Available online at www.sciencedirect.com
ScienceDirect
Automatica 44 (2008) 830–837
automatica
www.elsevier.com/locate/automatica

Technical communique

Further results on adaptive iterative learning control of robot manipulators ☆

Chiang-Ju Chien [a,*], Abdelhamid Tayebi [b]

[a] Department of Electronic Engineering, Huafan University, 223, Shihding, Taipei County, Taiwan
[b] Department of Electrical Engineering, Lakehead University, 995 Oliver Road, Thunder bay, Ont., Canada

Received 28 October 2005; received in revised form 8 April 2007; accepted 17 June 2007
Available online 26 November 2007

Abstract

Based on a combination of a PD controller and a switching type two-parameter compensation force, an iterative learning controller with a projection-free adaptive algorithm is presented in this paper for repetitive control of uncertain robot manipulators. The adaptive iterative learning controller is designed without any *a priori* knowledge of robot parameters under certain properties on the dynamics of robot manipulators with revolute joints only. This new adaptive algorithm uses a combined time-domain and iteration-domain adaptation law allowing to guarantee the boundedness of the tracking error and the control input, in the sense of the infinity norm, as well as the convergence of the tracking error to zero, without any *a priori* knowledge of robot parameters. Simulation results are provided to illustrate the effectiveness of the learning controller.

Keywords: Iterative learning control; Adaptive control; Robot manipulator

1. Introduction

Classical PD and PID linear controllers are widely used in robotics applications due to their implementation simplicity. In the early works (Arimoto, 1996; Spong & Vidyasagar, 1989; Takegaki & Arimoto, 1981), most of the controllers were designed to asymptotically stabilize the joint positions of rigid robot manipulators at a given set point. Owing to the physical property that the robot parameters enter linearly in the Lagrange equation, adaptive control strategies (Slotine & Li, 1991; Tomei, 1991) have been derived for trajectory tracking instead of set-point regulation.

Taking advantage of the fact that robot manipulators are generally used in repetitive tasks, several iterative learning control (ILC) schemes have been proposed for robot manipulators in the past two decades. The main objective of ILC approach is to enhance the tracking accuracy from operation to operation for systems executing repetitive tasks. Initially, ILC algorithms for robot manipulators were developed based on the contraction mapping theory and required a certain *a priori* knowledge of robot dynamics (Bondi, Casalino, & Gambardella, 1988; Horowitz, 1993; Kawamura, Miyazaki, & Arimoto, 1988; Norrlöf, 2002; Wang, Son, & Cheah, 1995). In the past decade, another type of ILC algorithms, namely adaptive iterative learning control (AILC), has been widely studied in the literature. Substantial efforts, in the area of AILC design for robot manipulators, have been deployed during the last decade (see, for instance, Choi & Lee, 2000; Kuc & Han, 2000; Park, Kuc, & Lee, 1996; Tayebi, 2004; Xu & Wiswanathan, 2000). The main feature of AILC is to iteratively estimate the uncertain parameters, which are in turn used to generate the current control input. Because of the iteration based control problem, the adaptive learning laws for the estimation of the unknown parameters are mostly designed in the iteration domain. In general, projection or deadzone mechanisms are necessary to construct the iteration-domain based adaptive laws in order to guarantee the tracking error convergence as well as the boundedness of all internal signals. In Choi and Lee (2000) both time-domain and iteration-domain adaptations were used. A time-domain adaptive law estimates the robot parameters so that the upper bounds on these parameters are not necessary. However, the iteration-domain learning law which learns the desired input and

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Vladimir O. Nikiforov under the direction of Editor André Tits.

* Corresponding author. Fax: +886 2 86606675.
E-mail addresses: cjc@huafan.hfu.edu.tw (C.-J. Chien), Abdelhamid.Tayebi@lakeheadu.ca (A. Tayebi).

0005-1098/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.06.023

disturbances still needs the upper bound and the projection mechanism. Recently, in Tayebi (2004), three AILC schemes have been proposed for the tracking problem of rigid robot manipulators without any *a priori* knowledge of the robot dynamics.

Based on Tayebi (2004), a combination of a PD controller and a two-parameter switching type compensation force, a new adaptive law using mixed time-domain and iteration-domain adaptation is developed in this paper for repetitive control of uncertain robot manipulators. This adaptive ILC system guarantees the boundedness of all signals in the sense of the infinity norm without using a projection mechanism in the adaptive law—note that in Tayebi (2004), the boundedness of the control input is guaranteed in the sense of the L_2 norm. From time-domain point of view, this adaptive law introduces a term similar to the typical σ-modification (Ioannou & Sun, 1996) which provides certain robust characteristics. Based on this new design the projection mechanism, which is widely applied in those related works (Choi & Lee, 2000; Kuc & Han, 2000; Park et al., 1996; Tayebi, 2004; Xu & Wiswanathan, 2000) can be relaxed such that the upper bounds on the unknown parameters are not required. Under some suitable properties on the dynamics of robot manipulators with revolute joints only, the adaptive iterative learning controller can be designed without any *a priori* knowledge of the robot dynamics. Due to the switching type control force and the robust projection-free adaptive law, the perfect tracking control performance of the robot manipulators can be achieved under an uncertain random disturbance environment. The adaptive law will become a pure time-domain learning law or iteration-domain learning law if a weighting gain is suitably chosen. Comparisons of advantages and disadvantages among the pure time domain, pure iteration-domain and mixed adaptive laws will be presented. A rigorous proof based on the Lyapunov-like approach is given to guarantee the stability and convergence of the closed-loop learning system. It is shown that all adjustable parameters as well as internal signals are bounded in time domain for each iteration. Furthermore, the position and velocity tracking error will asymptotically converge to zero in the iteration domain.

2. Problem formulation

In this paper, we consider an n degrees-of-freedom rigid manipulator with the equations of motion expressed, using the Lagrangian formulation, by

$$\begin{aligned} &M(q_k(t))\ddot{q}_k(t) + C(q_k(t), \dot{q}_k(t))\dot{q}_k(t) + G(q_k(t)) \\ &\quad = \tau_k(t) + d_k(t), \end{aligned} \tag{1}$$

where $t \in [0, T]$ denotes the time index and $k \in Z_+$ denotes the iteration number. The signals $q_k(t), \dot{q}_k(t), \ddot{q}_k(t) \in R^n$ are the joint position, joint velocity and joint acceleration vectors, respectively, at the kth iteration. $M(q_k(t)) \in R^{n\times n}$ is the inertia matrix, $C(q_k(t), \dot{q}_k(t))\dot{q}_k(t) \in R^n$ is a vector resulting from Coriolis and centrifugal forces, and $G(q_k(t)) \in R^n$ is the vector resulting from the gravitational forces. $\tau_k(t) \in R^n$ is the control input vector containing the torques and forces to be applied at each joint. $d_k(t) \in R^n$ is the vector containing the unknown external disturbances.

