on IOS small gain theorem☆

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Abstract

The problem of stabilization of a force-reflecting telerobotic system in presence of time delay in the communication channel is addressed. We introduce an approach that is based on application of the input-to-output stability (IOS) small gain theorem for functional differential equations (FDEs). A version of the stabilization algorithm as well as its two adaptive extensions are proposed. For all these control schemes, the input-to-state stability (ISS) of the overall telerobotic system is guaranteed in the global, global practical, or semiglobal practical sense for any constant communication delay under the assumption that the environmental dynamics satisfy a weak form of finite-gain stability property. As an intermediate step, we formulate and prove a general IOS (ISS) small gain result for FDEs.

Keywords: Telerobotics; Time delay systems; Small gain theorem; Input-to-state stability

1. Introduction

According to Sheridan (1989), teleoperation can be defined as the extension of a person's sensing and manipulation capabilities to a remote location. In a teleoperator system, two manipulators called master and slave are connected via a communication channel. The master is moved by a human operator, and the slave is programmed to follow the motion of the master. In the so-called force-reflecting or bilateral teleoperator systems, the contact force due to the environment is transmitted back through the communication channel and applied to the master manipulator without alteration. This makes the human operator feel the interaction with the remote environment. In Ferrell (1966), it is shown experimentally that the existence of even a small delay in the communication channel may cause instability of a force-reflecting teleoperator system. A control scheme that guarantees, under certain assumptions, stability of a bilaterally controlled teleoperator for any communication delay was first presented in Anderson and Spong (1989). This solution, however, has several restrictions, in particular, the control scheme does not provide the human operator with the exact information about environmental forces on the slave side. Since then, the problem of stabilization of bilaterally controlled teleoperators in presence of communication delay has attracted considerable attention in the literature (see for example, Alvarez-Gallegos et al., 1997; Leung et al., 1995; Niemeyer & Slotine, 2004; Zhu & Salcudean, 2000, among other papers). A comparative study of different control schemes for teleoperation with communication delay can be found in Arcara and Melchiorri (2002).

The contribution of this paper is the following. We consider the problem of stabilization of a force-reflecting teleoperator system that interacts with an environment whose dynamics satisfy a weak form of the finite-gain stability assumption. To analyze the stability properties of such a teleoperator system in the presence of communication delay, we introduce an approach that is based on a new version of the IOS small gain theorem for functional differential equations. We present a basic

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non-adaptive version of the stabilization algorithm as well as its two adaptive extensions. The non-adaptive version of this control law is virtually the same as the one proposed in Polushin and Marquez (2003), however, in this paper we prove the corresponding stability result under an essentially more general finite-gain assumption on the environmental dynamics. The development of the adaptive extensions is motivated mainly by our intention to make the algorithms suitable for practical implementation. Indeed, the basic algorithm proposed requires the exact models of both the master and the slave manipulators to be available, and it also utilizes the exact measurement of the joint velocities. In practical situations, however, the nonlinear structure of the dynamical equations of a manipulator expressed in terms of the so-called regressor function is usually known, while the parameters are unknown or (and) subject to changes, and the joint velocities are not available for direct measurement. Thus, we address a situation where the parameters of both the master and the slave manipulators are unknown, and provide an adaptive version of the stabilization algorithm. Furthermore, we assume that only the joint positions of both the master and the slave manipulators are available for measurement subject to small measurement disturbances. In the latter case, we consider an adaptive stabilization algorithm where the velocity measurements are replaced by the estimates obtained using the so-called "dirty derivative" filters. In all three cases we show that the overall telerobotic system, being considered as a system of functional differential equations (FDEs), is input-to-state stable (ISS) (in the global, global practical, or semiglobal practical sense) with respect to external forces for any constant communication delay. As an intermediate step of our stability analysis, we formulate and prove a general input-to-output stable (IOS) (ISS) small gain result for FDEs with restrictions (Theorem 1). This result, which is essentially an extension of the results of Jiang et al. (1994), Teel (1996) to the case of FDEs, plays a central role in our proofs.

The paper is organized as follows. In Section 2, the mathematical model of the force-reflecting telerobotic system is given. In Section 3, we formulate and prove a new version of the IOS (ISS) small gain theorem for systems of FDEs which is the main tool in the stability analysis of the telerobotic system with communication delay. In Section 4, we present three control schemes (basic non-adaptive, adaptive, and adaptive without velocity measurements) and formulate the corresponding stability results. Proofs of these results are given in Section 5. Simulation results are presented in Section 6, while in Section 7 some concluding remarks are given. Finally, Appendices A and B contain formulations and proofs of several technical facts.

2. Force-reflecting teleoperator system with communication delay

In this paper, we consider a bilaterally controlled teleoperator system where both the master and the slave are n-dimensional fully actuated manipulators with compact configuration spaces $\mathbb{Q}_{\rm m}$, $\mathbb{Q}_{\rm s}$, respectively. For simplicity of presentation, assume $\mathbb{Q}_{\rm m}=\mathbb{Q}_{\rm s}=\mathbb{T}^n$, where $\mathbb{T}^n$ is n-dimensional torus. Let $q_{\rm m}\in\mathbb{T}^n$, $q_{\rm s}\in\mathbb{T}^n$ be generalized coordinates of the master and the slave, respectively. The dynamics of both the master and the slave manipulators are described by the Euler–Lagrange equations as follows:

$$H_{\rm m}(q_{\rm m})\ddot{q}_{\rm m}+C_{\rm m}(q_{\rm m},\dot{q}_{\rm m})\dot{q}_{\rm m}+G_{\rm m}(q_{\rm m}) = F_{\rm h}+\hat{F}_{\rm e}+u_{\rm m}, \quad (1)$$

$$H_{\rm s}(q_{\rm s})\ddot{q}_{\rm s}+C_{\rm s}(q_{\rm s},\dot{q}_{\rm s})\dot{q}_{\rm s}+G_{\rm s}(q_{\rm s})=F_{\rm e}+u_{\rm s}. \quad (2)$$

Here, $H_{\rm m}, H_{\rm s}:\mathbb{T}^n\to\mathbb{R}^{n\times n}$ are the inertia matrices of the master and the slave, respectively, $C_{\rm m}, C_{\rm s}:\mathbb{T}^n\times\mathbb{R}^n\to\mathbb{R}^{n\times n}$ are the matrices of centrifugal and Coriolis forces, $G_{\rm m}, G_{\rm s}:\mathbb{T}^n\to\mathbb{R}^n$ are the vectors of potential forces, all assumed to be smooth functions of their arguments, $F_{\rm h}\in\mathbb{R}^n$ is a force (torque) applied by the human operator, $F_{\rm e}\in\mathbb{R}^n$ is the contact force (torque) due to environment applied to the slave, $\hat{F}_{\rm e}\in\mathbb{R}^n$ represents the measurement of $F_{\rm e}$ transmitted back to the motors of the master, and $u_{\rm m}, u_{\rm s}\in\mathbb{R}^n$ are the control inputs of the master and the slave, respectively. Eqs. (1), (2) are assumed to satisfy a set of standard properties (see, for example, Spong, 1996, Section 2.1). The signals transmitted through the communication channel between the master and the slave are subjects to communication delay. More precisely, let $\tau_1\geqslant 0$ be a communication delay of the forward communication channel (from the master to the slave). We assume that the (delayed) position and velocity of the master are available on the slave side starting from the moment $t=0$, i.e.,

$$\hat{q}_{\rm m}(t)=q_{\rm m}(t-\tau_1),\quad \hat{\dot{q}}_{\rm m}(t)=\dot{q}_{\rm m}(t-\tau_1) \quad (3)$$

for $t\geqslant 0$, and $\hat{q}_{\rm m}(t)\equiv 0, \hat{\dot{q}}_{\rm m}(t)\equiv 0$ for $t<0$. On the other hand, assume that a contact force due to environment is measured on the slave side and is sent back to the master with communication delay $\tau_2\geqslant 0$, so that the following force:

$$\hat{F}_{\rm e}(t)=F_{\rm e}(t-\tau_2) \quad (4)$$

for $t\geqslant 0$, and $\hat{F}_{\rm e}(t)\equiv 0$ for $t<0$, is applied to the motors of the master manipulator. Throughout the paper, we assume that the contact force due to environment satisfies a weak finite-gain assumption with respect to the variables of the slave, as follows.

Assumption 1. The contact force $F_{\rm e}$ satisfies

$$|F_{\rm e}(t)|\leqslant\gamma_{\rm e}(\max\{|\dot{q}_{\rm s}(t)|, |q_{\rm s}(t)|\})+|F_{\rm e}^*(t)|, \quad (5)$$

for some $\gamma_{\rm e}>0$ and for almost all $t\geqslant 0$, where $F_{\rm e}^*(t)$ is an arbitrary measurable essentially bounded function. The term $F_{\rm e}^*$ represents the disturbances and the globally bounded part of the environmental forces.

3. IOS (ISS) small gain theorem for systems of FDEs

To describe appropriately a telerobotic system with communication delay, we need to invoke a mathematical object more general than ordinary differential equations, namely, systems of FDEs. Following the notation of Teel (1998), for a given function $x : [-t_d, \infty) \to \mathbb{R}^n$, $t_d \geqslant 0$, and given $t \geqslant 0$, let us define a function $x_d(t)(\cdot) : [0, t_d] \to \mathbb{R}^n$ as $x_d(t)(s) := x(t-s)$. Consider a system described by FDE of the form

$$\dot{x}(t) = F(x_d(t), u_d(t), w_d(t)),$$
$$y(t) = H(x_d(t), u_d(t), w_d(t)), \tag{6}$$

where $x_d(\cdot)$ is the state, $u \in \mathbb{R}^l$, $w \in \mathbb{R}^m$ are the inputs, and $y \in \mathbb{R}^p$ is the output. The notation $|x_d(t)| = \sup_{t-t_d \leqslant s \leqslant t} |x(s)|$ will be used throughout the section, and $|u_d(t)|$, $|w_d(t)|$ are defined analogously. Below, we reformulate the IOS notion (Sontag & Wang, 1999) to the case of systems described by FDEs of the form (6). The analogous definition for ISS of FDE is presented in Teel (1998).

Definition 1. System (6) is said to be IOS with IOS gains $\gamma_u, \gamma_w \in \mathscr{K}$, restriction $(\Delta_x, \Delta_u, \Delta_w)$, $\Delta_x > 0$, $\Delta_u > 0$, $\Delta_w > 0$, and offset $\delta > 0$, if $|x_d(0)| \leqslant \Delta_x$, $\sup_{s \geqslant 0} |u_d(s)| \leqslant \Delta_u$, and $\sup_{s \geqslant 0} |w_d(s)| \leqslant \Delta_w$ imply that the solutions of (6) are defined for all $t \in [-t_d, +\infty)$, and the following two properties hold:

(i) *boundedness*: there exists a function $\beta \in \mathscr{K}_\infty$ such that

$$\sup_{t \geqslant 0} |y(t)| \leqslant \max \left\{ \begin{array}{c} \beta(|x_d(0)|), \gamma_u \left(\sup_{t \geqslant 0} |u_d(t)| \right), \\ \gamma_w \left(\sup_{t \geqslant 0} |w_d(t)| \right), \delta \end{array} \right\};$$

(ii) *convergence*:

$$\limsup_{t \to \infty} |y(t)| \leqslant \max \left\{ \begin{array}{c} \gamma_u \left(\limsup_{t \to \infty} |u_d(t)| \right), \\ \gamma_w \left(\limsup_{t \to \infty} |w_d(t)| \right), \delta \end{array} \right\}.$$

The system is called ISS if it is IOS with respect to the output $y = x$.

