Brief paper

All-pass filtering in iterative learning control[☆]

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ABSTRACT

In iterative learning control (ILC), it is highly desirable to have a learning compensator with a unit-gain for all frequencies, in order to avoid noise amplification and learning speed degradation during the learning process. In this paper, we show that the realization of a unit-gain compensator is straightforward in ILC, using both forward and backward filtering. As an illustrative example, a unit-gain derivative is proposed to overcome the drawbacks of the conventional derivative. The proposed scheme is equivalent to an all-pass unit-gain phase shifter; the forward filtering uses a 0.5-order derivative and the backward filtering employs a 0.5-order integral. The all-pass phase shifter is deployed in a unit-gain D-type ILC. The advantages of the unit-gain feature are demonstrated by some experimental results on a robot manipulator.

1. Introduction

Iterative learning control (ILC) (Arimoto, Kawamura, & Miyazaki, 1984; Middleton, Goodwin, & Longman, 1989) is a suitable technique for uncertain systems that operate in a repetitive manner. It aims to iteratively reduce the tracking error, over a finite time interval, by incorporating past experience in the actual control input. In general, ILC schemes are based on a batch update,[1] and the generation of the control input to be applied in the current cycle can be obtained "offline" using data from the previous cycles. Hence noncausal operations are allowed in terms of time *t* for the previous cycles. In Elci, Longman, Phan, Juang, and Ugoletti (2002), a simple linear anticipatory operator (phase lead) is adopted. In Jeong and Choi (2002), an input update law, which depends on the number of non-minimum phase zeros, using advanced output data has been proposed. A contraction mapping based learning law using all the "future" error data has been proposed in Jang and Longman (1994, 1996b). In Chen, Moore, and Bahl (2004), an average window is used as a zero-phase filter, and the learning law in Tsai, Lin, and Yau (2006) employs a zero-phase Butterworth filter. Backward filtering of the previous cycle data has also been used in Kinoshita, Sogo, and Adachi (2002), Yamakita and Furuta (1991) and Ye and

Wang (2005b). From the phase point of view, backward filtering essentially generates phase lead to compensate the phase lag of a real plant so that an overall approximate zero-phase effect is achieved. In Plotnik and Longman (1999), the previous error history is zero-phase filtered via a combination of forward and backward filtering. The learning compensators in Ghosh and Paden (2002) and Ye and Wang (2005a) contain a part in the form of a zero-phase filter, also via both forward and backward filtering of the previous error history.

It is well known that phase compensation in ILC is highly desirable as it can compensate for the plant phase (delay) leading to a wider learnable band and consequently improving the tracking accuracy (Ye & Wang, 2005b). Therefore, a learning compensator is typically chosen as a phase compensator to provide phase (lead) compensation. A phase compensator with a unit-gain or all-pass feature is welcome as it neither attenuates the learning speed nor amplifies the noise. To the best of the authors' knowledge, in the existing literature, three digital learning compensators have the unit-gain feature. The first one is the linear phase lead learning compensator (Elci et al., 2002). The linear phase lead will provide phase compensation increasing linearly with frequency, which will be excessive at high frequency, and cutoff has to be introduced to stop phase compensation at high frequencies. The second one is the learning compensator using the partial isometry of system Markov parameter matrix, calculated by singular value decomposition (Jang & Longman, 1996a). Nevertheless, the computation burden of the learning law is heavy, if the size of a system Markov parameter matrix is large, *i.e.*, the trajectory length is long. The third learning law is the phase cancelation learning law that constructs its learning compensator (matrix) by calculation of an Inverse Discrete Fourier Transform (IDFT) from the phase characteristics of the plant (Lee-Glauser, Juang, & Longman, 1996;

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[1] One can also find in the literatures the so-called current-cycle ILC using tracking errors from the actual operation or iteration (Kuc, Lee, & Nam, 1992).

Longman & Wang, 1996). Again, the computation burden may be heavy. Computational enhancement is proposed in Lee-Glauser et al. (1996) to reduce the calculation cost.

In this paper, the idea of noncausal unit-gain/all-pass filtering, which is also achieved via both forward and backward filtering, is firstly formalized in the ILC setting. Thereafter, a straightforward approach in a non-matrix form and with less computational complexity is proposed for the realization of noncausal unit-gain phase compensators. To illustrate the method, a unit-gain derivative scheme is presented as an example. In the forward filtering procedure, a 0.5-order derivative is performed while in the backward filtering procedure, a 0.5-order integral is carried out. The overall effect is a noncausal all-pass phase shifter. Employing the noncausal all-pass phase shifter, a unit-gain D-type ILC is proposed. For computer implementation, frequency-band synthesis of non-integer differentiator is used to approximate the 0.5-order derivative and 0.5-order integral. Experiments on a 6 DOF rigid robot manipulator have been carried out to show the advantages of the unit-gain feature.

2. Analysis of learning compensators

2.1. Necessity of phase advancement

Although ILC is a finite-time problem, it is a common practice to analyze it in the frequency domain (Chen et al., 2004; Goh, 1994; Longman, 2000). The standard convergence analysis from the phase point of view has been given by many authors in the literature (e.g., Wang and Ye (2004) and Ye and Wang (2005a)), and is provided here for the sake of completeness.

Consider the control system modeled by a transfer function $G_p(s)$, and described by the input-output relationship at the iteration k as follows

$$Y_k(s) = G_p(s)U_k(s). \tag{1}$$

The frequency characteristics of $G_p(s)$ are given as follows:

$$G_p(j\omega) = N_p(\omega)\exp(j\theta_p(\omega)) \tag{2}$$

where $N_p(\omega)$ and $\theta_p(\omega)$ are the magnitude and phase characteristics, respectively.