Assuming that the joint positions and velocities are measurable for feedback design, the control objective is to design a bounded adaptive iterative learning controller $\tau_k(t)$ ensuring the boundedness of $q_k(t), \dot{q}_k(t), \ddot{q}_k(t)$, $\forall t \in [0, T]$ and $\forall k \in Z_+$ and the convergence of $q_k(t), \dot{q}_k(t)$ to the desired reference position and velocity trajectories $q_d(t), \dot{q}_d(t)$ $\forall t \in [0, T]$ as k tends to infinity. To achieve the control objective, we make the following assumptions (Tayebi, 2004) for the robot manipulators:

- (A1) The reference trajectory $q_d(t)$ is achievable for the robot manipulators considered.
- (A2) $q_d(t), \dot{q}_d(t)$ and $\ddot{q}_d(t)$, as well as the external disturbance $d_k(t)$ are bounded $\forall t \in [0, T]$ and $\forall k \in Z_+$.
- (A3) The resetting initial condition is satisfied, i.e., $\dot{q}_d(0) - \dot{q}_k(0) = q_d(0) - q_k(0) = 0$, $\forall k \in Z_+$.

We also need the following properties the same as those stated in Tayebi (2004), which are common to robot manipulators.

- (P1) $M(q_k) \in R^{n\times n}$ is symmetric, bounded, and positive definite.
- (P2) The matrix $\dot{M}(q_k) - 2C(q_k, \dot{q}_k)$ is skew symmetric, hence $x^\top(\dot{M}(q_k) - 2C(q_k, \dot{q}_k))x = 0$, $\forall x \in R^n$.
- (P3) $\|C(q_k, \dot{q}_k)\| \leqslant k_c\|\dot{q}_k(t)\|$, $\|G(q_k)\| \leqslant k_g$, and $\|d_k(t)\| \leqslant k_d$, $\forall q_k, \dot{q}_k$, $\forall t \in [0, T]$ and $\forall k \in Z_+$, where k_c, k_g and k_d are unknown positive parameters.

Under Assumption (A2) and Property (P1), there exists an unknown positive constant k_{md} such that $\|M(q_k)\ddot{q}_d(t)\| \leqslant k_{md}$ $\forall t \in [0, T]$, $\forall k \in Z_+$. If we define the tracking joint position error and joint velocity error as $\tilde{q}_k(t) = q_d(t) - q_k(t)$ and $\dot{\tilde{q}}_k(t) = \dot{q}_d(t) - \dot{q}_k(t)$, respectively, then we have

$$\begin{aligned} &\dot{\tilde{q}}_k^\top(t)(M(q_k)\ddot{q}_d(t) + C(q_k, \dot{q}_k)\dot{q}_d(t) + G(q_k) - d_k(t)) \\ &\quad \leqslant \|\dot{\tilde{q}}_k(t)\|(k_{md} + k_g + k_d + k_c\|\dot{q}_d(t)\|\ \|\dot{q}_k(t)\|) \\ &\quad \leqslant \|\dot{\tilde{q}}_k(t)\|(k_{md} + k_g + k_d + k_c\|\dot{q}_d(t)\|^2 \\ &\qquad + k_c\|\dot{q}_d(t)\|\ \|\dot{\tilde{q}}_k(t)\|) \\ &\quad \leqslant \alpha\|\dot{\tilde{q}}_k(t)\|^2 + \delta'\|\dot{\tilde{q}}_k(t)\|, \end{aligned} \tag{2}$$

where $\alpha = k_c \sup_{t\in[0,T]}\|\dot{q}_d(t)\|$, $\delta' = k_{md} + k_g + k_d + k_c \sup_{t\in[0,T]}\|\dot{q}_d(t)\|^2$. Note that there exists a positive constant δ such that $\delta'\|\dot{\tilde{q}}_k(t)\| \leqslant \delta\|\dot{\tilde{q}}_k(t)\|_1 = \delta\dot{\tilde{q}}_k^\top(t)\ \mathrm{sgn}(\dot{\tilde{q}}_k(t))$ where $\|\dot{\tilde{q}}_k(t)\|_1$ is the one-norm of $\dot{\tilde{q}}_k(t)$ and $\mathrm{sgn}(\dot{\tilde{q}}_k(t)) = [\mathrm{sgn}(\dot{\tilde{q}}_{1,k}(t)), \ldots, \mathrm{sgn}(\dot{\tilde{q}}_{n,k}(t))]^\top$. This implies that inequality (2) can be rewritten as

$$\begin{aligned} &\dot{\tilde{q}}_k^\top(t)(M(q_k)\ddot{q}_d(t) + C(q_k, \dot{q}_k)\dot{q}_d(t) + G(q_k) - d_k(t)) \\ &\quad \leqslant \dot{\tilde{q}}_k^\top(t)(\alpha\dot{\tilde{q}}_k(t) + \delta\,\mathrm{sgn}(\dot{\tilde{q}}_k(t))) \\ &\quad \equiv \dot{\tilde{q}}_k^\top(t)\eta(\dot{\tilde{q}}_k)\theta, \end{aligned} \tag{3}$$

where $\eta(\dot{\tilde{q}}_k) = [\dot{\tilde{q}}_k(t), \mathrm{sgn}(\dot{\tilde{q}}_k(t))] \in R^{n\times 2}$, and $\theta = [\alpha, \delta]^\top \in R^2$.

3. Iterative learning controller with hybrid adaptive algorithm

To achieve the error convergence of $\tilde{q}_k(t)$ and $\dot{\tilde{q}}_k(t)$ with all the internal signals being bounded, we propose a projection-free hybrid adaptive iterative learning controller as follows:

$$\tau_k(t) = K_p\tilde{q}_k(t) + K_D\dot{\tilde{q}}_k(t) + \eta(\dot{\tilde{q}}_k(t))\hat{\theta}_k(t), \tag{4}$$

where $K_p \in R^{n\times n}$, $K_D \in R^{n\times n}$ are symmetric positive definite, and the adaptive law

$$(1-\gamma)\dot{\hat{\theta}}_k(t) = -\gamma\hat{\theta}_k(t) + \gamma\hat{\theta}_{k-1}(t) + \beta\eta^\top(\dot{\tilde{q}}_k(t))\dot{\tilde{q}}_k(t), \tag{5}$$

where $0<\gamma<1$, $\beta>0$ are defined as the weighting gain and learning gain, respectively. The initial value of the parameter vector is set to be $\hat{\theta}_k(0) = \hat{\theta}_{k-1}(T)\ \forall k \in Z_+$, and the initial parameter profile for $k=0$ is chosen as $\hat{\theta}_0(t) = \theta_{\mathrm{ini}}, \forall t \in [0,T]$ where θ_{ini} is a constant parameter vector. In general, the adaptive law (5) will become a pure time-domain adaptive law if $\gamma = 0$, or a pure iteration-domain adaptive law if $\gamma = 1$. In addition to the convergence of $\tilde{q}_k(t)$ and $\dot{\tilde{q}}_k(t)$ to zero when k tends to infinity, we will also guarantee the boundedness of all the internal signals, especially the boundedness of the parameter vector $\hat{\theta}_k(t)$ and control torque $\tau_k(t)$. To study the stability and convergence of the proposed adaptive iterative learning controller for robot manipulators, we use the concept of $L_{pe}[0,T]$ in the subsequent discussions to denote the set of Lebesgue measurable (or piecewise continuous) real valued (vector) functions with L_{pe} norm (Tayebi, 2004)

$$\|x(t)\|_{pe} = \begin{cases} \left(\int_0^T \|x(t)\|^p\,\mathrm{d}t\right)^{1/p} & \text{if } p \in [1,\infty), \\ \sup\limits_{0\leqslant t\leqslant T} \|x(t)\| & \text{if } p = \infty. \end{cases}$$

We say that $x(t) \in L_{pe}[0,T]$ when $\|x(t)\|_{pe}$ exists (i.e., when $\|x(t)\|_{pe}$ is finite).