To investigate stability properties of the telerobotic system, we need to formulate and prove a version of the IOS (ISS) small gain theorem for FDEs (6). Consider two systems of FDE of the following form:

$$\Sigma_i : \quad \begin{cases} \dot{x}_i(t) = F_i(x_{id}(t), u_{id}(t), w_{id}(t)), \\ y_i(t) = H_i(x_{id}(t), u_{id}(t), w_{id}(t)), \end{cases} \tag{7}$$

where $t_{di} = \tau_i \geqslant 0$, $i = 1, 2$. We assume that $F_i(0_{id}, 0_{id}, 0_{id}) = 0$, $H_i(0_{id}, 0_{id}, 0_{id}) = 0$ for $i = 1, 2$, where 0_{id} is a function essentially equal to 0 over $[0, t_{di}]$. We will address the stability of the overall system (Σ_1, Σ_2) subject to constraints on inputs u_1, u_2 described as follows: $u_1(t) \equiv u_2(t) \equiv 0$ for $t < 0$, and

$$|u_1(t)| \leqslant \chi_2(|y_2(t)|), \quad |u_2(t)| \leqslant \chi_1(|y_1(t)|), \tag{8}$$

for (almost all) $t \geqslant 0$, where $\chi_1(\cdot), \chi_2(\cdot) \in \mathscr{K}$. For simplicity, it is assumed that the system (7) subject to constraints (8) satisfies the following well-posedness assumption: for each initial data $x_{1d}(t_0)$, $x_{2d}(t_0)$, each admissible inputs $w_{1d}(\cdot)$, $w_{2d}(\cdot)$, and each admissible inputs $u_{1d}(\cdot)$, $u_{2d}(\cdot)$ satisfying the constraints (8), there exists an unique maximal solution defined on $[t_0, t_0 + t_{\max})$ for some $t_{\max} > 0$; this solution depends continuously on $x_{1d}(t_0)$, $x_{2d}(t_0)$, $w_{1d}(\cdot)$, $w_{2d}(\cdot)$, $u_{1d}(\cdot)$, $u_{2d}(\cdot)$. Note that the system (Σ_1, Σ_2) subject to constraints (8) is not a feedback system in the classical sense since u_1 and u_2 are not completely determined by outputs y_2, y_1, respectively. However, for such a system it is still possible to define the notions of the input-to-output (input-to-state) stability, and to establish small gain results. To be precise, let us consider $x_d := (x_1^{\mathrm{T}}, x_2^{\mathrm{T}})_d^{\mathrm{T}}$, $t_d = \max\{\tau_1, \tau_2\}$as a state, w_1, w_2 as inputs, and $y := (y_1^{\mathrm{T}}, y_2^{\mathrm{T}})^{\mathrm{T}}$ as an output of the closed-loop system (7), (8). Then we will say that the system (7) subject to constraints (8) is IOS (ISS) if the boundedness and the convergence properties of Definition 1 hold uniformly for any measurable $u_1(\cdot)$, $u_2(\cdot)$ satisfying the constraints (8). The following small gain theorem will be our main tool in the subsequent sections.

Theorem 1 (*IOS (ISS) small gain theorem for FDE*). *Consider system* (7) *subject to constraints* (8). *Suppose each subsystem Σ_i, $i = 1, 2$, is IOS with IOS gains $\gamma_{iu}, \gamma_{iw} \in \mathscr{K}$ and restrictions $(\Delta_{ix}, \Delta_{iu}, \Delta_{iw})$. Suppose also that the gains γ_{iu}, χ_i form a strict contraction, i.e.,*

$$\chi_1 \circ \gamma_{1u} \circ \chi_2 \circ \gamma_{2u}(s) < s \quad \textit{for all } s > 0. \tag{9}$$

Then, given $\Delta_x \leqslant \min\{\Delta_{1x}, \Delta_{2x}\}$, if $\Delta_{1u} > \Delta_{1u}^*$, $\Delta_{2u} > \Delta_{2u}^*$, where

$$\Delta_{1u}^* := \chi_2 \left(\max \left\{ \begin{array}{l} \beta_2(\Delta_x), \gamma_{2u} \circ \chi_1 \circ \gamma_{1w}(\Delta_{1w}), \\ \gamma_{2u} \circ \chi_1 \circ \beta_1(\Delta_x), \gamma_{2w}(\Delta_{2w}) \end{array} \right\} \right), \tag{10}$$

$$\Delta_{2u}^* := \chi_1 \left(\max \left\{ \begin{array}{l} \beta_1(\Delta_x), \gamma_{1u} \circ \chi_2 \circ \gamma_{2w}(\Delta_{2w}), \\ \gamma_{1u} \circ \chi_2 \circ \beta_2(\Delta_x), \gamma_{1w}(\Delta_{1w}) \end{array} \right\} \right), \tag{11}$$

then the system (7), (8) *with $t_d = \max\{\tau_1, \tau_2\}$ is IOS with restriction $(\Delta_x, \Delta_{1w}, \Delta_{2w})$, and the IOS gains for inputs w_1, w_2* are

$$\tilde{\gamma}_{1w}(\cdot) := \max\{\gamma_{1w}(\cdot), \gamma_{2u} \circ \chi_1 \circ \gamma_{1w}(\cdot)\}, \tag{12}$$

$$\tilde{\gamma}_{2w}(\cdot) := \max\{\gamma_{2w}(\cdot), \gamma_{1u} \circ \chi_2 \circ \gamma_{2w}(\cdot)\}. \tag{13}$$

If, additionally, each subsystem is ISS with ISS gains $\nu_{iu}, \nu_{iw} \in \mathscr{K}$ and the same restrictions $(\Delta_{ix}, \Delta_{iu}, \Delta_{iw})$, then the system (7), (8) *is also ISS with restriction $(\Delta_x, \Delta_{1w}, \Delta_{2w})$, and the ISS gains for inputs w_1, w_2 are*

$$\nu_1(\cdot) := \max \left\{ \begin{array}{c} \nu_{2u} \circ \chi_1 \circ \gamma_{1w}(\cdot), \nu_{1w}(\cdot), \\ \nu_{1u} \circ \chi_2 \circ \gamma_{2u} \circ \chi_1 \circ \gamma_{1w}(\cdot) \end{array} \right\}, \tag{14}$$

$$\nu_2(\cdot) := \max \left\{ \begin{array}{c} \nu_{1u} \circ \chi_2 \circ \gamma_{2w}(\cdot), \nu_{2w}(\cdot), \\ \nu_{2u} \circ \chi_1 \circ \gamma_{1u} \circ \chi_2 \circ \gamma_{2w}(\cdot) \end{array} \right\}. \tag{15}$$

Proof. The proof follows the line of reasoning close to the one presented in Teel (1996) for the case of systems of ordinary differential equations. Suppose assumptions of Theorem 1 hold. Consider the system (7) subject to constraints (8) with initial state $x_d^\lambda(0) := \lambda x_d(0)$ and input $w^\lambda(\cdot) := \lambda w(\cdot)$, where $|x_d(0)| \leqslant \Delta_x$, $\sup_{t \geqslant 0} |w_d(t)| \leqslant \Delta_w$, and $\lambda \in [0, 1]$ is a parameter. Note that for $\lambda = 0$ we have $y_1(t) \equiv 0$, $y_2(t) \equiv 0$

for all $t \geqslant 0$. Since trajectories of the system depend continuously on λ, we see that the following holds: given $\tau > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, there exists $\lambda^* \in (0, 1]$ such that

$$\sup_{t\in[0,\tau]} |y_1^{\lambda}(t)| \leqslant \varepsilon_2, \qquad \sup_{t\in[0,\tau]} |y_2^{\lambda}(t)| \leqslant \varepsilon_1, \tag{16}$$

hold for all $\lambda \in [0, \lambda^*]$. Now, let Δ_{1u}^*, Δ_{2u}^* be defined by (10), (11). Take any ε_1, ε_2 such that $\Delta_{1u}^* < \varepsilon_1 < \Delta_{1u}$, $\Delta_{2u}^* < \varepsilon_2 < \Delta_{2u}$, and let $\lambda^* \in (0, 1]$ be maximal number such that (16) holds for all $\lambda \in [0, \lambda^*]$. We have $\sup_{t\in[0,\tau]}|y_1^{\lambda^*}(t)| < \Delta_{2u}$, and $\sup_{t\in[0,\tau]}|y_2^{\lambda^*}(t)| < \Delta_{1u}$, therefore, we can write

$$\sup_{t\in[0,\tau]} |y_1^{\lambda^*}(t)| \leqslant \max\left\{ \begin{array}{c} \beta_1(|x_{1d}^{\lambda^*}(0)|), \gamma_{1w}\left(\sup_{t\in[0,\tau]} |w_1^{\lambda^*}(t)|\right), \\ \gamma_{1u}\circ\chi_2\left(\sup_{t\in[0,\tau]} |y_2^{\lambda^*}(t)|\right) \end{array} \right\}, \tag{17}$$

and

$$\sup_{t\in[0,\tau]} |y_2^{\lambda^*}(t)| \leqslant \max\left\{ \begin{array}{c} \beta_2(|x_{2d}^{\lambda^*}(0)|), \gamma_{2w}\left(\sup_{t\in[0,\tau]} |w_2^{\lambda^*}(t)|\right), \\ \gamma_{2u}\circ\chi_1\left(\sup_{t\in[0,\tau]} |y_1^{\lambda^*}(t)|\right) \end{array} \right\}. \tag{18}$$

Substituting (18) into (17), taking into account that the condition $\gamma_{1u}\circ\chi_2\circ\gamma_{2u}\circ\chi_1(s) < s$ for $s > 0$ is equivalent to (9), and using the fact that $|y| \leqslant \max\{a\cdot|y|, b\}$, $a \in [0, 1)$, $b \geqslant 0$, implies $|y| \leqslant b$, we get

$$\sup_{t\in[0,\tau]} |y_1^{\lambda^*}(t)| \leqslant \max\left\{ \begin{array}{c} \beta_1(|x_{1d}^{\lambda^*}(0)|), \gamma_{1u}\circ\chi_2\circ\beta_2(|x_{2d}^{\lambda^*}(0)|), \\ \gamma_{1u}\circ\chi_2\circ\gamma_{2w}\left(\sup_{t\in[0,\tau]} |w_2^{\lambda^*}(t)|\right), \\ \gamma_{1w}\left(\sup_{t\in[0,\tau]} |w_1^{\lambda^*}(t)|\right) \end{array} \right\}.$$