Let the Laplace transform of the learning law be:

$$U_k(s) = U_{k-1}(s) + \Gamma\Phi_c(s)E_{k-1}(s) \tag{3}$$

where Γ is a scalar learning gain and $\Phi_c(s)$ is a learning compensator in the Laplace domain. The learning compensator $\Phi_c(s)$ has frequency characteristics $\Phi_c(j\omega) = N_c(\omega)\exp(j\theta_c(\omega))$ with $N_c(\omega)$ and $\theta_c(\omega)$ being its magnitude and phase characteristics, respectively.

Using (1) and (3), we get

$$E_k(s) = [1 - \Gamma G_p(s)\Phi_c(s)]E_{k-1}(s). \tag{4}$$

$[1 - \Gamma G_p(s)\Phi_c(s)]$ can be viewed as a transfer function from the tracking error at iteration $(k-1)$ to the tracking error at iteration k. The condition for tracking error contraction at steady state is (Goh, 1994; Hideg & Judd, 1988),

$$\left|1 - \Gamma G_p(j\omega)\Phi_c(j\omega)\right| < 1. \tag{5}$$

Using the characteristics of $G_p(j\omega)$ and $\Phi_c(j\omega)$, (5) leads to:

$$\Gamma N_p(\omega)N_c(\omega) < 2\cos(\theta_p(\omega) + \theta_c(\omega)). \tag{6}$$

If $\Gamma > 0$, (6) necessarily requires

$$-90° < \theta_p(\omega) + \theta_c(\omega) + i \times 360° < 90°,$$
$$i = 0, \pm1, \pm2\ldots. \tag{7}$$

The frequency range where (5) or (6) hold is termed the *learnable band*. We cutoff the frequencies outside the learnable band to prevent a bad learning transient as in Longman (2000). In this paper, cutoff is done on the input signal $u_k(t)$.

Condition (7) is vital because if (7) is satisfied we can always find a learning gain Γ small enough to satisfy (6). Therefore the frequency range where (7) holds is named the *utmost learnable band*. For minimum phase systems, $\theta_p(\omega)$ is typically negative, and with $i = 0$ in (7), $\Phi_c(j\omega)$ should generally provide positive phase $\theta_c(\omega)$. For non-minimum phase systems whose phase starts from 360° (or its multiples) at dc, i is set as -1 (or its multiples) so that $\theta_p(\omega) + i \times 360°$ is 0 at dc and negative at other frequencies. Again, $\theta_c(\omega)$ should generally be positive. In both cases, phase advancement is required to ensure a wider learnable band. And a wider learnable band can ensure higher tracking accuracy (Wang & Ye, 2004).

2.2. Desirable unit-gain feature

If a learning compensator has a magnitude $N_c(\omega) = 1$, it has a unit-gain or all-pass feature. $N_c(\omega) = 1$ means that the noise will not be amplified if there is noise contained in error $e_{k-1}(t)$. From (5), the error contraction rate between two successive iterations is $\left|1 - \Gamma N_p(\omega)N_c(\omega)e^{j(\theta_p(\omega)+\theta_c(\omega))}\right|$. If a learning compensator has $N_c(\omega) \to 0$ at some frequencies, at these frequencies, the contraction rate will tend to 1 and one can expect a very slow learning speed. For example, a low-pass learning compensator has $N_c(\omega) \to 0$ at high frequency and the classic D-type learning compensator (Arimoto et al., 1984) has $N_c(\omega) \to 0$ at low frequency. On the contrary, if $N_c(\omega) = 1$, the learning speed will not be sacrificed at any frequency.

3. Realization of noncausal unit-gain

The noncausal unit-gain is realized by first passing through a filter $\phi(s)$ with stable zeros and then reverse filtering the result by $1/\phi(s)$, as follows,

$$\begin{cases} \text{step 1}: \phi(s) & \text{runs in the forward time} \\ \text{step 2}: 1/\phi(-s) & \text{runs in the backward time} \end{cases} \tag{8}$$

$-s$ indicates that the Laplace transform is defined in backward time, according to the definition of one-sided Laplace transform (Le Page, 1980). To implement step 2, the result of step 1 is firstly reversed in sequence, the reversed result is then passed through $1/\phi(s)$, and the final result is reversed back. According to the definition of one-sided Laplace transform (Le Page, 1980), the magnitude of $1/\phi(-j\omega)$ is still $1/|\phi(j\omega)|$. Combination of the two sub-equations in (8) leads to an interesting fact: the overall magnitude is 1.

If $\phi(s)$ is chosen with phase (lead) characteristics $\theta(\omega)$, the overall phase (lead) compensation effect of (8), i.e., $\phi(s)/\phi(-s)$, is $2 \times \theta(\omega)$ which is double of the original phase. Supposing that ideally the plant is fully known, if $\theta(\omega) = 0.5 \times (-\theta_p(\omega))$, from (6), exact-phase-cancelation is achieved. This provides a new approach of phase cancelation learning control (Longman & Wang, 1996) with no attenuation of learning speed.

4. A unit-gain derivative example

To demonstrate more effectively the proposed unit-gain realization method, a simple illustrative example is given.