At first, we will derive the boundedness of $\tilde{q}_1(t), \dot{\tilde{q}}_1(t), \hat{\theta}_1(t), \tau_1(t)$ at the first iteration in a way different from that for $\tilde{q}_k(t), \dot{\tilde{q}}_k(t), \hat{\theta}_k(t), \tau_k(t)$ with $k\geqslant 2$.

Proposition 1. *Consider the robot manipulator system* (1) *with properties* (P1)–(P3) *under the control torque* (4) *and parameter adaptive law* (5). *If assumptions* (A1)–(A3) *are satisfied, then we have* $\tilde{q}_1(t), \dot{\tilde{q}}_1(t), \hat{\theta}_1(t), \tau_1(t) \in L_{\infty e}[0,T]$.

Proof. Let us consider the following Lyapunov-like positive definite function:

$$V_k(t) = \frac{1}{2}\dot{\tilde{q}}_k^\top(t)M(q_k)\dot{\tilde{q}}_k(t) + \frac{1}{2}\tilde{q}_k^{\top}(t)K_p\tilde{q}_k(t) + \frac{1-\gamma}{2\beta}\tilde{\theta}_k^{\top}(t)\tilde{\theta}_k(t), \tag{6}$$

where $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta$ is the parameteric estimation error. Its derivative with respective to time t along (1) can be computed as follows[1]:

$$\begin{aligned}\dot{V}_k &= \dot{\tilde{q}}_k^\top M(q_k)\ddot{\tilde{q}}_k + \frac{1}{2}\dot{\tilde{q}}_k^\top\dot{M}(q_k)\dot{\tilde{q}}_k + \dot{\tilde{q}}_k^\top K_p\tilde{q}_k + \frac{1-\gamma}{\beta}\tilde{\theta}_k^\top\dot{\tilde{\theta}}_k \\ &= \dot{\tilde{q}}_k^\top(-M(q_k)\ddot{q}_k + M(q_k)\ddot{q}_d) \\ &\quad + \frac{1}{2}\dot{\tilde{q}}_k^\top\dot{M}(q_k)\dot{\tilde{q}}_k + \dot{\tilde{q}}_k^\top K_p\tilde{q}_k + \frac{1-\gamma}{\beta}\tilde{\theta}_k^\top\dot{\tilde{\theta}}_k \\ &= \dot{\tilde{q}}_k^\top(M(q_k)\ddot{q}_d + C(q_k,\dot{q}_k)\dot{q}_d + G(q_k) - d_k) \\ &\quad - \dot{\tilde{q}}_k^\top\tau_k + \dot{\tilde{q}}_k^\top K_p\tilde{q}_k + \frac{1-\gamma}{\beta}\tilde{\theta}_k^\top\dot{\tilde{\theta}}_k,\end{aligned} \tag{7}$$

where property (P2) is applied. Substituting inequality (3) and the control torque (4) into (7), we have

$$\dot{V}_k \leqslant -\dot{\tilde{q}}_k^\top K_D\dot{\tilde{q}}_k - \dot{\tilde{q}}_k^\top\eta(\dot{\tilde{q}}_k)\tilde{\theta}_k + \frac{1-\gamma}{\beta}\tilde{\theta}_k^\top\dot{\tilde{\theta}}_k. \tag{8}$$

Using the adaptive law (5) and the fact that $-\gamma\hat{\theta}_k + \gamma\hat{\theta}_{k-1} = -\gamma\tilde{\theta}_k + \gamma\tilde{\theta}_{k-1}$, Eq. (8) leads to

$$\begin{aligned}\dot{V}_k &\leqslant -\dot{\tilde{q}}_k^\top K_D\dot{\tilde{q}}_k - \dot{\tilde{q}}_k^\top\eta(\dot{\tilde{q}}_k)\tilde{\theta}_k \\ &\quad + \frac{1}{\beta}\tilde{\theta}_k^{\top}(-\gamma\tilde{\theta}_k + \gamma\tilde{\theta}_{k-1} + \beta\eta^\top(\dot{\tilde{q}}_k)\dot{\tilde{q}}_k) \\ &= -\dot{\tilde{q}}_k^\top K_D\dot{\tilde{q}}_k - \frac{\gamma}{\beta}\tilde{\theta}_k^\top\tilde{\theta}_k + \frac{\gamma}{\beta}\tilde{\theta}_k^\top\tilde{\theta}_{k-1} \\ &= -\dot{\tilde{q}}_k^\top K_D\dot{\tilde{q}}_k + \frac{\gamma}{4\beta}\tilde{\theta}_{k-1}^\top\tilde{\theta}_{k-1} \\ &\quad - \frac{\gamma}{\beta}\left(\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}\right)^\top\left(\tilde{\theta}_k - \frac{1}{2}\tilde{\theta}_{k-1}\right) \\ &\leqslant \frac{\gamma}{4\beta}\tilde{\theta}_{k-1}^\top\tilde{\theta}_{k-1}.\end{aligned} \tag{9}$$

Now, consider the first iteration of $k=1$. Since $\hat{\theta}_0(t)$ in the adaptive law (5) is chosen as a constant vector $\theta_{\mathrm{ini}}\ \forall t \in [0,T]$, we have $\tilde{\theta}_0(t) = \hat{\theta}_0(t) - \theta = \theta_{\mathrm{ini}} - \theta \equiv \bar{\theta}_0$ and $\tilde{\theta}_1(0) = \hat{\theta}_1(0) - \theta = \hat{\theta}_0(T) - \theta = \theta_{\mathrm{ini}} - \theta \equiv \bar{\theta}_0$. This implies that the initial condition of $V_1(0) = \frac{1}{2}\dot{\tilde{q}}_1^\top(0)M(q_1)\dot{\tilde{q}}_1(0) + \frac{1}{2}\tilde{q}_1^\top(0)K_p\tilde{q}_1(0) + \frac{1-\gamma}{2\beta}\tilde{\theta}_1^\top(0)\tilde{\theta}_1(0) = \frac{1-\gamma}{2\beta}\bar{\theta}_0^\top\bar{\theta}_0$ is bounded due to Assumption (A2). The Lyapunov-like function (6) at the first iteration will now satisfy

$$\dot{V}_1(t) \leqslant \frac{\gamma}{4\beta}\tilde{\theta}_0^\top(t)\tilde{\theta}_0(t) = \frac{\gamma}{4\beta}\bar{\theta}_0^\top\bar{\theta}_0 \tag{10}$$

which readily concludes that $V_1(t), \tilde{q}_1(t), \dot{\tilde{q}}_1(t), \tilde{\theta}_1(t) \in L_{\infty e}[0,T]$ and hence, $\tau_1(t) \in L_{\infty e}[0,T]$. □

Based on the results given in Proposition 1, we next prove the boundedness of $\tilde{q}_k(T), \dot{\tilde{q}}_k(T), \hat{\theta}_k(T)$ at the end of each iteration and the convergence of $\dot{\tilde{q}}_k^\top\dot{\tilde{q}}_k$ in the sense of L_{1e} norm.