On the other hand, substituting (17) into (18), and using the same line of reasoning, we have

$$\sup_{t\in[0,\tau]} |y_2^{\lambda^*}(t)| \leqslant \max\left\{ \begin{array}{c} \beta_2(|x_{2d}^{\lambda^*}(0)|), \gamma_{2u}\circ\chi_1\circ\beta_1(|x_{1d}^{\lambda^*}(0)|), \\ \gamma_{2u}\circ\chi_1\circ\gamma_{1w}\left(\sup_{t\in[0,\tau]} |w_1^{\lambda^*}(t)|\right), \\ \gamma_{2w}\left(\sup_{t\in[0,\tau]} |w_2^{\lambda^*}(t)|\right) \end{array} \right\}.$$

Taking into account (10), (11), we see that $\sup_{t\in[0,\tau]}|y_1^{\lambda^*}(t)| \leqslant \Delta_{2u}^* < \varepsilon_2$, and $\sup_{t\in[0,\tau]}|y_2^{\lambda^*}(t)| \leqslant \Delta_{1u}^* < \varepsilon_1$. Now, we can see that $\lambda^* = 1$. Indeed, if $\lambda^* < 1$, then in view of the fact that both $y_1^{\lambda}(\cdot)$ and $y_1^{\lambda}(\cdot)$ depend continuously on λ, we see that there exists $\tilde{\lambda} \in (\lambda^*, 1]$ such that $\sup_{t\in[0,\tau]}|y_1^{\tilde{\lambda}}(t)| \leqslant \varepsilon_2$, $\sup_{t\in[0,\tau]}|y_2^{\tilde{\lambda}}(t)| \leqslant \varepsilon_1$, which contradicts our assumption that λ^* is maximal number such that (16) holds for all $\lambda \in [0, \lambda^*]$. Thus, $\lambda^* = 1$. Taking into account the arbitrary choice of $\tau > 0$, we conclude that the uniform boundedness property holds for the system (7), (8) with $\beta(\cdot) := \max\{\beta_1(\cdot), \gamma_{1u}\circ\chi_2\circ\beta_2(\cdot), \beta_2(\cdot), \gamma_{2u}\circ\chi_1\circ\beta_1(\cdot)\}$, and $\tilde{\gamma}_{1w}(\cdot)$, $\tilde{\gamma}_{2w}(\cdot)$ defined by (12), (13). To prove the convergence, note that

$$\limsup_{t\to+\infty} |y_1(t)| \leqslant \max\left\{ \begin{array}{c} \gamma_{1u}\circ\chi_2\left(\limsup_{t\to\infty} |y_2(t)|\right), \\ \gamma_{1w}\left(\limsup_{t\to+\infty} |w_1(t)|\right) \end{array} \right\},$$

$$\limsup_{t\to+\infty} |y_2(t)| \leqslant \max\left\{ \begin{array}{c} \gamma_{2u}\circ\chi_1\left(\limsup_{t\to\infty} |y_1(t)|\right), \\ \gamma_{2w}\left(\limsup_{t\to+\infty} |w_2(t)|\right) \end{array} \right\}.$$

Combining the last inequalities and using (9), it is easy to see that the system enjoys the convergence property. Thus, the system is IOS. The ISS part of the result is now obvious. This completes the proof of Theorem 1.

4. Stabilization schemes for force-reflecting teleoperation

In this section, a stabilization scheme for force-reflecting teleoperators as well as its two adaptive extensions are given, and the corresponding stability results are formulated. Proofs of these results are presented in Section 5.

4.1. Basic (nonadaptive) stabilization scheme

Below, we introduce a control scheme that stabilizes the bilaterally controlled teleoperator system (1)–(5) independently on communication delays τ_1, $\tau_2 \geqslant 0$. Consider the following control law:

$$\begin{aligned} u_{\rm m} = &-H_{\rm m}(q_{\rm m})\sigma_{\rm m}\dot{q}_{\rm m} - C_{\rm m}(q_{\rm m}, \dot{q}_{\rm m})\sigma_{\rm m}q_{\rm m} \\ &+ G_{\rm m}(q_{\rm m}) - K_{\rm m}(\dot{q}_{\rm m} + \sigma_{\rm m}q_{\rm m}), \end{aligned} \tag{19}$$

$$\begin{aligned} u_{\rm s} = &H_{\rm s}(q_{\rm s})\sigma_{\rm s}(\dot{\hat{q}}_{\rm m} - \dot{q}_{\rm s}) + C_{\rm s}(q_{\rm s}, \dot{q}_{\rm s})\sigma_{\rm s}(\hat{q}_{\rm m} - q_{\rm s}) \\ &+ G_{\rm s}(q_{\rm s}) - K_{\rm s}(\dot{q}_{\rm s} + \sigma_{\rm s}(q_{\rm s} - \hat{q}_{\rm m})), \end{aligned} \tag{20}$$

where $K_{\rm m}, K_{\rm s}, \sigma_{\rm m}, \sigma_{\rm s} \in \mathbb{R}^{n\times n}$ are symmetric positive definite matrices. This control law was proposed in Polushin and Marquez (2003), however, we are now going to prove the corresponding stability result under essentially more general "finite-gain" assumption imposed on environmental dynamics. In the following, given a symmetric matrix K, its minimal (maximal) eigenvalue is denoted by $\lambda_{\min}(K)$ ($\lambda_{\max}(K)$). Also, denote $\tilde{q}_{\rm s} = q_{\rm s} - \hat{q}_{\rm m}$. Our main result is presented in the following theorem.

Theorem 2. *There exist* $\kappa_{\rm m}, \kappa_{\rm s} > 0$ *such that if* $\lambda_{\min}(K_{\rm m}) \geqslant \kappa_{\rm m}$, $\lambda_{\min}(K_{\rm s}) \geqslant \kappa_{\rm s}$, *then the closed-loop telerobotic system* (1)–(5), (19), (20), *with state* $\mathbf{x}_d := (q_{\rm m}^{\rm T}, \dot{q}_{\rm m}^{\rm T}, \tilde{q}_{\rm s}^{\rm T}, \dot{q}_{\rm s}^{\rm T})_d^{\rm T}$, $t_d = \max\{\tau_1, \tau_2\}$ *and input* $u = (F_{\rm h}^{\rm T}, F_{\rm e}^{*\rm T})^{\rm T}$ *is ISS.*

4.2. Adaptive stabilization scheme

A standard statement of the adaptive stabilization problem for robotic manipulators utilizes the so-called linear parameterization property of the manipulator's dynamics (Spong, 1996). Using this property, let us denote

$$-H_m(q_m)\sigma_m\dot{q}_m - C_m(q_m,\dot{q}_m)\sigma_m q_m + G_m(q_m) = Y_{cm}(q_m,\dot{q}_m)\theta_m,$$

where $Y_{cm}(q_m,\dot{q}_m)\in\mathbb{R}^{n\times r}$ is the regressor of the master manipulator, and $\theta_m\in\mathbb{R}^r$ is the vector of parameters of the master manipulator. For the slave manipulator, let us write

$$H_s(q_s)\sigma_s(\dot{\hat{q}}_m - \dot{q}_s) + C_s(q_s,\dot{q}_s)\sigma_s(\hat{q}_m - q_s) + G_s(q_s) = Y_{cs}(q_s,\dot{q}_s,\hat{q}_m,\dot{\hat{q}}_m)\theta_s,$$

where $Y_{cs}(q_s,\dot{q}_s,\hat{q}_m,\dot{\hat{q}}_m)\in\mathbb{R}^{n\times r}$ is the regressor of the slave, and $\theta_s\in\mathbb{R}^r$ is the vector of the slave's parameters. In the following, it is assumed that both θ_m, θ_s are unknown but constant. An adaptive version of the stabilization algorithm (19), (20), can be defined by setting $u_m(t)\equiv 0$, $u_s(t)\equiv 0$ for $t<0$, and

$$u_m = Y_{cm}(q_m,\dot{q}_m)\hat{\theta}_m - K_m(\dot{q}_m + \sigma_m q_m), \tag{21}$$

$$u_s = Y_{cs}(q_s,\dot{q}_s,\hat{q}_m,\dot{\hat{q}}_m)\hat{\theta}_s - K_s(\dot{q}_s + \sigma_s(q_s - \hat{q}_m)), \tag{22}$$

for $t\geqslant 0$, where $\hat{\theta}_m, \hat{\theta}_s\in\mathbb{R}^r$ are estimates for θ_m and θ_s, respectively, that satisfy $\hat{\theta}_m(t)\equiv\hat{\theta}_s(t)\equiv 0$ for $t<0$ and

$$\dot{\hat{\theta}}_m = -\Gamma_m Y_{cm}^T(q_m,\dot{q}_m)(\dot{q}_m + \sigma_m q_m) - \varepsilon_m(\hat{\theta}_m - \theta_m^*), \tag{23}$$

$$\dot{\hat{\theta}}_s = -\Gamma_s Y_{cs}^T(q_s,\dot{q}_s,\hat{q}_m,\dot{\hat{q}}_m)(\dot{q}_s + \sigma_s(q_s - \hat{q}_m)) - \varepsilon_s(\hat{\theta}_s - \theta_s^*), \tag{24}$$

for $t\geqslant 0$, where Γ_m, Γ_s are symmetric positive definite matrices, $\theta_m^*, \theta_s^*\in\mathbb{R}^r$ are vectors that represent nominal values of the parameters θ_m, θ_s respectively, and $\varepsilon_m, \varepsilon_s>0$ are arbitrary constants. Also, denote $\tilde{\theta}_m := \hat{\theta}_m - \theta_m$, $\tilde{\theta}_s := \hat{\theta}_s - \theta_s$, and $\tilde{q}_s = q_s - \hat{q}_m$. Now, one can state the following result:

Theorem 3. *Given $\delta>0$, there exist $\kappa_m, \kappa_s, g_m, g_s>0$, such that if $\lambda_{\min}(K_m)\geqslant\kappa_m$, $\lambda_{\min}(K_s)\geqslant\kappa_s$, $\lambda_{\min}(\Gamma_m)\geqslant g_m$, and $\lambda_{\min}(\Gamma_s)\geqslant g_s$, then the closed-loop telerobotic system* (1)–(5), (21)–(24) *with state $\mathbf{x}_d := (q_m^T, \dot{q}_m^T, \tilde{\theta}_m^T, \tilde{q}_s^T, \dot{q}_s^T, \tilde{\theta}_s^T)_d^T$, input (F_h, F_e^*), and output $y := (q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T)^T$ is* (i) *ISS with some offset $D\geqslant 0$, and* (ii) *IOS with offset less than or equal to δ.*