4.1. D-type ILC revisited

In Arimoto et al. (1984), the input update utilizes the derivative of the previous error signal and the learning law is termed D-type ILC,

$$u_k(t) = u_{k-1}(t) + \Gamma \frac{\mathrm{d}}{\mathrm{d}t}(e_{k-1}(t)). \tag{9}$$

D-type ILC is a simple but effective law. In the frequency domain, the differentiation brings in a 90° phase advancement which helps to compromise the phase lag of the system and extend the learnable band of ILC. However, at low frequencies, the magnitude is small and the learning speed is attenuated and in fact, D-type ILC cannot learn dc components; at high frequencies, differentiation amplifies the magnitude significantly and brings in substantial noise, leading to possible divergence. Since the Laplace transfer of the derivative is s, the convergence condition for the D-type ILC (9), derived from (5), is (Chen & Moore, 2001; Wang & Ye, 2005)

$$\left|1 - \Gamma j\omega G_p(j\omega)\right| < 1. \tag{10}$$

And correspondingly, (6) and (7) become

$$\Gamma \omega N_p(\omega) < 2\cos(\theta_p(\omega) + 90°) \tag{11}$$

and

$$-180° < \theta_p(\omega) + i \times 360° < 0, \quad i = 0, \pm1, \pm2\ldots. \tag{12}$$

Inequality (12) shows that the differential action provides 90° phase compensation so that, when $i = 0$, the phase lag restriction of the system is relaxed from $-90°$ to $-180°$. Hence phase advancement is a desirable feature of D-type ILC. However, the magnitude of the derivative, i.e., ω, becomes very large at high frequencies. Since the right-hand-side of (11) is upper-bounded by 2, the increasing magnitude will inevitably violate the convergence condition. Therefore, the ω magnitude is an undesirable feature of D-type ILC. To tackle this undesirable feature, one can either use a much smaller learning gain Γ or employ a zero-phase low-pass filter on the derivative signal to attenuate the magnitude, so that (11) is satisfied over a wider learnable band. But a small learning gain will degrade the learning speed, and the introduction of a zero-phase low-pass filter puts more burdens on the design. Aiming at removing the undesirable magnitude feature while keeping the desirable phase advancement feature, the derivative action is modified.

4.2. All-pass derivative

By observing that $s = s^{0.5} \times s^{0.5}$, it is noted that the phase of s is double of the phase of $s^{0.5}$. The aforementioned idea (8) is applied to the special case of $\phi(s) = s^{0.5}$, and an all-pass unit-gain derivative is proposed as follows

$$\begin{cases} \text{step 1} : s^{0.5} & \text{runs in the forward time} \\ \text{step 2} : 1/(-s)^{0.5} & \text{runs in the backward time} \end{cases} \tag{13}$$

$s^{0.5}$ is the 0.5-order derivative, running in the forward time. $1/(-s)^{0.5}$ is the 0.5-order integral running in the backward time. A fractional-order derivative and integral is a generalization of integration and differentiation to non-integer order operators (Oldham & Spanier, 1974). The Laplace transform of an α-order derivative or integral is defined as s^α (Petráš & Dorčák, 1999). In this paper, only the situation where α is a real number is considered. If $\alpha > 0$, s^α is an α-order derivative. And if $\alpha < 0$, s^α is an $|\alpha|$-order integral.

The magnitude characteristics of step 1 and magnitude characteristics of step 2 are reciprocal so that the complete gain is 1. The overall phase is 90°: step 1 and step 2 both contribute half, i.e., 45°. An all-pass unit-gain derivative scheme is then obtained, keeping the phase advancement feature and eliminating the frequency dependant magnitude feature. Note that the overall transfer function of steps 1 and 2 is $s/|s|$ whose frequency response is simply j.

Remark 1. In theory, we have realized a noncausal *all-pass phase shifter*. The all-pass unit-gain derivative scheme is mathematically equivalent to using a zero-phase filter with magnitude characteristics of $1/\omega$ on the conventional derivative signal. Since a zero-phase filter with such magnitude restriction is difficult to realize, the use of (13) becomes a viable choice.

4.3. Unit-gain D-type ILC

Based on the proposed all-pass derivative scheme, a unit-gain D-type ILC is formulated as follows,

$$\begin{cases} e1(t) = e_{k-1}(t) \longrightarrow s^{0.5} \\ e2(t) = e1(T - t) \\ e3(t) = e2(t) \longrightarrow \dfrac{1}{s^{0.5}} \\ e4(t) = e3(T - t) \\ u_k(t) = u_{k-1}(t) + \Gamma e4(t) \end{cases} \tag{14}$$

where the operator $\longrightarrow$ denotes passing the signal on its left side through the system represented by the transfer function on its right. The Laplace transform of (14) is

$$\begin{cases} E1(s) = E_{k-1}(s)s^{0.5} \\ E2(s) = e^{Ts}E1(-s) \\ E3(s) = E2(s)\dfrac{1}{s^{0.5}} \\ E4(s) = e^{Ts}E3(-s) \\ U_k(s) = U_{k-1}(s) + \Gamma E4(s). \end{cases} \tag{15}$$

Note that the Laplace transform of $e1(T - t)$ is $e^{Ts}E1(-s)$ (Le Page, 1980).

From (15), the unit-gain D-type ILC can be written simply in the Laplace domain as

$$U_k(s) = U_{k-1}(s) + \Gamma \frac{s}{|s|}E_{k-1}(s), \tag{16}$$

or in the frequency domain as

$$U_k(j\omega) = U_{k-1}(j\omega) + \Gamma j E_{k-1}(j\omega). \tag{17}$$

Remark 2. A pioneer work dealing with fractional-order derivative based ILC can be found in Chen and Moore (2001). In this paper, an in-between P-type (Saab, 1994) and D-type scheme, using a fractional-order derivative (s^α, $\alpha \in (0, 1]$) of the tracking error, is proposed and termed D$^\alpha$-type ILC. But this scheme does not change the undesirable features of small magnitude at low frequency and large magnitude at high frequency. Moreover, the phase compensation is $\alpha \times 90°$, less than that of the conventional derivative action. And the possible extension of the utmost learnable band is also limited according to (7).