[1] Throughout this proof, the argument t will be omitted if it does not lead to any confusion.

Proposition 2. *Consider the problem set-up in Proposition* 1. *The proposed adaptive iterative learning system ensures that* $\dot{\tilde{q}}_k(T), \tilde{q}_k(T), \tilde{\theta}_k(T), \int_0^T \tilde{\theta}_k^\top \tilde{\theta}_k \,\mathrm{d}t$, *and* $\int_0^T \dot{\tilde{q}}_k^\top \dot{\tilde{q}}_k \,\mathrm{d}t$ *are bounded* $\forall k \in Z_+$ *and*

$$\lim_{k\to\infty} \dot{\tilde{q}}_k(T) = \lim_{k\to\infty} \tilde{q}_k(T) = \lim_{k\to\infty} \int_0^T \dot{\tilde{q}}_k^\top \dot{\tilde{q}}_k \,\mathrm{d}t = 0.$$

Proof. Define a positive definite functional $W_k(T)$ as

$$W_k(T) = \int_0^T \frac{\gamma}{\beta} \tilde{\theta}_k^\top \tilde{\theta}_k \,\mathrm{d}t + \frac{1-\gamma}{2\beta} \tilde{\theta}_k^\top(T)\tilde{\theta}_k(T). \tag{11}$$

The difference between $W_k(T)$ and $W_{k-1}(T)$ can be derived by using integration by parts and the fact that $\tilde{\theta}_k(0) = \tilde{\theta}_{k-1}(T)$ as follows:

$$\begin{aligned}
\Delta W_k(T) &= W_k(T) - W_{k-1}(T) \\
&= \int_0^T \frac{\gamma}{2\beta}[\tilde{\theta}_k^\top \tilde{\theta}_k - \tilde{\theta}_{k-1}^\top \tilde{\theta}_{k-1}] \,\mathrm{d}t + \frac{1-\gamma}{2\beta}\tilde{\theta}_k^\top(T)\tilde{\theta}_k(T) \\
&\quad - \frac{1-\gamma}{2\beta}\tilde{\theta}_{k-1}^\top(T)\tilde{\theta}_{k-1}(T) \\
&= \int_0^T \frac{\gamma}{2\beta}[\tilde{\theta}_k^\top \tilde{\theta}_k - \tilde{\theta}_{k-1}^\top \tilde{\theta}_{k-1}] \,\mathrm{d}t + \frac{1-\gamma}{\beta}\int_0^T \tilde{\theta}_k^\top \dot{\tilde{\theta}}_k \,\mathrm{d}t \\
&\quad + \frac{1-\gamma}{2\beta}\tilde{\theta}_k^\top(0)\tilde{\theta}_k(0) - \frac{1-\gamma}{2\beta}\tilde{\theta}_{k-1}^\top(T)\tilde{\theta}_{k-1}(T) \\
&= \int_0^T \frac{\gamma}{2\beta}[\tilde{\theta}_k^\top \tilde{\theta}_k - \tilde{\theta}_{k-1}^\top \tilde{\theta}_{k-1}] \,\mathrm{d}t \\
&\quad + \frac{1}{\beta}\int_0^T \tilde{\theta}_k^\top[-\gamma\tilde{\theta}_k + \gamma\tilde{\theta}_{k-1} + \beta\eta(\dot{\tilde{q}}_k)^\top \dot{\tilde{q}}_k] \,\mathrm{d}t \\
&= \int_0^T -\frac{\gamma}{2\beta}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^\top(\tilde{\theta}_k - \tilde{\theta}_{k-1}) \,\mathrm{d}t \\
&\quad + \int_0^T \tilde{\theta}_k^\top \eta^\top(\dot{\tilde{q}}_k)\dot{\tilde{q}}_k \,\mathrm{d}t.
\end{aligned} \tag{12}$$

Now, define another positive definite function U_k as

$$U_k = \tfrac{1}{2}\dot{\tilde{q}}_k^\top M(q_k)\dot{\tilde{q}}_k + \tfrac{1}{2}\tilde{q}_k^\top K_p \tilde{q}_k,$$

i.e., $V_k = U_k + ((1-\gamma)/2\beta)\tilde{\theta}_k^\top\tilde{\theta}_k$. The time derivative of U_k with respective to time t will satisfy

$$\dot{U}_k \leqslant -\dot{\tilde{q}}_k^\top K_D \dot{\tilde{q}}_k - \dot{\tilde{q}}_k^\top \eta(\dot{\tilde{q}}_k)\tilde{\theta}_k \tag{13}$$

according to the result of (8). Integrating (13) from 0 to T gives

$$U_k(T) - U_k(0) \leqslant -\int_0^T \dot{\tilde{q}}_k^\top K_D \dot{\tilde{q}}_k \,\mathrm{d}t - \int_0^T \dot{\tilde{q}}_k^\top \eta(\dot{\tilde{q}}_k)\tilde{\theta}_k \,\mathrm{d}t,$$

which implies that

$$\begin{aligned}
\int_0^T \tilde{\theta}_k^\top \eta^\top(\dot{\tilde{q}}_k)\dot{\tilde{q}}_k \,\mathrm{d}t &= \int_0^T \dot{\tilde{q}}_k^\top \eta(\dot{\tilde{q}}_k)\tilde{\theta}_k \,\mathrm{d}t \\
&\leqslant -U_k(T) - \int_0^T \dot{\tilde{q}}_k^\top K_D \dot{\tilde{q}}_k \,\mathrm{d}t,
\end{aligned} \tag{14}$$

where we use the fact that $U_k(0) = 0$ due to Assumption (A2). Substituting (14) into (12), yields

$$\begin{aligned}
\Delta W_k(T) &= W_k(T) - W_{k-1}(T) \\
&\leqslant -U_k(T) - \int_0^T \dot{\tilde{q}}_k^\top K_D \dot{\tilde{q}}_k \,\mathrm{d}t \\
&\quad - \int_0^T \frac{\gamma}{2\beta}(\tilde{\theta}_k - \tilde{\theta}_{k-1})^\top(\tilde{\theta}_k - \tilde{\theta}_{k-1}) \,\mathrm{d}t
\end{aligned} \tag{15}$$

$$\leqslant -U_k(T) - \int_0^T \dot{\tilde{q}}_k^\top K_D \dot{\tilde{q}}_k \,\mathrm{d}t \leqslant 0. \tag{16}$$