4.3. Adaptive stabilization without velocity measurement

Now, let us address the adaptive stabilization problem for bilaterally controlled teleoperators in the situation where the joint velocities are not available for measurement. Moreover, we assume that the joint positions of both master and slave manipulators are available for measurement subject to (small) measurement disturbances. More precisely, let

$$\bar{q}_m(t) = q_m(t) + \Omega_m(t), \quad \bar{q}_s(t) = q_s(t) + \Omega_s(t), \tag{25}$$

be measured joint angles of the master and the slave manipulator, respectively, where q_m, q_s are the actual positions, and Ω_m, Ω_s are measurement disturbances. In the following, we assume that $\omega_m(t) := \mathrm{d}\Omega_m(t)/\mathrm{d}t$ exists for almost all t, and $\int_a^b \omega_m = \Omega_m(b) - \Omega_m(a)$, and analogously for $\omega_s(t) := \mathrm{d}\Omega_s(t)/\mathrm{d}t$. This can be guaranteed, for example, by assuming that $\Omega_m(\cdot)$, $\Omega_s(\cdot)$ are absolutely continuous (Shilov, 1974). The estimates v_m, v_s for the master's and slave's velocities can be defined in the Laplace domain as follows:

$$v_m(s) := \frac{\rho_m s}{s+\rho_m}\bar{q}_m(s), \quad v_s(s) := \frac{\rho_s s}{s+\rho_s}\bar{q}_s(s), \tag{26}$$

where $\rho_m>0$, $\rho_s>0$ are constants to be determined. The initial conditions of the filters (26) in the time domain are set $v_m(-t_d)=v_s(-t_d)=0$, where $t_d\geqslant 0$ is defined below. The signals

$$\hat{q}_m(t) = \bar{q}_m(t-\tau_1), \quad \hat{v}_m(t) = v_m(t-\tau_1) \tag{27}$$

are available on the slave side for $t\geqslant 0$ (it is assumed that $\hat{q}_m(t)\equiv 0$, $\hat{v}_m(t)\equiv 0$ for $t<0$). Substituting the estimates v_m, v_s for the velocities in the control law of the previous section, we get the following control algorithm:

$$u_m = Y_{cm}(\bar{q}_m, v_m)\hat{\theta}_m - K_m(v_m + \sigma_m\bar{q}_m), \tag{28}$$

$$u_s = Y_{cs}(\bar{q}_s, v_s, \hat{q}_m, \hat{v}_m)\hat{\theta}_s - K_s(v_s + \sigma_s(\bar{q}_s - \hat{q}_m)) \tag{29}$$

for $t\geqslant 0$, where $Y_{cm}(\cdot)$, $Y_{cs}(\cdot)$ are regressor matrices defined in Section 4.2, $\hat{\theta}_m, \hat{\theta}_s\in R^r$ are estimates for θ_m and θ_s, respectively, and $u_m(t)\equiv 0$, $u_s(t)\equiv 0$ for $t<0$. The estimates $\hat{\theta}_m$, $\hat{\theta}_s$ are assumed to satisfy $\hat{\theta}_m(t)\equiv\hat{\theta}_s(t)\equiv 0$ for $t\leqslant 0$, and

$$\dot{\hat{\theta}}_m = -\Gamma_m Y_{cm}^T(\bar{q}_m, v_m)(v_m + \sigma_m\bar{q}_m) - \varepsilon_m(\hat{\theta}_m - \theta_m^*), \tag{30}$$

$$\dot{\hat{\theta}}_s = -\Gamma_s Y_{cs}^T(\bar{q}_s, v_s, \hat{q}_m, \hat{v}_m)(v_s + \sigma_s(\bar{q}_s - \hat{q}_m)) - \varepsilon_s(\hat{\theta}_s - \theta_s^*) \tag{31}$$

for $t\geqslant 0$, where Γ_m, Γ_s are symmetric positive definite matrices, and $\theta_m^*, \theta_s^*\in\mathbb{R}^r$ are nominal (expected) values of the parameters θ_m, θ_s, respectively. Denote $w_m := v_m - \dot{q}_m$, $w_s := v_s - \dot{q}_s$. The main result of this section is as follows.

Theorem 4. *Given $\Delta_x, \Delta_F\geqslant 0$, $\delta>0$, there exist constants $\kappa_m, \kappa_s, g_m, g_s>0$, such that the following holds. Suppose $\lambda_{\min}(K_m)\geqslant\kappa_m$, $\lambda_{\min}(K_s)\geqslant\kappa_s$, $\lambda_{\min}(\Gamma_m)\geqslant g_m$, and $\lambda_{\min}(\Gamma_s)\geqslant g_s$. Then there exists $\rho_m^*>0$, $\rho_s^*>0$ dependent on K_m, K_s, and $\Omega_m^*>0$, $\Omega_s^*>0$ dependent on K_m, K_s, Γ_m, Γ_s such that if $\rho_m\geqslant\rho_m^*$, $\rho_s\geqslant\rho_s^*$, and the measurement disturbances satisfy $|\Omega_m(t)|\leqslant\Omega_m^*$, $|\Omega_s(t)|\leqslant\Omega_s^*$ for almost all $t\geqslant -\max\{\tau_1,\tau_2\}$, then the overall telerobotic system* (1), (2), (4), (25)–(31) *with state $\mathbf{x}_d := (q_m^T, \dot{q}_m^T, \tilde{\theta}_m^T, \tilde{q}_s^T, \dot{q}_s^T, \tilde{\theta}_s^T, w_m^T, w_s^T)_d^T$, $t_d = \max\{\tau_1,\tau_2\}$, is ISS with restriction (Δ_x, Δ_F), where Δ_x corresponds to state, and Δ_F is a restriction for F_h, F_e^*, and some offset $D>0$. Moreover, the offset for output $y := (q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T, w_m^T, w_s^T)^T$ is less than or equal to δ.*

5. Proofs

This section contains proofs of the results presented above in Section 4. Essentially, the following line of reasoning is utilized in all the proofs presented. First, we show that the proposed control algorithms make both the master and the slave closed-loop subsystems input-to-state (input-to-output) stable with respect to a certain set of input and possibly output signals. Also, some of the ISS (IOS) gains can be assigned arbitrarily small by appropriate choice of the control algorithm's parameters. Further, we consider both the master and the slave closed-loop subsystems together with the corresponding communication delay in the input channel. Due to Lemma 2 (Appendix B), both the subsystems inherit the ISS (IOS) property in the sense of Definition 1, with the same ISS (IOS) gains. To finalize the proof of stability, the version of the IOS small gain theorem for FDE (Theorem 1) is applied.

5.1. Proof of Theorem 2

Consider the closed-loop master subsystem (1), (19).

Proposition 1. *Given $\gamma^*_{Fm} > 0$, there exists $\kappa_m \geqslant 0$ such that if $\lambda_{\min}(K_m) \geqslant \kappa_m$ then the closed-loop "master" subsystem (1), (19) with state $x_M := (q_m^T, \dot{q}_m^T)^T$ and input $u_M := (F_h^T, \hat{F}_e^T)^T$ is ISS with ISS gain less than or equal to γ^*_{Fm}.*

Proof. Denote $e_m = \sigma_m q_m + \dot{q}_m$. Consider the ISS-Lyapunov function candidate $V_m = \frac{1}{2} e_m^T H_m(q_m) e_m + \frac{1}{2} q_m^T q_m$. It is easy to check that $\alpha_{1m}(|\dot{q}_m|^2 + |q_m|^2) \leqslant V_m \leqslant \alpha_{2m}(|\dot{q}_m|^2 + |q_m|^2)$, for some $\alpha_2 \geqslant \alpha_1 > 0$. Further, using properties of Euler–Lagrange equations (Spong, 1996) and completing the squares, for $\lambda_{\min}(K_m) \geqslant 2$ we see that the time derivative of V along the trajectories of (1), (19) satisfies

$$\dot{V}_m \leqslant -\frac{c_{1m}}{2}(|\dot{q}_m|^2 + |q_m|^2) + \frac{1}{\lambda_{\min}(K_m)}|F_h + \hat{F}_e|^2$$
$$\leqslant -\frac{c_{1m}}{2\alpha_2} V_m + \frac{1}{\lambda_{\min}(K_m)}|F_h + \hat{F}_e|^2.$$

Applying the results of Sontag and Wang (1995), we see that an arbitrary ISS gain can be assigned by increasing $\lambda_{\min}(K_m) > 0$. This completes the proof of Proposition 1. □

Now, one can consider the "master + input delay" subsystem (1), (4), (19) as a system of FDE with input $(F_h^T, F_e^T)^T$ and $t_d = \tau_2 \geqslant 0$. We see that the following result is valid by combination of Lemma 2 (Appendix B) and Proposition 1.

Proposition 2. *Given $\gamma^*_{Fm} > 0$, there exists $\kappa_m > 0$ such that if $\lambda_{\min}(K_m) \geqslant \kappa_m$, then the closed-loop "master + input delay" subsystem (1), (4), (19) with input $(F_h^T, F_e^T)^T$ is ISS with $t_d = \tau_2 \geqslant 0$, and the corresponding ISS gain is less than or equal to γ^*_{Fm}.*

Now, consider the "slave + environment" subsystem (2), (5), (20).

Proposition 3. *Given $\gamma^*_{F_e}, \gamma^*_{\hat{q}} > 0$, there exists $\kappa_s \geqslant 0$ such that if $\lambda_{\min}(K_s) \geqslant \kappa_s$ then the subsystem (2), (5), (20) with state $x_S := (\tilde{q}_s^T, \dot{q}_s^T)^T$ and inputs F_e^*, $\hat{q}_m$, and $\hat{\dot{q}}_m$, is ISS, and the ISS gains with respect to the first two inputs are less than or equal to $\gamma^*_{F_e}$ and $\gamma^*_{\hat{q}}$, respectively. Moreover, there exists $\gamma^*_{\hat{\dot{q}}} > 0$ independent on $\gamma^*_{F_e}$, $\gamma^*_{\hat{q}}$ such that the ISS gain with respect to input $\hat{\dot{q}}_m$ is less than or equal to $\gamma^*_{\hat{\dot{q}}}$.*

Proof. Denote $e_s = \sigma_s \tilde{q}_s + \dot{q}_s$. Consider the ISS-Lyapunov function candidate $V_s = \frac{1}{2} e_s^T H_s(q_s) e_s + \frac{1}{2} \tilde{q}_s^T \tilde{q}_s$. Again, it is easy to check that $\alpha_{1s}(|\dot{q}_s|^2 + |\tilde{q}_s|^2) \leqslant V_s \leqslant \alpha_{2s}(|\dot{q}_s|^2 + |\tilde{q}_s|^2)$, for some $\alpha_{2s} \geqslant \alpha_{1s} > 0$. Calculating the time derivative of V_s along the trajectories of the closed-loop system (2), (5), (20), and completing the squares, one can get the following inequality:

$$\frac{d}{dt} V_s \leqslant -\left(\frac{1}{2}\lambda_{\min}(K_s) - \frac{\gamma_e}{c_{1s}}\right)|e_s|^2 - \left(\frac{3\sigma_s}{4} - \frac{2(\gamma_e^2 + c_{1s}^2)}{\lambda_{\min}(K_s) c_{1s}^2}\right)|\tilde{q}_s|^2 + \frac{2(|F_e^*|^2 + \gamma_e^2 |\hat{q}_m|^2)}{\lambda_{\min}(K_s)} + \frac{|\hat{\dot{q}}_m|^2}{\sigma_s}.$$

Again, applying the results of Sontag and Wang (1995), it is easy to check that the system can be made ISS and arbitrary ISS gains can be assigned for the inputs F_e^*, $\hat{q}_m$ by choosing $\lambda_{\min}(K_s) > 0$ sufficiently large. The statement of Proposition 3 follows. □

Combining Lemma 2 with Proposition 3, and using simple calculations, it is easy to get the following fact.