Remark 3. The proposed unit-gain D-type scheme can be extended to using an all-pass $G_p^{0.5}(-s)/G_p^{0.5}(s)$ as the learning compensator if the plant $G_p(s)$ has only stable zeros. The procedure will phase-shift the error signal by the inverse of the plant phase, i.e., $-\theta_p(\omega)$ and an overall zero-phase is achieved by fractional-order filtering. In practice, by using the plant model $G_m(s)$, approximate zero-phase can be achieved by means of $G_m^{0.5}(-s)/G_m^{0.5}(s)$.

4.4. Implementation of fractional-order operator

One can use analog devices for the realization of true fractional-order operations (Bohannan, 2002). However, computer implementation needs a suitable approximation to the fractional-order operator. In ILC, phase lead compensation in a designated frequency band is enough to substantially extend the learnable band. Fortunately, Oustaloup, Levron, Mathieu, and Nanot (2000) synthesizes fractional-order differentiators whose action is limited to any given frequency bandwidth. The authors use a recursive approximation scheme to approximate the fractional-order derivative.

Using a $2N + 1$ order approximation, s^α can be approximated by

$$D_N(s) = \left(\frac{\omega_u}{\omega_h}\right)^\alpha \prod_{m=-N}^{N} \frac{1 + s/\omega'_m}{1 + s/\omega_m} \tag{18}$$

with $\omega_u = \sqrt{\omega_l \omega_h}$ and

$$\omega'_m = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(m+N+0.5-0.5\alpha)/(2N+1)}, \tag{19}$$

$$\omega_m = \omega_l \left(\frac{\omega_h}{\omega_l}\right)^{(m+N+0.5+0.5\alpha)/(2N+1)}. \tag{20}$$

α is the fractional number and N is a nonnegative integer number. ω_l is the low transitional frequency where the designer wants the phase compensation effect to appear and ω_h is the high transitional frequency where the designer wants the phase compensation effect to disappear. $1/s^\alpha$ is then approximated by $1/D_N(s)$.

5. Experimental study

To demonstrate the benefits of the unit-gain feature, experimental study has been performed on a 6-DOF robot manipulator CRS465. The six links are independently controlled by a PD feedback control with

$K_p = [2.5, 2.5, 2.5, 0.5, 0.5, 0.3]$,

$K_d = [0.05, 0.05, 0.05, 0.005, 0.005, 0.002]$.

System identification is performed on a single axis while keeping the others locked, one by one. The closed-loop transfer functions, from the reference joint position to actual joint position, around the ready configuration of the robot manipulator, have been identified by the system identification toolbox of MATLAB, as follows:

$$\begin{cases} \text{J1}: & G_{p1}(s) = \dfrac{-0.2622s + 1624}{s^2 + 39.42s + 1761} \\ \text{J2}: & G_{p2}(s) = \dfrac{-0.2101s + 1312}{s^2 + 35.15s + 1374} \\ \text{J3}: & G_{p3}(s) = \dfrac{0.6385s + 1285}{s^2 + 35.74s + 1380} \\ \text{J4}: & G_{p4}(s) = \dfrac{-1.067s + 797.5}{s^2 + 91.32s + 880.8} \\ \text{J5}: & G_{p5}(s) = \dfrac{-0.5967s + 564.5}{s^2 + 61.34s + 603.5} \\ \text{J6}: & G_{p6}(s) = \dfrac{-0.2123s + 481.2}{s^2 + 55.56s + 545.2}. \end{cases} \tag{21}$$

Remark 4. Note that some of the identified transfer functions are non-minimum phase systems with phase starting from 360° at dc. Noncausal learning control laws have been successfully applied to non-minimum phase systems, with bounded input, e.g., the α-pseudo inverse based ILC (Ghosh & Paden, 2002) and the approach
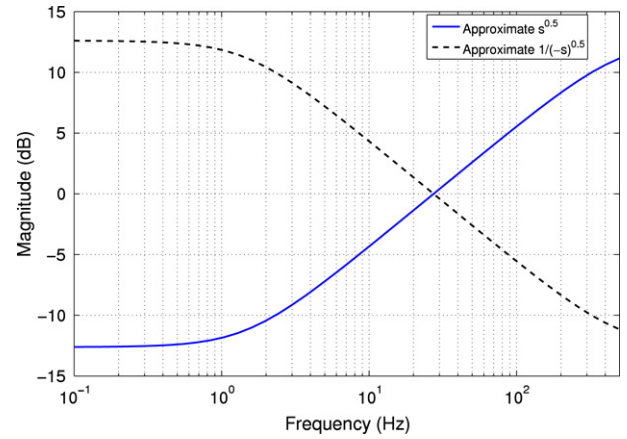

Fig. 1. Magnitude responses of the two semi-filters.

in Ye and Wang (2005a). The basic idea in these publications is to avoid using an inverse of the system as the learning compensator and to invert only the phase (phase cancelation) so that the input is bounded. In this paper, system inversion is not used either and approximate phase cancelation is adopted, similar to that in Ghosh and Paden (2002) and Ye and Wang (2005a).

Remark 5. The robot is in fact a nonlinear system. However, being closed-loop controlled, each joint can be locally approximated by a linear system. The effect of coupling and nonlinearities can be treated as a repetitive disturbance term applied to each joint. Linear learning control laws have been very successful in dealing with nonlinear robots with repetitive disturbances (Chen et al., 2004; Gunnarsson & Norrlöf, 2001; Longman, 2000; Tang, Cai, & Huang, 2000).