The boundedness of $W_k(T)$ and hence, $\tilde{\theta}_k^\top(T)\tilde{\theta}_k(T)$ and $\int_0^T \tilde{\theta}_k^\top\tilde{\theta}_k \,\mathrm{d}t$, is guaranteed $\forall k \in Z_+$ since $W_1(T)$ is bounded according to Proposition 1. Furthermore, Eq. (16) implies that

$$\begin{aligned}
U_k(T) + \int_0^T \dot{\tilde{q}}_k^\top K_D \dot{\tilde{q}}_k \,\mathrm{d}t &\leqslant W_{k-1}(T) - W_k(T) \\
&\leqslant W_1(T)
\end{aligned} \tag{17}$$

which ensures the boundedness of $U_k(T)$, $\dot{\tilde{q}}_k(T)$, $\tilde{q}_k(T)$ and $\int_0^T \dot{\tilde{q}}_k^\top \dot{\tilde{q}}_k \,\mathrm{d}t$, $\forall k \in Z_+$. Note that Eq. (16) also gives

$$W_k(T) \leqslant W_1(T) - \sum_{j=2}^k U_j(T) - \sum_{j=2}^k \int_0^T \dot{\tilde{q}}_j^\top K_D \dot{\tilde{q}}_j \,\mathrm{d}t \tag{18}$$

or equivalently

$$\begin{aligned}
\sum_{j=2}^k U_j(T) + \sum_{j=2}^k \int_0^T \dot{\tilde{q}}_j^\top K_D \dot{\tilde{q}}_j \,\mathrm{d}t &\leqslant W_1(T) - W_k(T) \\
&\leqslant W_1(T).
\end{aligned} \tag{19}$$

Hence, we conclude from (16), (17) and (19) that $\dot{\tilde{q}}_k(T)$, $\tilde{q}_k(T)$, $\tilde{\theta}_k(T)$, $\int_0^T \tilde{\theta}_k^\top\tilde{\theta}_k \,\mathrm{d}t$, and $\int_0^T \dot{\tilde{q}}_k^\top \dot{\tilde{q}}_k \,\mathrm{d}t$ are bounded $\forall k \in Z_+$ and

$$\begin{aligned}
\lim_{k\to\infty} U_k(T) &= \lim_{k\to\infty} \dot{\tilde{q}}_k(T) = \lim_{k\to\infty} \tilde{q}_k(T) \\
&= \lim_{k\to\infty} \int_0^T \dot{\tilde{q}}_k^\top \dot{\tilde{q}}_k \,\mathrm{d}t = 0. \quad \square
\end{aligned} \tag{20}$$

In Propositions 1 and 2, we have shown that all the internal signals for the first iteration are bounded, and $\dot{\tilde{q}}_k(T)$, $\tilde{q}_k(T)$, $\tilde{\theta}_k(T)$, or equivalently $\dot{\tilde{q}}_k(0)$, $\tilde{q}_k(0)$, $\tilde{\theta}_k(0)$, are bounded $\forall k \in Z_+$. In the following theorem, the boundedness of all the internal signals at each iteration and convergence of $\dot{\tilde{q}}_k(t)$, $\tilde{q}_k(t)$ will be established.

Theorem. *Consider the same problem set-up in Proposition* 1. *If Assumptions* (A1)–(A3) *are satisfied, then we have* $\tilde{q}_k(t)$, $\dot{\tilde{q}}_k(t)$, $\hat{\theta}_k(t)$, $\tau_k(t) \in L_{\infty e}[0, T]$ *for all* $k \in Z_+$ *and* $\lim_{k\to\infty} \tilde{q}_k(t) = \lim_{k\to\infty} \dot{\tilde{q}}_k(t) = 0$, $\forall t \in [0, T]$.

Proof. Consider the Lyapunov-like function $V_k(t)$ in (6) again and note that its time derivative satisfies (9). Integrating (9) from 0 to t for any $t \in [0, T]$, we have

$$\begin{aligned} V_k(t) &\leqslant V_k(0) + \int_0^t \frac{\gamma}{4\beta}\widetilde{\theta}_{k-1}^\top(t')\widetilde{\theta}_{k-1}(t')\,\mathrm{d}t' \\ &\leqslant V_k(0) + \int_0^T \frac{\gamma}{4\beta}\widetilde{\theta}_{k-1}^\top(t)\widetilde{\theta}_{k-1}(t)\,\mathrm{d}t. \end{aligned} \tag{21}$$

As $V_k(0)=((1-\gamma)/2\beta)\widetilde{\theta}_k^\top(0)\widetilde{\theta}_k(0)=((1-\gamma)/2\beta)\widetilde{\theta}_{k-1}^\top(T)\widetilde{\theta}_{k-1}(T)$ and $\int_0^T(\gamma/4\beta)\widetilde{\theta}_{k-1}^\top(t)\widetilde{\theta}_{k-1}(t)\,\mathrm{d}t$ are finite for all $k \in Z_+$ as shown in Proposition 2, the finiteness of $V_k(t)$, $\widetilde{q}_k(t)$, $\dot{\widetilde{q}}_k(t)$, $\widetilde{\theta}_k(t)$ and $\tau_k(t)$ is guaranteed by (21) $\forall t \in [0, T]$ and $\forall k \in Z_+$. Moreover, $\ddot{\widetilde{q}}_k(t)$ is also bounded due to the equation of motion (1) for robot manipulators. Now, we have $\widetilde{q}_k(t), \dot{\widetilde{q}}_k(t), \ddot{\widetilde{q}}_k(t) \in L_{\infty e}[0, T]$ and

$$\lim_{k\to\infty}\int_0^T \dot{\widetilde{q}}_k^\top(t)\dot{\widetilde{q}}_k(t)\,\mathrm{d}t = 0. \tag{22}$$

Consequently, $\lim_{k\to\infty}\dot{\widetilde{q}}_k(t)=0$, $\forall t \in [0, T]$ by using similar argument for Barbalat's lemma (e.g., Ioannou & Sun, 1996, Lemma 3.2.6). Finally, it is easy to show that $\lim_{k\to\infty}\widetilde{q}_k(t)=0$, $\forall t \in [0, T]$ since $\widetilde{q}_k(0)=0$ and $\widetilde{q}_k(t)$ is uniformly continuous over $[0, T]$, $\forall k \in Z_+$ by (22). This completes the proof. □

Remark 1. In this paper, we present a general adaptive learning algorithm combining time-domain and iteration-domain adaptation for adaptive ILC of robot manipulators. In the main theorem, we show that $\dot{\widetilde{q}}_k(t)$ and $\widetilde{q}_k(t)$ converge to zero for all $t \in [0, T]$ as the iteration number $k \to \infty$ with all the internal signals belonging to $L_{\infty e}[0, T]$. For the extreme case of $\gamma = 1$, the adaptive law (5) becomes a pure iteration-domain adaptive law as that in Tayebi (2004) as follows:

$$\widehat{\theta}_k(t) = \widehat{\theta}_{k-1}(t) + \beta\eta^\top(\dot{\widetilde{q}}_k(t))\dot{\widetilde{q}}_k(t) \tag{23}$$

with $\widehat{\theta}_0(t)$ being some specified initial vector. The main advantage of (23) is that it can be applied to systems with time-varying parameters and the learning convergence speed is in general faster than the case of $\gamma \in [0, 1)$. But, without a projection mechanism, only L_2 boundedness of control parameters is guaranteed. On the other hand, another extreme case of $\gamma = 0$ results in a pure time-domain adaptive law

$$\dot{\widehat{\theta}}_k(t) = \beta\eta^\top(\dot{\widetilde{q}}_k(t))\dot{\widetilde{q}}_k(t), \tag{24}$$

where $\widehat{\theta}_k(0) = \widehat{\theta}_{k-1}(T)$, $\forall k \in Z_+$ and $\widehat{\theta}_1(0) = \theta_{\mathrm{ini}}$ for some specified constant vector θ_{ini}. The technical results shown in our theorem are still valid for the case of $\gamma = 0$. In fact, Eq. (24) is similar to the adaptive law presented in French and Rogers (2000). The main feature of (24) is that the previous parameter profile during the time interval $[0, T]$ is no longer needed. However, the convergence speed of the learning error is in general slow, especially when compared with the cases of $\gamma \neq 0$. This can be easily seen from (15), where larger values of γ contribute to generate a more negative term in the right-hand side. By increasing γ will result in W_k much smaller than W_{k-1} indicating higher convergence rates.