Proposition 4. *Given $\gamma^*_{F_e}, \gamma^*_{\hat{q}} > 0$, there exists $\kappa_s \geqslant 0$ such that if $\lambda_{\min}(K_s) \geqslant \kappa_s$, then the closed-loop "slave + environment + input delay" subsystem (2), (3), (5), (20) with inputs F_e^*, q_m, $\dot{q}_m$, state $(\tilde{q}_s^T, \dot{q}_s^T)_d^T$, $t_d = \tau_1 \geqslant 0$, and output $y_S := (q_s^T, \dot{q}_s^T)^T$ is IOS, and the IOS gains for inputs F_e^*, q_m are less than or equal to $\gamma^*_{F_e}$, $\gamma^*_{\hat{q}} + 1$, respectively. Moreover, there exists $\gamma^*_{\hat{\dot{q}}} > 0$ independent on $\gamma^*_{F_e}$, $\gamma^*_{\hat{q}}$, such that the IOS gain for input $\dot{q}_m$ is less than or equal to $\gamma^*_{\hat{\dot{q}}}$.*

Now, the proof of Theorem 2 can be finalized as follows. Consider the force-reflecting telerobotic system as a feedback interconnection of two input-to-state (input-to-output) stable subsystems of FDE, namely, the closed-loop "master + input delay" subsystem (1), (4), (19), and the closed-loop "slave + environment + input delay" subsystem (2), (3), (5), (20). Applying Theorem 1 (assuming all restrictions are infinite), we see that the overall telerobotic system is ISS, if

$$\gamma^*_{Fm} \cdot \max\{\gamma^*_{\hat{q}} + 1, \gamma^*_{\hat{\dot{q}}}\} \cdot \gamma_e < 1. \quad (32)$$

Since $\gamma^*_{Fm} > 0$ can be assigned arbitrarily, the last inequality can always be satisfied. This completes the proof of Theorem 2.

5.2. Proof of Theorem 3

Consider the closed-loop master subsystem (1), (21), (23). Using the notation $\bar{\theta}^*_{\mathrm{m}} = \theta_{\mathrm{m}} - \theta^*_{\mathrm{m}}$, one can rewrite the control/adaptation law (21), (23) as follows:

$$u_{\mathrm{m}} = Y_{\mathrm{cm}}(q_{\mathrm{m}}, \dot{q}_{\mathrm{m}})(\tilde{\theta}_{\mathrm{m}} + \theta_{\mathrm{m}}) - K_{\mathrm{m}}(\dot{q}_{\mathrm{m}} + \sigma_{\mathrm{m}} q_{\mathrm{m}}) \quad (33)$$

$$\dot{\tilde{\theta}}_{\mathrm{m}} = -\Gamma_{\mathrm{m}} Y^{\mathrm{T}}_{\mathrm{cm}}(q_{\mathrm{m}}, \dot{q}_{\mathrm{m}})(\dot{q}_{\mathrm{m}} + \sigma_{\mathrm{m}} q_{\mathrm{m}}) - \varepsilon_{\mathrm{m}}(\tilde{\theta}_{\mathrm{m}} + \bar{\theta}^*_{\mathrm{m}}). \quad (34)$$

Below, consider $\bar{\theta}^*_{\mathrm{m}}$ formally as an additional input.

Proposition 5. *Given $\gamma^*_{F\mathrm{m}}, \gamma^*_{\theta\mathrm{m}} > 0$, there exist $\kappa_{\mathrm{m}}, g_{\mathrm{m}} \geqslant 0$ such that if $\lambda_{\min}(K_{\mathrm{m}}) \geqslant \kappa_{\mathrm{m}}$, and $\lambda_{\min}(\Gamma) \geqslant g_{\mathrm{m}}$, then the closed-loop "master + input delay" subsystem* (1), (4), (33), (34), *with state $(q^{\mathrm{T}}_{\mathrm{m}}, \dot{q}^{\mathrm{T}}_{\mathrm{m}}, \tilde{\theta}^{\mathrm{T}}_{\mathrm{m}})^{\mathrm{T}}_d$, $t_d = \tau_2 \geqslant 0$, and inputs F_{h}, F_{e}, $\bar{\theta}^*_{\mathrm{m}}$ is ISS. Moreover, the IOS gains from inputs F_{h}, F_{e}, $\bar{\theta}^*_{\mathrm{m}}$ to output $y_{\mathrm{m}} = (q^{\mathrm{T}}_{\mathrm{m}}, \dot{q}^{\mathrm{T}}_{\mathrm{m}})^{\mathrm{T}}$ are less than or equal to $\gamma^*_{F\mathrm{m}}$, $\gamma^*_{F\mathrm{m}}$, and $\gamma^*_{\theta\mathrm{m}}$, respectively.*

Proof. Based on using the ISS Lyapunov function candidate $V_{\mathrm{m}} = \frac{1}{2} e^{\mathrm{T}}_{\mathrm{m}} H_{\mathrm{m}}(q_{\mathrm{m}}) e_{\mathrm{m}} + \frac{1}{2} q^{\mathrm{T}}_{\mathrm{m}} q_{\mathrm{m}} + \frac{1}{2} \tilde{\theta}^{\mathrm{T}}_{\mathrm{m}} \Gamma^{-1} \tilde{\theta}_{\mathrm{m}}$, and subsequent application of Lemma 2. Detailed proof is omitted due to space limitation reasons. □

Now, using the notation $\dot{\tilde{q}}_{\mathrm{s}} = \dot{q}_{\mathrm{s}} - \dot{\hat{q}}_{\mathrm{m}}$, $\bar{\theta}^*_{\mathrm{s}} = \theta_{\mathrm{s}} - \theta^*_{\mathrm{s}}$, the control/adaptation law for the slave subsystem (22), (24) can be rewritten for $t \geqslant 0$ as follows

$$u_{\mathrm{s}} = Y_{\mathrm{cs}}(q_{\mathrm{s}}, \dot{q}_{\mathrm{s}}, \hat{q}_{\mathrm{m}}, \dot{\hat{q}}_{\mathrm{m}})\tilde{\theta}_{\mathrm{s}} + Y_{\mathrm{cs}}(q_{\mathrm{s}}, \dot{q}_{\mathrm{s}}, \hat{q}_{\mathrm{m}}, \dot{\hat{q}}_{\mathrm{m}})\theta_{\mathrm{s}} - K_{\mathrm{s}}(\dot{q}_{\mathrm{s}} + \sigma_{\mathrm{s}} \tilde{q}_{\mathrm{s}}), \quad (35)$$

$$\dot{\tilde{\theta}}_{\mathrm{s}} = -\Gamma_{\mathrm{s}} Y^{\mathrm{T}}_{\mathrm{cs}}(q_{\mathrm{s}}, \dot{q}_{\mathrm{s}}, \hat{q}_{\mathrm{m}}, \dot{\hat{q}}_{\mathrm{m}})(\dot{q}_{\mathrm{s}} + \sigma_{\mathrm{s}} \tilde{q}_{\mathrm{s}}) - \varepsilon_{\mathrm{s}}(\tilde{\theta}_{\mathrm{s}} + \bar{\theta}^*_{\mathrm{s}}). \quad (36)$$

Again, let us consider formally $\bar{\theta}^*_{\mathrm{s}}$ as an additional input. The ISS of the closed-loop slave subsystem is stated in the following proposition.

Proposition 6. *Given $\gamma^*_{F_{\mathrm{e}}}, \gamma^*_{\hat{q}}, \gamma^*_{\theta} > 0$, there exist $\kappa_{\mathrm{s}}, g_{\mathrm{s}} \geqslant 0$ such that if $\lambda_{\min}(K_{\mathrm{s}}) \geqslant \kappa_{\mathrm{s}}$, and $\lambda_{\min}(\Gamma) \geqslant g_{\mathrm{s}}$, then the closed-loop "slave + environment + input delay" subsystem* (2), (3), (5), (22), (24), *with state $(\tilde{q}^{\mathrm{T}}_{\mathrm{s}}, \dot{q}^{\mathrm{T}}_{\mathrm{s}}, \theta^{\mathrm{T}}_{\mathrm{s}})^{\mathrm{T}}_d$, $t_d = \tau_1 \geqslant 0$, inputs F^*_{e}, q_{m}, $\bar{\theta}^*_{\mathrm{s}}$, $\dot{q}_{\mathrm{m}}$, and output $y_{\mathrm{s}} = (q^{\mathrm{T}}_{\mathrm{s}}, \dot{q}^{\mathrm{T}}_{\mathrm{s}})^{\mathrm{T}}$ is ISS* (*and IOS*), *and the corresponding IOS gains from inputs F^*_{e}, $\hat{q}_{\mathrm{m}}$, $\bar{\theta}^*_{\mathrm{s}}$ to output $y_{\mathrm{S}} := (\tilde{q}^{\mathrm{T}}_{\mathrm{s}}, \dot{q}^{\mathrm{T}}_{\mathrm{s}})^{\mathrm{T}}$ are less than or equal to $\gamma^*_{F_{\mathrm{e}}}$, $\gamma^*_{\hat{q}} + 1$, γ^*_{θ}, respectively. Moreover, there exists $\gamma^*_{\dot{\hat{q}}} > 0$ independent on $\gamma^*_{F_{\mathrm{e}}}, \gamma^*_{\hat{q}}, \gamma^*_{\theta\mathrm{s}}$, such that the IOS gain from input $\dot{\hat{q}}_{\mathrm{m}}$ to output y_{s} is less than or equal to $\gamma^*_{\dot{\hat{q}}}$.*

Proof. Utilizes the ISS-Lyapunov function candidate $V_{\mathrm{s}} = \frac{1}{2} e^{\mathrm{T}}_{\mathrm{s}} H_{\mathrm{s}}(q_{\mathrm{s}}) e_{\mathrm{s}} + \frac{1}{2} \tilde{q}^{\mathrm{T}}_{\mathrm{s}} \tilde{q}_{\mathrm{s}} + \frac{1}{2} \tilde{\theta}^{\mathrm{T}}_{\mathrm{s}} \Gamma^{-1} \tilde{\theta}_{\mathrm{s}}$, where $e_{\mathrm{s}} = \sigma_{\mathrm{s}} \tilde{q}_{\mathrm{s}} + \dot{q}_{\mathrm{s}}$. Calculating the time derivative of V_{s} along the trajectories of the closed-loop "slave-environment" subsystem (2), (5), (22), (22) (i.e., the system without input delay), and applying Lemma 2, one can obtain the result of Proposition 6.Again, detailed proof is omitted due to space limitation reasons. □

Now, applying Theorem 1 (assuming all restrictions are infinite), we see that the overall telerobotic system is ISS, if the small gain condition (32) is satisfied. The last can always be achieved, since $\gamma^*_{F\mathrm{m}} > 0$ can be assigned arbitrarily. Moreover, calculating the IOS gains for inputs $\bar{\theta}^*_{\mathrm{m}}$, $\bar{\theta}^*_{\mathrm{s}}$ according to formulas (12), (13), one get $\tilde{\gamma}^*_{\theta\mathrm{m}} := \gamma^*_{\theta\mathrm{m}} \max\{1, \gamma^*_{\hat{q}}, \gamma^*_{\dot{\hat{q}}}\}$, and $\tilde{\gamma}^*_{\theta\mathrm{s}} := \gamma^*_{\theta\mathrm{s}} \max\{1, \gamma^*_{F\mathrm{m}} \cdot \gamma^*_{\hat{q}}\}$. We see that both $\tilde{\gamma}^*_{\theta\mathrm{m}} > 0$, $\tilde{\gamma}^*_{\theta\mathrm{s}} > 0$ can be assigned arbitrarily by decreasing $\gamma^*_{\theta\mathrm{m}} > 0$, $\gamma^*_{\theta\mathrm{s}} > 0$. Now, given $\delta > 0$, let $\tilde{\gamma}^*_{\theta\mathrm{m}} > 0$, $\tilde{\gamma}^*_{\theta\mathrm{s}} > 0$ be such that

$$\delta \geqslant \max\{\tilde{\gamma}^*_{\theta\mathrm{m}} \cdot \bar{\theta}^*_{\mathrm{m}}, \tilde{\gamma}^*_{\theta\mathrm{s}} \cdot \bar{\theta}^*_{\mathrm{s}}\}. \quad (37)$$

In this case, the offset of the output $y = (q^{\mathrm{T}}_{\mathrm{m}}, \dot{q}^{\mathrm{T}}_{\mathrm{m}}, \tilde{q}^{\mathrm{T}}_{\mathrm{s}}, \dot{q}^{\mathrm{T}}_{\mathrm{s}})^{\mathrm{T}}$ due to parameters $\bar{\theta}^*_{\mathrm{m}}$, $\bar{\theta}^*_{\mathrm{s}}$ is less than or equal to δ. The proof of Theorem 3 is complete.