To examine the tracking ability of all joints under fast motion, the same desired trajectory is used for all joints

$$y_d(t) = \begin{cases} \displaystyle\sum_{n=1}^{51} a_n[1 - \cos(2\omega_n t)] & 0 \le t \le 0.5 \text{ s} \\ \displaystyle\sum_{n=1}^{51} a_n[\cos(2\omega_n t) - 1] & 0.5 < t \le 1 \text{ s} \end{cases} \tag{22}$$

where the ω_n are 0π, 2π, 4π, 6π, ..., 100π, and the amplitude is $a_n = 2400e^{-\omega_n}$. Swinging quickly in 1 s, the desired trajectory is really a challenge for each joint.

Remark 6. Since the desired trajectory starts with zero and ends with zero, the effects of initial/final conditions of filtering are trivial. For general reference trajectories, extension of the initial/final points may be necessary to suppress the initial condition effects, as having been extensively studied in Plotnik and Longman (1999).

The sampling rate is 1 kHz. Cutoff is realized by DFT/IDFT with no end-extension. Learning control is applied to the six DOF closed-loop systems independently. Learning gain should be no more than the reciprocal of the dc gain of closed-loop system (Longman, 2000) and smaller gain will result in a smaller final error level (Wirkander & Longman, 1999). Hence the learning gains are chosen as $\Gamma_1 = 0.5$, $\Gamma_2 = 0.5$, $\Gamma_3 = 0.5$, $\Gamma_4 = 0.2$, $\Gamma_5 = 0.2$, $\Gamma_6 = 0.2$. s^α with $\alpha = 0.5$ is approximated by (18), given $N = 3$. $1/s^\alpha$ is approximated by $1/D_N(s)$ with the same parameters. Zero degree is the initial position for all joints. After each trial, feedback control drives all joints back to their initial positions.

Fig. 1 shows the magnitude responses of the approximate $s^{0.5}$ and approximate $1/(-s)^{0.5}$. They are reciprocal and their product
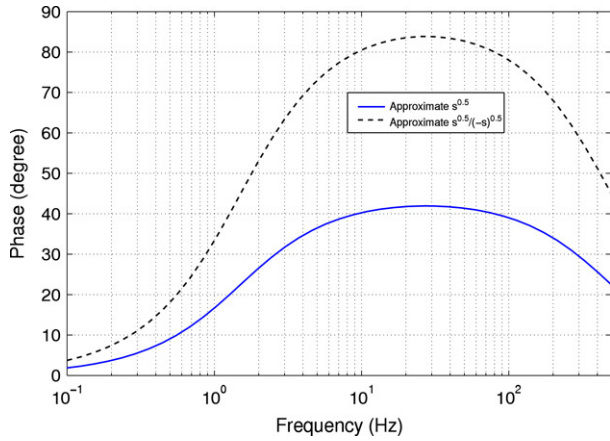
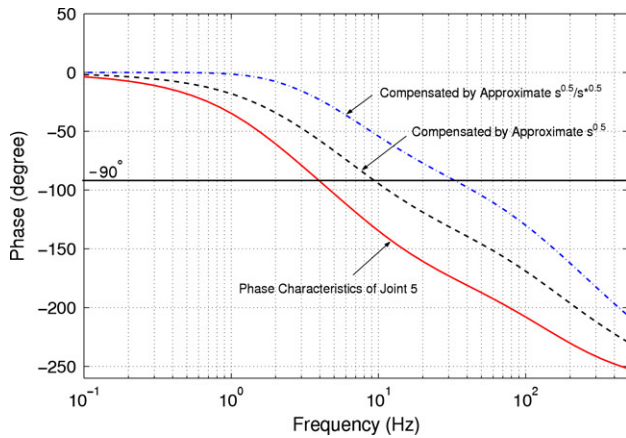**Fig. 2.** Phase responses of the two semi-filters.

Fig. 3. Phase compensation effects for joint 5.

Fig. 4. Convergence conditions for joint 5.

Table 1
Cutoff frequencies (Hz).

	J1	J2	J3	J4	J5	J6
Unit-gain D	36	27	100	15	27	34
Approximate D	7	7	10	5	7	4
Conventional D	2	3	3	2	2	3

Fig. 7 depicts the output profiles of joint 5 at iteration 0, 5, 10, and 20. At iteration 20, the difference between y_{20} and the desired trajectory is very small.

Fig. 8 depicts the input profiles of joint 5 at iteration 0, 5, 10, 50, and 150 (input $u_0(t) = y_d(t)$). At iteration 50, the difference between u_{50} and u_{150} is tiny which means the input converges towards its final profile.

Remark 7. The proposed scheme can actually be model-free. One can tune ω_l (and possibly ω_h) and the cutoff frequency in experiments until good performance is achieved.

Remark 8. It is worth noting that it is possible to use $s^\alpha/(-s)^\alpha$ with $\alpha > 0.5$ instead of $s^{0.5}/(-s)^{0.5}$ to introduce more phase compensation. The learnable band can be further extended. Since the rms errors have reached the reproducibility level of the robot, there will be no significant improvement in the tracking errors for this desired trajectory. For trajectories rich in high frequency components and systems with high relative degree, further extension of the learnable band may be necessary.