Remark 2. It is worth noting that the updating term $\beta\eta^\top(\dot{\widetilde{q}}_k(t))\dot{\widetilde{q}}_k(t)$ is always positive. This implies that a possible parameter drift may occur for both pure iteration-domain adaptive law (23) and pure time-domain adaptive law (24) when the velocity tracking error $\dot{\widetilde{q}}_k(t)$ cannot be exactly zero in a real physical environment. To solve the possible parameter drift, it is necessary to introduce a certain mechanism such as a projection or a deadzone for a practical realization. The combined adaptive law (5), however, is stable in time domain as the eigenvalue $-\gamma/(1-\gamma) < 0$. Once the control parameters at the previous trial and the velocity tracking error are both bounded, the control parameters at the current trial will be bounded.

Remark 3. For the problem of nonzero initial position or velocity errors at the beginning of each iteration, a possible solution is to apply the technique such as that in Chien and Yao (2004) where a time-varying boundary layer based saturation function approach is utilized to compensate for the uncertainties from initial errors. As the position and velocity variables are measurable, a rectifying action such as that in Xu and Yan (2005) can also be applied to solve the problem. More precisely, let the reference position and velocity trajectories be revised as

$$q_d^*(t) = \begin{cases} q_d(t) & \text{if } t \in [h, T], \\ q_r(t) & \text{if } t \in [0, h), \end{cases}$$

$$\dot{q}_d^*(t) = \begin{cases} \dot{q}_d(t) & \text{if } t \in [h, T], \\ \dot{q}_r(t) & \text{if } t \in [0, h), \end{cases}$$

where $h \in [0, T]$ can be chosen arbitrary. Here $q_r(t)$ and $\dot{q}_r(t)$ are certain smooth functions that connect $q_k(0)$ and $q_d(h)$ as

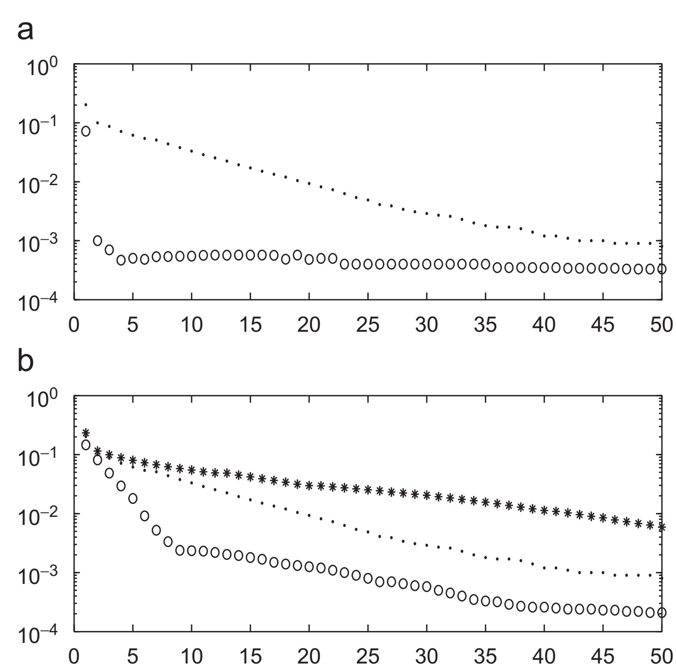

Fig. 1. Convergence of the tracking errors. (a) $\sup_{t\in[0,1]}\widetilde{q}_k(t)$ (rad) versus iteration number k; $\cdots$ for link 1, $\circ\circ\circ$ for link 2, $\gamma=0.5$. (b) $\sup_{t\in[0,1]}\widetilde{q}_{1,k}(t)$ (rad) versus iteration number k; $\cdots$ for $\gamma=0$, $***$ for $\gamma=0.5$, $\circ\circ\circ$ $\gamma=1$.