5.3. Proof of Theorem 4

The equations of control/adaptation law (28), (30) of the master subsystem can be rewritten as follows

$$u_{\mathrm{m}} = Y_{\mathrm{cm}}(q_{\mathrm{m}} + \Omega_{\mathrm{m}}, \dot{q}_{\mathrm{m}} + w_{\mathrm{m}})(\tilde{\theta}_{\mathrm{m}} + \theta_{\mathrm{m}}) - K_{\mathrm{m}}(\dot{q}_{\mathrm{m}} + w_{\mathrm{m}} + \sigma_{\mathrm{m}} q_{\mathrm{m}} + \sigma_{\mathrm{m}} \Omega_{\mathrm{m}}), \quad (38)$$

$$\dot{\tilde{\theta}}_{\mathrm{m}} = -\Gamma_{\mathrm{m}} Y^{\mathrm{T}}_{\mathrm{cm}}(q_{\mathrm{m}} + \Omega_{\mathrm{m}}, \dot{q}_{\mathrm{m}} + w_{\mathrm{m}}) \times (\dot{q}_{\mathrm{m}} + w_{\mathrm{m}} + \sigma_{\mathrm{m}} q_{\mathrm{m}} + \sigma_{\mathrm{m}} \Omega_{\mathrm{m}}) - \varepsilon_{\mathrm{m}}(\tilde{\theta}_{\mathrm{m}} + \bar{\theta}^*_{\mathrm{m}}), \quad (39)$$

where $\bar{\theta}^*_{\mathrm{m}} = \theta_{\mathrm{m}} - \theta^*_{\mathrm{m}}$. At this point, consider $\bar{\theta}^*_{\mathrm{m}}$, Ω_{m}, and w_{m} as additional inputs. The following result is valid.

Proposition 7. *Given $\gamma^*_{F\mathrm{m}}, \gamma^*_{\theta\mathrm{m}} > 0$, $\Delta_{x\mathrm{m}} \in (0, +\infty)$, $\Delta_{F\theta\mathrm{m}} \in (0, +\infty)$, there exist $\kappa_{\mathrm{m}} \geqslant 0$, $g_{\mathrm{m}} \geqslant 0$, and $\Delta_{\Omega w\mathrm{m}} \in (0, +\infty)$, such that if $\lambda_{\min}(K_{\mathrm{m}}) \geqslant \kappa_{\mathrm{m}}$, and $\lambda_{\min}(\Gamma_{\mathrm{m}}) \geqslant g_{\mathrm{m}}$, then the following hold*: (i) *the "master + input delay" subsystem* (1), (4), (38), (39) *with state $(q^{\mathrm{T}}_{\mathrm{m}}, \dot{q}^{\mathrm{T}}_{\mathrm{m}}, \tilde{\theta}^{\mathrm{T}}_{\mathrm{m}})^{\mathrm{T}}_d$, $t_d = \tau_2 \geqslant 0$, output $y_{\mathrm{m}} = (q^{\mathrm{T}}_{\mathrm{m}}, \dot{q}^{\mathrm{T}}_{\mathrm{m}})^{\mathrm{T}}$, and inputs F_{h}, F_{e}, $\bar{\theta}^*_{\mathrm{m}}$, Ω_{m}, w_{m}, is ISS* (*and therefore IOS*) *with restriction* $(\Delta_{x\mathrm{m}}, \Delta_{F\theta\mathrm{m}}, \Delta_{\Omega w\mathrm{m}})$, *where $\Delta_{x\mathrm{m}}$ corresponds to state, $\Delta_{F\theta\mathrm{m}}$ corresponds to inputs F_{h}, F_{e}, $\bar{\theta}^*_{\mathrm{m}}$, and $\Delta_{\Omega w\mathrm{m}}$ is the restriction for Ω_{m}, w_{m}*; (ii) *the IOS gains for inputs F_{h}, $\hat{F}_{\mathrm{e}}$ are less than or equal to $\gamma^*_{F\mathrm{m}}$, and* (iii) *the IOS gain for input $\bar{\theta}^*_{\mathrm{m}}$ is less than or equal to $\gamma^*_{\theta\mathrm{m}}$.*

Proof. Consider the system (1), (38), (39) with $\Omega_{\mathrm{m}}(t) \equiv 0$, $w_{\mathrm{m}}(t) \equiv 0$. It is already shown in Proposition 5 that this system can be made input-to-state stable, and the IOS gains are arbitrary prescribed, if K_{m}, Γ_{m} are chosen sufficiently large.The statement of Proposition 7 now follows from Lemma 3 (Appendix B), and Lemma 2. □

Now, similar result can be obtained for the slave subsystem. Denote $w_{\mathrm{s}} = v_{\mathrm{s}} - \dot{q}_{\mathrm{s}}$, $\bar{\theta}^*_{\mathrm{s}} = \theta_{\mathrm{s}} - \theta^*_{\mathrm{s}}$. The equations of control/adaptation law for the slave subsystem (29), (31) can be rewritten as follows:

$$u_s = Y_{cs}(q_s + \Omega_s, \dot{q}_s + w_s, \hat{q}_m, \hat{v}_m)(\theta_s + \tilde{\theta}_s) - K_s(\dot{q}_s + w_s + (q_s + \Omega_s - \hat{q}_m)), \tag{40}$$

$$\dot{\tilde{\theta}}_s = -\Gamma_s Y_{cs}^T(q_s + \Omega_s, \dot{q}_s + w_s, \hat{q}_m, \hat{v}_m) \times (\dot{q}_s + w_s + (q_s + \Omega_s - \hat{q}_m - \hat{\Omega}_m)) - \varepsilon_s(\tilde{\theta}_s + \bar{\theta}_s^*). \tag{41}$$

Combining Proposition 6 with Lemma 3, i.e., using the same line of reasoning as in the case of the master subsystem, one can obtain the following result.

Proposition 8. *Given $\gamma_{F_e}^*, \gamma_{\hat{q}}^*, \gamma_{\theta}^*, \Delta_x, \Delta_{Fq\theta} > 0$, there exist $\kappa_s, g_s \geqslant 0$, and $\Delta_{\Omega w} > 0$ such that if $\lambda_{\min}(K_s) \geqslant \kappa_s$, and $\lambda_{\min}(\Gamma_s) \geqslant g_s$, then the following hold*: (i) *the closed-loop "slave + environment + input delay" subsystem* (2), (3), (5), (40), (41) *with state $(\tilde{q}_s^T, \dot{q}_s^T, \theta_s^T)_d^T$, $t_d = \tau_1 \geqslant 0$, inputs F_e^*, q_m, $\bar{\theta}_s^*$, $\dot{q}_m$, Ω_s, Ω_m, w_s, w_m, and output $y_s = (q_s^T, \dot{q}_s^T)^T$ is ISS* (*and therefore IOS*) *with restriction* $(\Delta_x, \Delta_{Fq\theta}, \Delta_{\Omega w})$, *where Δ_x is the restriction for the state, $\Delta_{Fq\theta}$ is for the inputs F_e^*, q_m, $\bar{\theta}_s^*$, $\dot{q}_m$, while $\Delta_{\Omega w}$ is for the inputs Ω_s, Ω_m, w_s, w_m*; (ii) *the IOS gains for inputs F_e^*, q_m, $\bar{\theta}_s^*$ are less than or equal to $\gamma_{F_e}^*$, $\gamma_{\hat{q}}^* + 1$, and $\gamma_{\theta s}^*$, respectively*; (iii) *there exists $\gamma_{\dot{\hat{q}}}^* < +\infty$ independent on $\gamma_{F_e}^*$, $\gamma_{\hat{q}}^*$, $\gamma_{\theta s}^*$, such that the IOS gain for input $\dot{q}_m$ is less than or equal to $\gamma_{\dot{\hat{q}}}^*$.*

Now, combining Propositions 7 and 8 and applying the IOS (ISS) small gain theorem (Theorem 1) and Corollary 1, the following statement can be obtained.

Proposition 9. *Given $\tilde{\gamma}_{\theta m}^*, \tilde{\gamma}_{\theta s}^* > 0$, $\Delta_x, \Delta_F \geqslant 0$, $\Delta_{\theta s} \geqslant \max\{|\bar{\theta}_m^*|, |\bar{\theta}_s^*|\}$, there exist $\kappa_m, \kappa_s, g_m, g_s > 0$, $\Delta_{\Omega w} > 0$ such that if $\lambda_{\min}(K_m) \geqslant \kappa_m$, $\lambda_{\min}(K_s) \geqslant \kappa_s$, $\lambda_{\min}(\Gamma_m) \geqslant g_m$, and $\lambda_{\min}(\Gamma_s) \geqslant g_s$, then the following hold*: (i) *the system* (1)–(5), (38)–(41) *with state $(q_m^T, \dot{q}_m^T, \tilde{\theta}_m^T, \tilde{q}_s^T, \dot{q}_s^T, \tilde{\theta}_s^T)_d^T$, output $y = (q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T)^T$, and the following set of inputs F_h, F_e^*, $\bar{\theta}_m^*$, $\bar{\theta}_s^*$, Ω_m, Ω_s, w_m, w_s, is ISS with $t_d = \max\{\tau_1, \tau_2\}$, and restrictions $(\Delta_x, \Delta_F, \Delta_\theta, \Delta_{\Omega w})$, where Δ_x corresponds to the state, Δ_F is for the inputs F_h, F_e^*, Δ_θ is for the "inputs" $\bar{\theta}_m^*$, $\bar{\theta}_s^*$, while $\Delta_{\Omega w}$ is the restriction for the inputs Ω_m, Ω_s, w_m, w_s*; (ii) *the IOS gain for "inputs" $\bar{\theta}_m^*$, $\bar{\theta}_s^*$ are less than or equal to $\tilde{\gamma}_{\theta m}^*$, $\tilde{\gamma}_{\theta s}^*$, respectively.*