For comparison, the conventional D-type ILC is also performed. Even under very small learning gains $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = \Gamma_5 = \Gamma_6 = 0.05$, the cutoff frequencies have to be set very low to ensure stability of the learning control, see Table 1. Moreover, the robot generates a loud noise. The tracking performance is rather poor, Fig. 9.

One may think to approximate s using (18) with $\alpha = 1$ to replace the severe action of the conventional differentiation to improve the performance. Fig. 10 shows that the phase characteristics of $D_N(s)$ with $\alpha = 1$, is nearly the same as that of the all-pass $D_N(s)/D_N(-s)$ with $\alpha = 0.5$, given $N = 3$, $\omega_l = 1.5$ Hz, and $\omega_h = 500$ Hz for both cases. The learning law can be written as

$$U_k(s) = U_{k-1}(s) + \Gamma D_N(s)E_{k-1}(s) \tag{23}$$

where $D_N(s)$ is defined by (18) given $\alpha = 1$, $N = 3$, $\omega_l = 1.5$ Hz, and $\omega_h = 500$ Hz. The learning gains are again chosen as $\Gamma_1 = 0.5$, $\Gamma_2 = 0.5$, $\Gamma_3 = 0.5$, $\Gamma_4 = 0.2$, $\Gamma_5 = 0.2$, $\Gamma_6 = 0.2$. The tuning results of the cutoff frequencies are also listed in Table 1. This intuitive scheme performs much better than the conventional

is one. Fig. 2 illustrates the phase response of the approximate $s^{0.5}$. The phase response of the approximate $1/(-s)^{0.5}$ is exactly the same as the phase response of the approximate $s^{0.5}$. The phase response of the approximate $s^{0.5}/(-s)^{0.5}$, which is double of that of the approximate $s^{0.5}$, is also shown in Fig. 2. Figs. 1 and 2 clearly demonstrate the unit-gain phase compensation effect of the combination of the two semi-filters. Joint 5 is taken as an example to illustrate the design procedure. Noting that joint 5 is a non-minimum phase system whose phase is 360° at dc, i is set as -1 in (7). $\theta_p(\omega) - 360°$ of joint 5 is plotted in Fig. 3. ω_h is simply set as the Nyquist frequency, *i.e.*, 500 Hz, because phase compensation is unnecessary after the Nyquist frequency. The selection of ω_l is to make the overall phase close to zero within a frequency band as wide as possible, so that we can have a fast convergence rate. ω_l is tuned to 1.5 Hz. Fig. 3 summarizes the intermediate and final effects of phase compensation by the approximate 0.5-order derivative and all-pass derivative. The utmost learnable bandwidth where the −90° crosses the phase characteristics increases significantly, from about 4 Hz to about 9 Hz, and to about 35 Hz, respectively.

The same ω_h and ω_l are used for all the other joints. To find the cutoff frequency, we plot $|1 - \Gamma G_p(j\omega)\Phi_c(j\omega)|$ with $\angle \Phi_c(j\omega) = 2\angle D_N(j\omega)$ and $|\Phi_c(j\omega)| = 1$, as shown in Fig. 4. Starting from the frequency points where $|1 - \Gamma G_p(j\omega)\Phi_c(j\omega)| = 1$, the cutoff frequencies are tuned in the experiments so that long-term slow divergence will not happen. The final tuning results are listed in Table 1. Figs. 5(a) and 6(a) are the rms error histories of 150 iterations for the unit-gain D-type ILC. The rms errors have reached close to the reproducibility level of the robot.
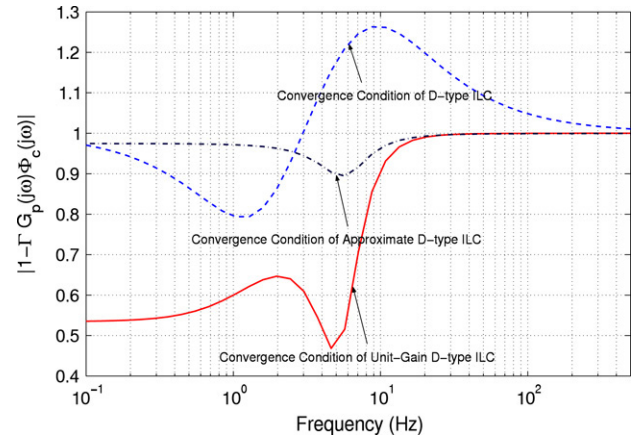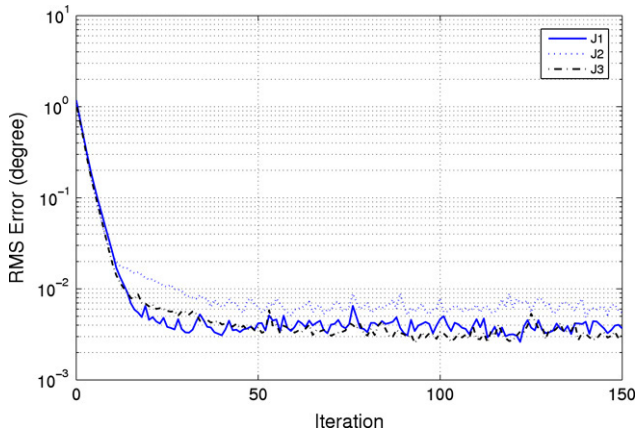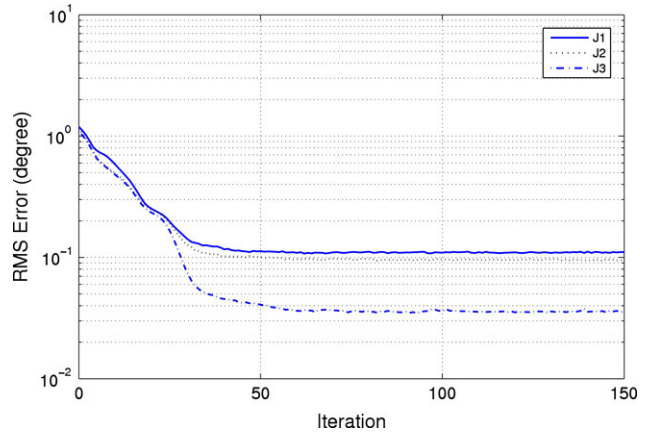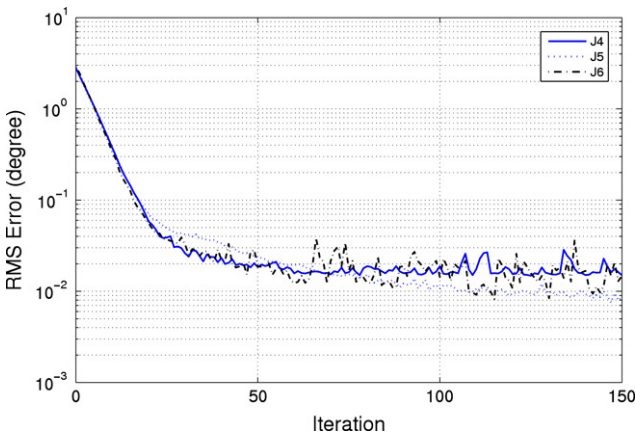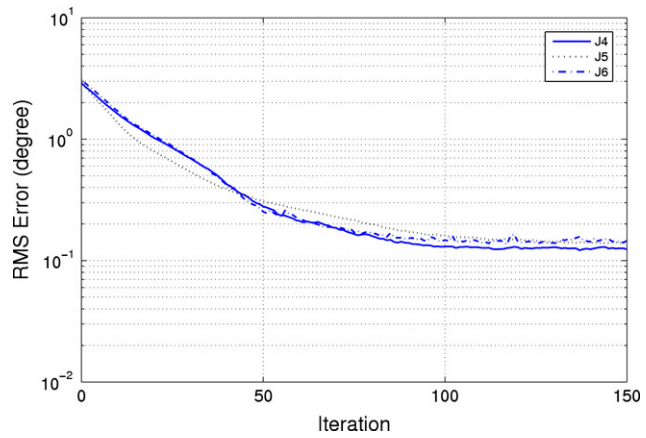
(a) Unit-gain D (J1, J2, J3).