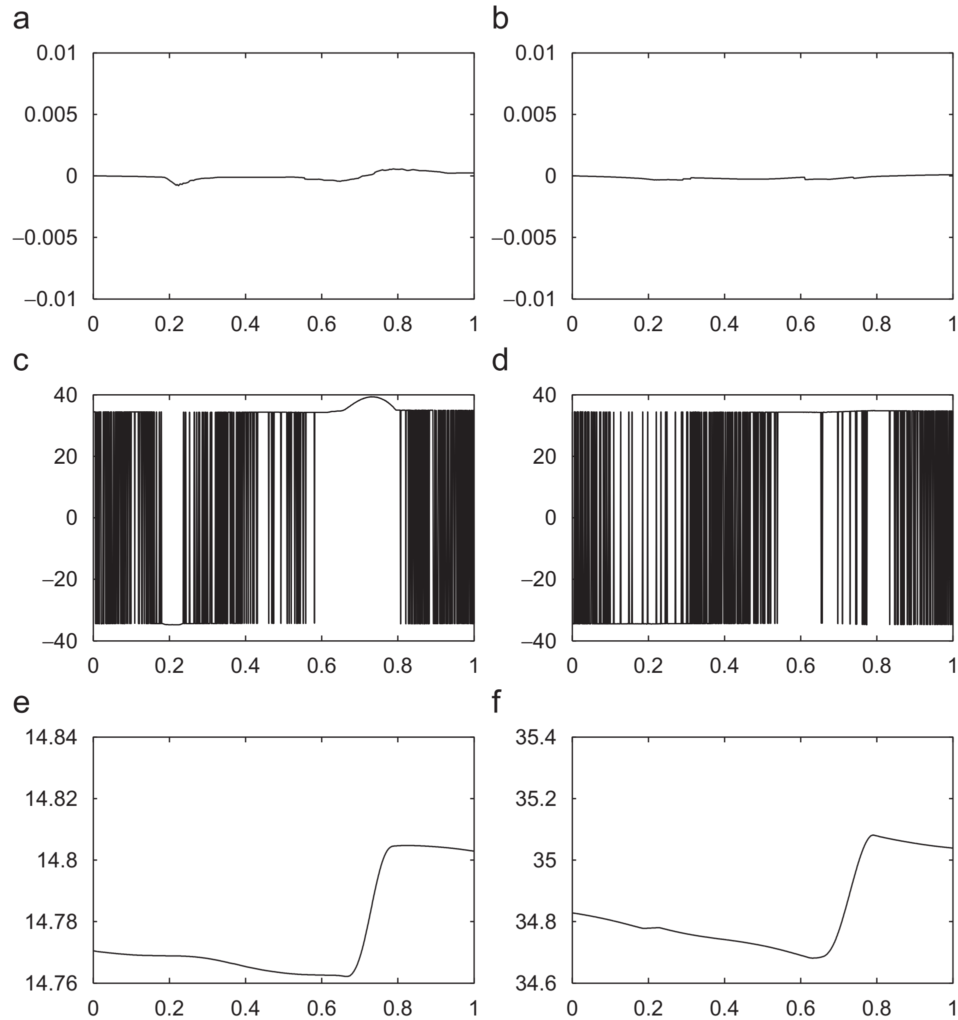

Fig. 2. Responses at the 50th iteration ($k=50$) under adaptive learning controller (4), (5) with sign function. (a) $\widetilde{q}_{1,k}(t)$ versus time t, (b) $\widetilde{q}_{2,k}(t)$ versus time t, (c) $\tau_{1,k}(t)$ versus time t, (d) $\tau_{2,k}(t)$ versus time t, (e) $\widehat{\theta}_{1,k}(t)$ versus time t, (f) $\widehat{\theta}_{2,k}(t)$ versus time t.

well as $\dot{q}_k(0)$ and $\dot{q}_d(h)$ at the time moment $t=h$. The less the h, the closer the trajectories $q_d^*(t)$ and $\dot{q}_d^*(t)$ to the original reference trajectories. Under these conditions, we have $q_d^*(0)-q_k(0)=\dot{q}_d^*(0)-\dot{q}_k(0)=0$ so that the technique proposed in this paper can be directly applied without modifications.

4. Simulation results

To demonstrate the effectiveness of the proposed adaptive learning controller, we consider a two degrees-of-freedom planar manipulator with revolute joints described by (1). The matrix $M=[m_{ij}]_{2\times 2}$ is given by $m_{11}=m_1\ell_{c1}^2+m_2(\ell_1^2+\ell_{c2}^2+2\ell_1\ell_{c2}\cos q_2)+I_1+I_2$, $m_{12}=m_{21}=m_2(\ell_{c2}^2+\ell_1\ell_{c2}\cos q_2)+I_2$, and $m_{22}=m_2\ell_{c2}^2+I_2$. The matrix $C=[c_{ij}]_{2\times 2}$ is given by $c_{11}=h\dot{q}_2$, $c_{12}=h\dot{q}_1+h\dot{q}_2$, $c_{21}=-h\dot{q}_1$, and $c_{22}=0$ where $h=-m_2\ell_1\ell_{c2}\sin q_2$. The vector $G=[G_1,G_2]^\top$ is given by $G_1=(m_1\ell_{c1}+m_2\ell_1)g\cos q_1+m_2\ell_{c2}g\cos(q_1+q_2)$ and $G_2=m_2\ell_{c2}g\cos(q_1+q_2)$. The robot parameters shown above are given by $m_1=m_2=1\,\text{kg}$, $\ell_1=\ell_2=0.5\,\text{m}$, $\ell_{c1}=\ell_{c2}=0.25\,\text{m}$, $I_1=I_2=0.1\,\text{kg}\,\text{m}^2$, $g=9.81\,\text{m/s}^2$. The disturbances are assumed to be $d_1=d_2=rand(k)\sin(t)$ where $rand(k)$ is a random function taking its value between 0 and 1. The formulation of the disturbances implies that they are time-varying and varying from iteration to iteration. The desired trajectories for q_1 and q_2 are chosen as $q_{1,d}(t)=\sin(2\pi t)$ and $q_{2,d}(t)=\cos(2\pi t)$ over the time interval is $[0,1]$ s.

At first, we investigate the convergence of the sup-norm for the joint position tracking errors versus the iteration number k. The PD control gains are set to be $K_p=K_D=20I_{2\times 2}$ where $I_{2\times 2}$ is a 2×2 identity matrix and the learning gain is set to be $\beta=20$, respectively. These parameters are easily chosen. According to the technical analysis, we understand that K_p, K_D should be chosen as positive definite matrices and β should be chosen as a positive constant. In general, when the eigenvalues of K_p, K_D and the value of β are larger, the convergence speed will be faster. Fig. 1(a) shows the evolutions of $\sup_{t\in[0,1]}\widetilde{q}_{1,k}(t)$ and $\sup_{t\in[0,1]}\widetilde{q}_{2,k}(t)$ versus the iteration number k when $\gamma=0.5$. After 50th trials, the sup-norm of position error is less than 0.0008 rad for joint one and less than 0.00033 rad for joint two.