Now, the proof of Theorem 4 can be finalized as follows. Let $\delta, \Delta_x, \Delta_F > 0$ be given. Let the IOS gains $\tilde{\gamma}_{\theta m}^*, \tilde{\gamma}_{\theta s}^* > 0$ be such that (37) holds. Then, by Proposition 9, the offset due to parameters $\bar{\theta}_m^*$, $\bar{\theta}_s^*$ is less than or equal to δ. Moreover, let $\gamma_\Omega \in \mathscr{K}$ be the IOS gain for inputs Ω_m, Ω_s, and $\gamma_w \in \mathscr{K}$ be the IOS gain for inputs w_m, w_s. Suppose $\Omega_0, w_0 \in (0, \Delta_{\Omega w}]$ are such that $\delta \geqslant \max\{\gamma_\Omega(\Omega_0), \gamma_w(w_0), w_0\}$. Denote $x := (q_m^T, \dot{q}_m^T, \tilde{\theta}_m^T, \tilde{q}_s^T, \dot{q}_s^T, \tilde{\theta}_s^T)^T$and $t_d = \max\{\tau_1, \tau_2\}$. Taking into account the ISS property as formulated in Proposition 9 and the equations of the telerobotic system, decreasing $\Omega_0 > 0$ if necessary, applying Lemma 1 (presented in Appendix A), and using simple contradiction arguments, one can conclude that the following fact holds. Suppose $|x(s)| \leqslant \Delta_x$ for all $s \in [-t_d, 0]$, and $\max\{|F_h(s)|, |F_e^*(s)|\} \leqslant \Delta_F$ for almost all $s \geqslant -t_d$. Suppose also $\max\{|\Omega_m(s)|, |\Omega_s(s)|\} \leqslant \Omega_0$ for almost all $s \geqslant -t_d$. Then there exist $\rho_m^* > 0$, $\rho_s^* > 0$, such that if the cutoff frequencies of the filters (26) satisfy $\rho_m \geqslant \rho_m^*$, $\rho_s \geqslant \rho_s^*$, then

$$\sup_{t \in [-t_d, +\infty)} \max\{|w_m(t)|, |w_s(t)|\} \leqslant w_0. \tag{42}$$

Details of this derivation are omitted here due to space limitation reasons. From (42), due to our choice of Ω_0 and w_0, we see that (i) the offset of the output $y = (q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T)^T$ due to Ω_m, Ω_s, w_m, w_s is less than or equal to δ and (ii) $\max\{|w_m(t)|, |w_s(t)|\} \leqslant \delta$ for almost all $s \geqslant -t_d$. Thus, the system with output $y := (q_m^T, \dot{q}_m^T, \tilde{q}_s^T, \dot{q}_s^T, w_m^T, w_s^T)^T$ is also IOS with offset less than or equal to δ. The proof of Theorem 4 is complete.

6. Simulation results

In this section, an example of computer simulations of a force-reflecting telerobotic system is presented. Consider a force-reflecting telerobotic system described by (1)–(4), with $H_m(q) = H_s(q) \in \mathbb{R}^{2\times 2}$, $C_m(q, \dot{q}) = C_s(q, \dot{q}) \in \mathbb{R}^{2\times 2}$, and $G_m(q) = G_s(q) \in \mathbb{R}^2$, where

$$h_{11} = (2l_1 \cos q_2 + l_2) l_2 m_2 + l_1^2(m_1 + m_2),$$
$$h_{12} = h_{21} = l_2^2 m_2 + l_1 l_2 m_2 \cos q_2, \quad h_{22} = l_2^2 m_2,$$
$$c_{11} = -l_1 l_2 m_2 \sin(q_2)\dot{q}_2, \quad c_{21} = l_1 l_2 m_2 \sin(q_2),$$
$$c_{12} = -l_1 l_2 m_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2), \quad c_{22} = 0,$$
$$g_1 = g(m_2 l_2 \sin(q_1 + q_2) + (m_1 + m_2) l_1 \sin(q_1)),$$
$$g_2 = g m_2 l_2 \sin(q_1 + q_2),$$

and the parameters are $m_1 = 10\,\text{kg}$, $m_2 = 5\,\text{kg}$, $l_1 = 0.7\,\text{m}$, $l_2 = 0.5\,\text{m}$, $g = 9.81\,\text{m/s}^2$. In our simulations, we evaluate the performance of the adaptive stabilization scheme presented in Section 4.2. A vector of the parameters for both the master and the slave manipulators can be chosen as $\theta_m = \theta_s \in \mathbb{R}^5$, where the components are $\theta_1 = l_1 l_2 m_2$, $\theta_2 = l_1^2(m_1 + m_2)$, $\theta_3 = l_2^2 m_2$, $\theta_4 = g m_2 l_2$, and $\theta_5 = g(m_1 + m_2) l_1$. The force (torque) applied by the human operator to both the joints of the master manipulators is as shown in Fig. 1. When the slave follows the resulting trajectory of the master, it contacts a rigid obstacle which is located at $x = 0.2\,\text{m}$. The obstacle is modeled by the following equations:

$$F_e = \begin{cases} -B\dot{x} - K(x - 0.2) & \text{if } x \geqslant 0.2\,\text{m}, \\ 0 & \text{otherwise}, \end{cases}$$

where $B > 0$, and $K > 0$ are damping and stiffness of the environment, respectively. In the simulations below, we put $B = 1$, and $K = 10\,000$, i.e., a contact with a very stiff environment is addressed. The parameters of the control law (21)–(24) are taken as follows: $K_m = 60 \cdot \mathbb{I}_{2\times 2}$, $\sigma_m = \mathbb{I}_{2\times 2}$, $\Gamma_m = 10 \cdot \mathbb{I}_{5\times 5}$, $\varepsilon_m = 1$, $K_s = 10 \cdot \mathbb{I}_{2\times 2}$, $\sigma_s = \text{diag}\{4, 3\}$, $\Gamma_s = \text{diag}\{20, 20, 20, 40, 40\}$, and $\varepsilon_s = 0.5$. Here, by $\mathbb{I}_{n\times n}$ the identity $n \times n$-matrix is denoted. All the initial conditions are set to be zero. An example

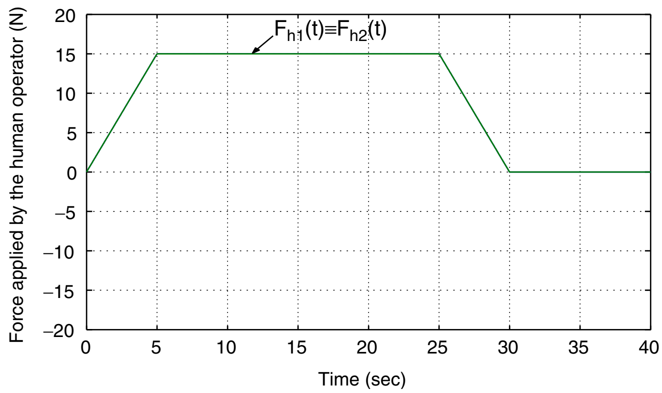

Fig. 1. Torques applied by the human operator, $F_{h1}(t) \equiv F_{h2}(t)$.

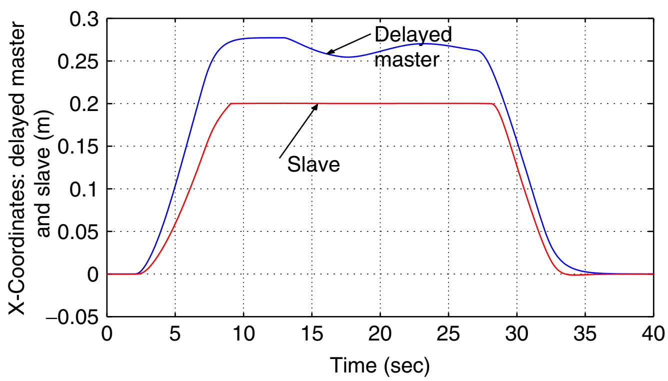

Fig. 2. X-coordinates of the delayed master and the slave.

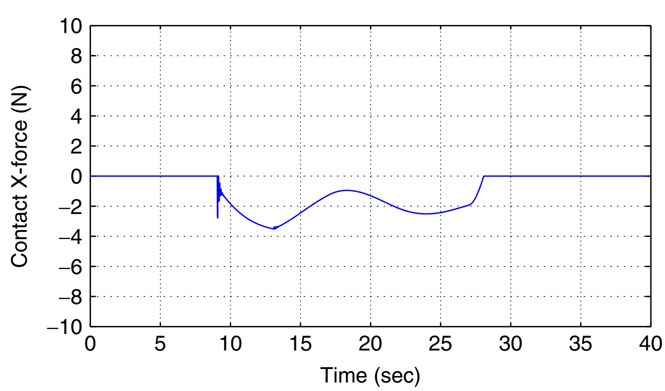

Fig. 3. Environmental X-forces applied to the slave.

of simulations is presented in Figs. 2–5. In this example, we address the case of parametric uncertainty, i.e., where the actual values of the mass/inertia parameters of the slave manipulator are different than the nominal ones. Namely, the actual mass of the second link of the slave manipulator is increased to $m_2 = 7\,\text{kg}$, while the nominal values of the slave parameters θ_s^* are still calculated for $m_2 = 5\,\text{kg}$. The communication delays are $\tau_1 = \tau_2 = 2\,\text{s}$. In general, our simulations show that the adaptive control scheme proposed in this paper provides a stable contact with the obstacle for different values of com-

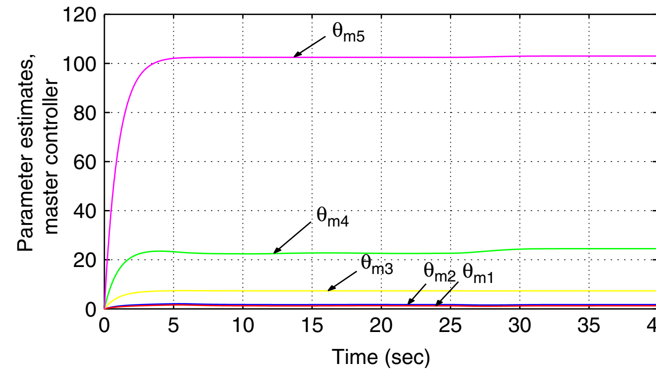

Fig. 4. Parameter estimates of the master controller $\hat{\theta}_{\mathrm{m}}$.

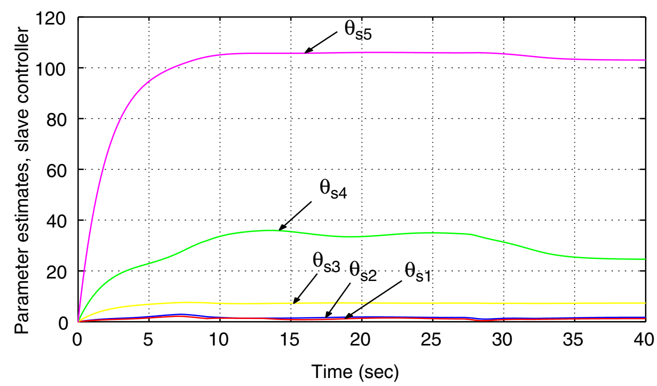

Fig. 5. Parameter estimates of the slave controller $\hat{\theta}_{\mathrm{s}}$.

munication delays as well as in the presence of parametric uncertainty. However, one can note that the trackingproperties of the proposed scheme needs some improvements. This will be a topic for future research.