(b) Approximate D (J1, J2, J3).

Fig. 5. RMS error histories of unit-gain D/approximate D-type ILC, J1–3.

(a) Unit-gain D (J4, J5, J6).

(b) Approximate D (J4, J5, J6).

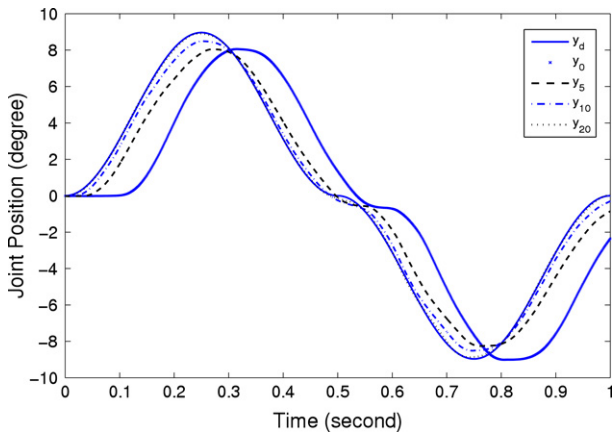**Fig. 6.** RMS error histories of unit-gain D/approximate D-type ILC, J4–6.

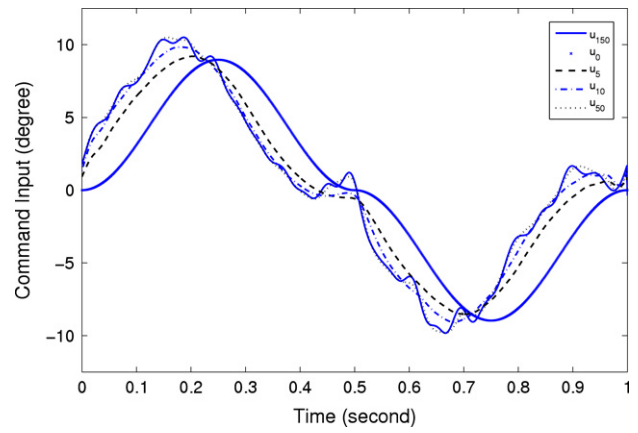
Fig. 7. Output profiles of joint 5.

Fig. 8. Input profiles of joint 5.

D-type ILC. But the rms errors are still one to two orders higher than their counterparts in the unit-gain D-type case, Fig. 5(b) and Fig. 6(b). Moreover, the learning is much slower than that of the unit-gain D-type.

The magnitude plot of the approximate s, shown by the solid curve in Fig. 11, can help to understand the performance difference. At low frequencies, the approximate derivative attenuates its input significantly. The attenuation makes the curve of $\left| 1 - \Gamma G_p(j\omega)\Phi_c(j\omega) \right|$ (joint 5) close to 1 in Fig. 4, and results in

a slow learning speed. While at high frequencies, the approximate derivative largely amplifies its input, and in turn, violates the convergence condition and brings in noise. The magnitude of s is much larger than that of the approximate derivative at high frequencies, as shown by the dash curve in Fig. 11.