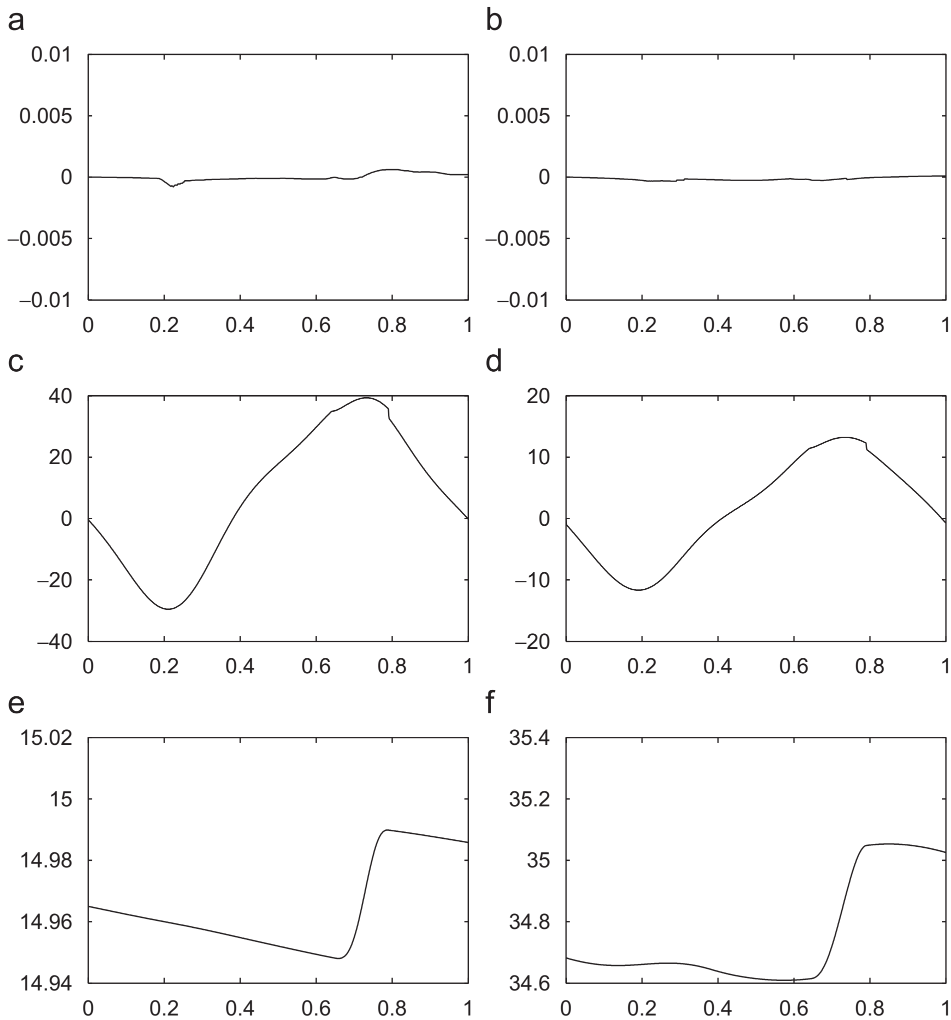

Fig. 3. Responses at the 50th iteration ($k = 50$) under adaptive learning controller (4), (5) with saturation function (25). (a) $\widetilde{q}_{1,k}(t)$ versus time t, (b) $\widetilde{q}_{2,k}(t)$ versus time t, (c) $\tau_{1,k}(t)$ versus time t, (d) $\tau_{2,k}(t)$ versus time t, (e) $\widehat{\theta}_{1,k}(t)$ versus time t, (f) $\widehat{\theta}_{2,k}(t)$ versus time t.

In order to study the effect of the parameter γ, we choose the sup-norm of the position error at joint one for comparisons. Fig. 1(b) illustrates the evolution of $\sup_{t\in[0,1]}\widetilde{q}_{1,k}(t)$ versus the iteration number k with $\gamma = 0, 0.5$ and 1, respectively. In other words, the pure time domain, combined domain and pure iteration-domain adaptive laws are applied for simulation. No matter what the value of γ is, the error convergence is guaranteed once $0 \leqslant \gamma \leqslant 1$. However, the convergence speed is faster if γ is larger.

Finally, the trajectories of position tracking errors $\widetilde{q}_{1,k}(t)$, $\widetilde{q}_{2,k}(t)$, input torques $\tau_{1,k}(t), \tau_{2,k}(t)$ as well as the control parameters $\widehat{\theta}_{1,k}(t)$ and $\widehat{\theta}_{2,k}(t)$ for the case of $\gamma = 0.5$ at 50th iteration are presented in Fig. 2(a)–(f), respectively. The chattering problem in the input torque $\tau_k(t)$ which is inherent to the use of the sign function (as in most robust control techniques) can be reduced by substituting the sign function by a saturation function leading to a practical convergence to a compact domain around zero. In Fig. 3, we performed the same simulations where the sign function has been substituted by the following saturation function:

$$sat(\dot{\tilde{q}}_{i,k}) = \begin{cases} 1 & \text{if } \dot{\tilde{q}}_{i,k} \geqslant 0.01, \\ 100\dot{\tilde{q}}_{i,k} & \text{if } |\dot{\tilde{q}}_{i,k}| < 0.01, \\ -1 & \text{if } \dot{\tilde{q}}_{i,k} \leqslant -0.01. \end{cases} \quad (25)$$

With this modification, we obtained a more practical control signal without chattering.

5. Conclusion

For repetitive control of robot manipulators, a new AILC strategy is proposed in this paper. The main feature of this learning control scheme is a switching-type robust controller and a mixed time-domain and iteration-domain adaptation law. Under this adaptive learning controller, the projection mechanism is not required in the adaptive law to ensure the boundedness of all signals involved in the control scheme. A rigorous proof, via a Lyapunov-like approach, is given to show the finiteness of tuning control parameters, rejection of the random input disturbance and the asymptotic error convergence along the iteration axis. Simulation results show that the error convergence and the bounded internal signals can be achieved. In the future work, we will study the implementation and experiment of the proposed approach for robot manipulators. We also would like to improve the adaptive iterative learning controller with simpler control structure using only one control parameter and without using sign function. The relaxation of identical initial resetting condition can also be considered.

Acknowledgments

This work is supported by the National Science Council, ROC, under Grant NSC95-2221-E-211-006 and by the Natural Sciences and Engineering Research Council (NSERC), Canada, under Grant RGPIN/238701-2001.

References

Arimoto, S. (1996). *Control theory of non-linear mechanical systems*. Oxford, UK: Oxford Science Publications.

Bondi, P., Casalino, G., & Gambardella, L. (1988). On the iterative learning control theory for robotic manipulators. *IEEE Journal of Robotics and Automation*, *4*, 14–22.

Chien, C. J., & Yao, C. Y. (2004). An output based adaptive iterative learning controller for high relative degree uncertain linear systems. *Automatica*, *40*, 145–153.

Choi, J. Y., & Lee, J. S. (2000). Adaptive iterative learning control of uncertain robotic systems. *IEE Proceedings D, Control Theory Application*, *147*, 217–223.

French, M., & Rogers, E. (2000). Non-linear iterative learning by an adaptive Lyapunov technique. *International Journal of Control*, *73*, 840–850.

Horowitz, R. (1993). Learning control of robot manipulators. *ASME Journal of Dynamic Systems Measurements and Control*, *115*, 402–411.

Ioannou, P. A., & Sun, J. (1996). *Robust adaptive control*. Englewood Cliffs, NJ: Prentice-Hall.

Kawamura, S., Miyazaki, F., & Arimoto, S. (1988). Realization of robot motion based on a learning method. *IEEE Transactions on Systems, Man and Cybernetics, Part-B*, *18*, 126–134.

Kuc, T. Y., & Han, W. G. (2000). An adaptive PID learning control of robot manipulators. *Automatica*, *36*, 717–725.

Norrlöf, M. (2002). An adaptive iterative learning control algorithm with experiments on an industrial robot. *IEEE Transactions on Robotics and Automation*, *18*, 245–251.

Park, P. H., Kuc, T. Y., & Lee, J. S. (1996). Adaptive learning control of uncertain robotic systems. *International Journal of Control*, *65*, 725–744.

Slotine, J. J., & Li, W. (1991). *Applied nonlinear control*. Englewood Cliffs, NJ: Prentice-Hall.

Spong, M. W., & Vidyasagar, M. (1989). *Robot dynamics and control*. New York: Wiley.

Takegaki, M., & Arimoto, S. (1981). A new feedback method for dynamic control of manipulators. *Journal of Dynamic Systems, Measurement and Control*, *103*, 119–125.

Tayebi, A. (2004). Adaptive iterative learning control for robot manipulators. *Automatica*, *40*, 1195–1203.

Tomei, P. (1991). Adaptive PD controller for robot manipulators. *IEEE Transactions on Robotics and Automation*, *7*, 565–570.

Wang, D., Son, Y. C., & Cheah, C. C. (1995). Robust motion and force control of constrained manipulators by learning. *Automatica*, *31*, 257–262.

Xu, J. X., & Wiswanathan, B. (2000). Adaptive robust iterative learning control with dead zone scheme. *Automatica*, *36*, 91–99.

Xu, J. X., & Yan, R. (2005). On initial conditions in iterative learning control. *IEEE Transactions on Automatic Control*, *50*, 1349–1354.