7. Conclusions

In this paper, the problem of stabilization of a force-reflecting teleoperator system is addressed under a finite-gain assumption imposed on environmental dynamics. We propose a non-adaptive version of the stabilization algorithm as well as its two adaptive extensions. These control schemes guarantee the ISS of the overall telerobotic system in the global, global practical, or semiglobal practical sense respectively for any (constant) delay in the communication channel. Stability analysis of the schemes proposed is based on a new version of the IOS (ISS) small gain theorem for FDEs. We would like to emphasize that the proposed method allows us to prove stability of the telerobotic system for an arbitrary constant communication delay and, in some sense, the stability is uniform with respect to communication delay. It is worth to mention that, because of increasing popularity of the Internet-based teleoperation, there is a strong interest in developing control schemes for force-reflecting teleoperation that can handle time-varying communication delay, see for example, Chopra et al. (2003). A particular extension of the results presented in this paper to the case of time-varying communication delay can be found in Polushin et al. (2005).

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Appendix A. Lemmas

The following simple statement is used in the proof of Theorem 4. Let $\bar{q}(t) = q(t) + \Omega(t)$, where $q(\cdot) : \mathbb{R} \to \mathbb{R}$ is a twice continuously differentiable function ($q \in C^2$), and $\Omega(\cdot) : \mathbb{R} \to \mathbb{R}$ is absolutely continuous. Further, denote $v(s) = (\rho s/(s + \rho))\bar{q}(s)$. In time domain, function $v(t)$ satisfies for almost all t the following differential equation:

$$\dot{v}(t) = \rho(\dot{q}(t) + \omega(t) - v(t)), \tag{43}$$

where $\omega(t) := \mathrm{d}\Omega/\mathrm{d}t$ is time derivative of $\Omega(t)$ (since, by assumption, $\Omega(t)$ is absolutely continuous, we see that $\omega(t)$ is defined for almost all $t \in \mathbb{R}$). The following fact is valid.

Lemma 1. *Given $D \geqslant 0$, $\Delta_{\mathrm{m}} \geqslant 0$, $\tau > 0$, and $\varepsilon_1 > 0$, there exist $\rho^* > 0$, $\Omega^* > 0$ such that the following holds. Suppose $\rho \geqslant \rho^*$* in (43). *Suppose also $|v(-\tau) - \dot{q}(-\tau)| \leqslant \Delta_{\mathrm{m}}$, and there exists $t \geqslant 0$ such that the inequalities $|\ddot{q}(s)| \leqslant D$ and $|\Omega(s)| \leqslant \Omega^*$ hold for almost all $s \in [-\tau, t)$. Then $|v(s) - \dot{q}(s)| \leqslant \varepsilon_1$ for all $s \in [0, t]$.*

Proof. Based on solving equation (43) and applying of the mean value theorem. Details are omitted due to space limitation reasons. □

Appendix B. Input-to-state stability for systems of ordinary differential equations (ODEs)

Definition 2 (*Sontag and Wang, 1996*). A system of the form $\dot{x} = F(x, w)$, where $x \in \mathbb{R}^n$, $w \in \mathbb{R}^{\mathrm{m}}$, is said to be input-to-state stable (ISS) with ISS gain $\gamma \in \mathscr{K}$ and restriction (Δ_x, Δ_w), if $|x(0)| \leqslant \Delta_x$, $\sup_{t \geqslant 0}|w(t)| \leqslant \Delta_w$ imply that the solutions of (2) are defined for all $t \geqslant 0$, and the following two properties hold.

(i) *Global stability*: there exist $\beta \in \mathscr{K}_\infty$ such that

$$\sup_{t \geqslant 0} |x(t)| \leqslant \max\left\{\beta(|x(0)|), \gamma\left(\sup_{t \geqslant 0} |w(t)|\right)\right\}.$$

(ii) *Convergence* (*asymptotic gain*):

$$\lim_{t \to +\infty} \sup |x(t)| \leqslant \gamma\left(\lim_{t \to +\infty} \sup |w(t)|\right).$$

The following simple relation between the IOS (ISS) properties of ODEs and FDEs is used in the proofs of Theorems 2–4.

Lemma 2. *Suppose a system*

$$\dot{x}(t) = F(x(t), u(t), w(t))$$

is ISS with ISS gains $\gamma_u, \gamma_w \in \mathscr{K}$ and restriction $(\Delta_x, \Delta_u, \Delta_w)$, $\Delta_x > 0$, $\Delta_u > 0$, $\Delta_w > 0$. Then the system

$$\dot{x}(t) = F(x(t), u(t - \tau), w(t)),$$

where $\tau \geqslant 0$, being considered as a system of FDE, is ISS for any $t_d \geqslant \tau$ with the same ISS gains γ_u, γ_w, and the same restriction $(\Delta_x, \Delta_u, \Delta_w)$.

It is well known that the ISS property can be characterized in terms of existence of an ISS-Lyapunov function (Sontag & Wang, 1995). We will utilize this fact to establish the following lemma which is used in the Proof of Theorem 4.

Lemma 3. *Consider a nonlinear system of the form*

$$\dot{x} = f(x, u, w), \tag{44}$$

where $f(\cdot, \cdot, \cdot)$ is a locally Lipschitz function of its arguments. Suppose the system with $w(\cdot) \equiv 0$ is ISS, i.e., there exists a smooth ISS Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}^+$ such that $\alpha_1(|x|) \leqslant V(x) \leqslant \alpha_2(|x|)$ holds for some $\alpha_1, \alpha_2 \in \mathscr{K}_\infty$, and there exist $\chi_u \in \mathscr{K}$, $\alpha_3 \in \mathscr{K}_\infty$ such that $|x| \geqslant \chi_u(|u|)$ implies $\nabla_x V f(x, u, 0) \leqslant -\alpha_3(|x|)$, where $\nabla_x V$ is gradient of V. Then, given $\Delta_x, \Delta_u \in [0, +\infty)$, there exist $\Delta_w > 0$ and $\chi_w \in \mathscr{K}$ such that the system (44) *ISS with restriction $(\Delta_x, \Delta_u, \Delta_w)$ and ISS gains $\gamma_u := \alpha_1^{-1} \circ \alpha_2 \circ \chi_u$, $\gamma_w := \alpha_1^{-1} \circ \alpha_2 \circ \chi_w$.*

Proof. Given $\Delta_x, \Delta_u \in [0, +\infty)$. Increasing Δ_x if necessary, assume without loss of generality that $\Delta_x > \chi_u(\Delta_u)$. Denote $D_x := \alpha_1^{-1} \circ \alpha_2(\Delta_x)$. By continuity, we see that there exists $\eta \in \mathscr{K}$ such that $|\nabla_x V(f(x, u, w) - f(x, u, 0))| \leqslant \eta(|w|)$ holds whenever $|x| \leqslant D_x$, $|u| \leqslant \Delta_u$. Thus, if $|u| \leqslant \Delta_u$, $\chi_u(|u|) \leqslant |x| \leqslant D_x$, then $\nabla_x V f(x, u, w) \leqslant -\alpha_3(|x|) + \eta(|w|)$. Moreover, define $\chi_w(\cdot) := \alpha_3^{-1}(2\eta(\cdot))$, then $|x| \geqslant \chi_w(|w|)$ implies $1/2\alpha_3(|x|) \geqslant \eta(|w|)$. Thus, we see that $\max\{\chi_u(|u|), \chi_w(|w|)\} \leqslant |x| \leqslant D_x$, $|u| \leqslant \Delta_u$ imply $\nabla_x V f(x, u, w) \leqslant -\frac{1}{2}\alpha_3(|x|)$. Now, choose $\Delta_w > 0$ such that $\Delta_x > \chi_w(\Delta_w)$. From here, the statement follows using standard line of reasoning (see, for example, Sontag & Wang, 1995, Proof of Lemma 2.14). □

References

Alvarez-Gallegos, J., Rodriguez, D. de C., & Spong, M. W. (1997). A stable control scheme for teleoperators with time delay. *International Journal of Robotics and Automation*, *12*(3), 73–79.

Anderson, R. J., & Spong, M. W. (1989). Bilateral control of teleoperators with time delay. *IEEE Transactions on Automatic Control*, *AC-34*(5), 494–501.

Arcara, P., & Melchiorri, C. (2002). Control schemes for teleoperation with time delay: A comparative study. *Robotics and Autonomous Systems*, *38*(1), 49–64.

Chopra, N., Spong, M. W., Hirche, S., & Buss, M. (2003). Bilateral teleoperation over the internet: The time varying delay problem. In '*American control conference*', Denver, CO.

Ferrell, W. R. (1966). Delayed force feedback. *Human Factors*, *8*, 449–455.

Jiang, Z.-P., Teel, A. R., & Praly, L. (1994). Small-gain theorem for ISS systems and applications. *Mathematics of Control, Signals, and Systems*, *7*, 95–120.

Leung, G. M., Francis, B. A., & Apkarian, J. (1995). Bilateral controller for teleoperators with time delay via mu-synthesis. *IEEE Transactions on Robotics and Automation*, *11*, 105–116.

Niemeyer, G., & Slotine, J.-J. E. (2004). Telemanipulation with time delays. *International Journal of Robotics Research*, *23*(9), 873–890.

Polushin, I. G., & Marquez, H. J. (2003). Stabilization of bilaterally controlled teleoperators with communication delay: An ISS approach. *International Journal of Control*, *76*(8), 858–870.

Polushin, I. G., Tayebi, A., & Marquez, H. J. (2005). Stabilization scheme for force reflecting teleoperation with time-varying communication delay based on IOS small gain theorem. In '*16th IFAC world congress*', Prague, Czech Republic.

Sheridan, T. B. (1989). Telerobotics. *Automatica*, *25*(4), 487–507.

Shilov, G. E. (1974). *Elementary functional analysis*. Cambridge, MA: MIT Press, (translated and edited by Richard A. Silverman).

Sontag, E. D., & Wang, Y. (1995). On characterizations of the input-to-state stability property. *Systems & Control Letters*, *24*, 351–359.

Sontag, E. D., & Wang, Y. (1996). New characterizations of input-to-state stability. *IEEE Transactions on Automatic Control*, *AC-41*, 1283–1294.

Sontag, E. D., & Wang, Y. (1999). Notions of input-to-output stability. *Systems & Control Letters*, *38*, 235–248.

Spong, M. W. (1996). Motion control of robot manipulators. In W. Levine (Ed.), *Handbook of control* (pp. 1339–1350). Boca Raton, FL: CRC Press.

Teel, A. R. (1996). A nonlinear small gain theorem for the analysis of control systems with saturation. *IEEE Transactions on Automatic Control*, *AC-41*(9), 1256–1270.

Teel, A. R. (1998). Connections between Razumikhin-type theorems and the ISS nonlinear small gain theorem. *IEEE Transactions on Automatic Control*, *AC-43*(7), 960–964.

Zhu, W.-H., & Salcudean, S. E. (2000). Stability guaranteed teleoperation: An adaptive motion/force control approach. *IEEE Transactions on Automatic Control*, *AC-45*(11), 1951–1969.

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