The convergence condition for the conventional D-type ILC (joint 5) is also depicted in Fig. 4. The learnable band is rather narrow and the cutoff frequency has to be set very low. Comparison shows that, below 10 Hz, the curve of $\left| 1 - \Gamma G_p(j\omega)\Phi_c(j\omega) \right|$ for the
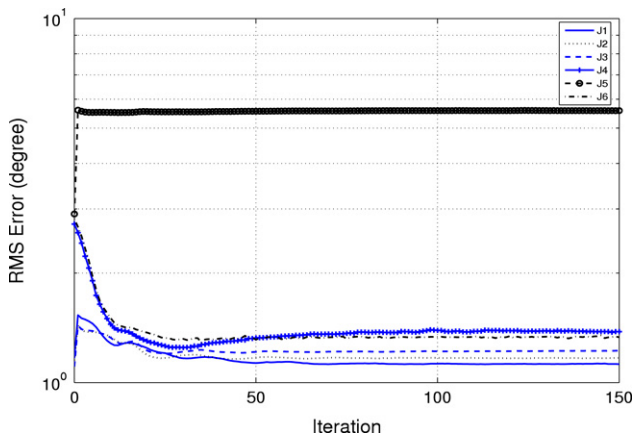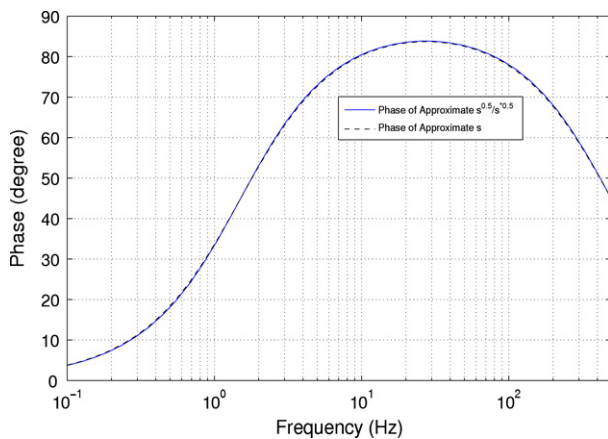
Fig. 9. RMS error histories of conventional D-type ILC.

Fig. 11. Magnitude of approximate s and s.
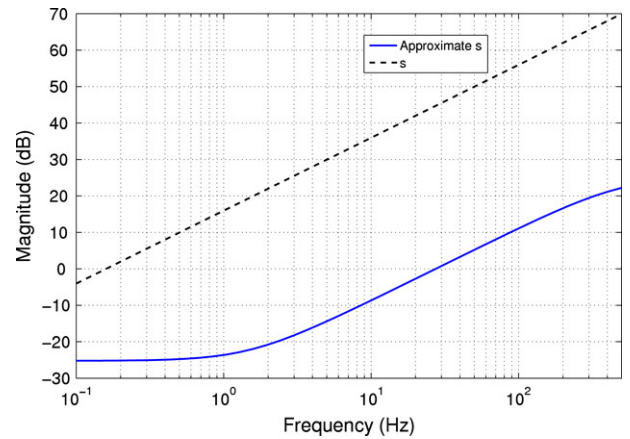

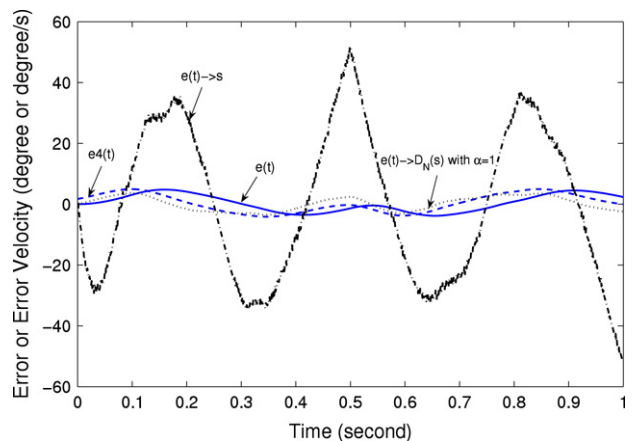**Fig. 10.** Phase characteristics of approximate D and all-pass D.

Fig. 12. Phase advances with respect to $e(t)$.

unit-gain D-type ILC is much more away from 1 than the other two curves, leading to good convergence rate for the unit-gain D-type ILC. It is demonstrated that the unit-gain feature does keep the learning speed unattenuated.

Fig. 12 compares $e(t)$, $\dot{e}(t)$, and $\dot{\bar{e}}(t)$ (*i.e.*, the filtered result of $e(t)$ by $D_N(s)$ with $\alpha = 1$), and the all-pass derivative $e4(t)$ in (14) after iteration 0.

It is noted that $\dot{e}(t)$, $\dot{\bar{e}}(t)$, and the all-pass derivatives $e4(t)$ are all phase-lead-shifted with respect to $e(t)$. The signal $\dot{e}(t)$ is noisy and its amplitude is large.

6. Conclusion

In this paper, the freedom of noncausal operation in ILC is exploited to achieve the unit-gain compensation feature which is highly desirable in ILC applications. It is found that the realization of a noncausal all-pass filter is as straightforward as that of a zero-phase filter. A method to generate a noncausal unit-gain learning compensator that can provide phase lead compensation is proposed. As an illustrative example, an all-pass unit-gain derivative is formulated. Based on this, a unit-gain D-type ILC is proposed. Experiments performed on a 6 DOF robot manipulator verify the advantages of the proposed approach. Future work will address the issues of a more general unit-gain learning compensator.

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