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Transient Performance Improvement In Model Reference Adaptive Control Via Iterative Learning

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Abstract— In this paper, we propose an iterative control strategy for the transient performance improvement of model reference adaptive control (MRAC) for continuous-time single-input single-output (SISO) linear time-invariant (LTI) systems with unknown parameters. The transient improvement is achieved through the introduction of a supplementary discrete-type parametric adaptation law along the iteration-axis, which is obtained in a straightforward manner from the continuous-time parametric adaptation law used in the MRAC scheme. This approach is referred to as the iterative model reference adaptive control (IMRAC). Initially, a standard MRAC scheme is applied to the system under consideration. Thereafter, the parameters are updated iteratively in order to enhance the tracking performance from iteration to iteration. In the case of systems with relative degree one, we obtain a pointwise convergence of the tracking error to zero, over the whole finite time-interval, when the number of iterations tends to infinity. In the general case, i.e., systems with arbitrary relative degree, we show that the tracking error converges to a prescribed small domain around zero, over the whole finite time interval, when the number of iterations tends to infinity. Simulation results are also carried out to support the theoretical development.

I. Introduction

Adaptive control is one of the most popular control techniques that has been fascinating the automatic control community for several years [6], [9]. In the standard adaptive control framework, the parametric adaptation rule is generally an integration along the time-axis which is commonly designed using the Lyapunov method in order to achieve asymptotic tracking. Hence, the tracking objective is achieved along an infinite time interval, and a transient tracking error will always be present. Model reference adaptive control is among the famous adaptive techniques that have been around for more than three decades. The major problem that one can attribute to this technique is the bad transient performance. To overcome this drawback, other alternatives, such as the backstepping approach [7], have been proposed in the literature. However, the benefit brought by those techniques (e.g., backstepping), in terms of transient improvement, is often eclipsed by the control law implementation complexity. On the other hand, in practical applications, the designed controller can be applied more than once to the plant under consideration, over a finite time interval. In this case, one can benefit from the information collected at the previous operations in order to enhance the transient performance for the subsequent operations. This technique is known as the iterative learning control (ILC) [1]. Most of the existing ILC schemes in the literature are based upon the contraction mapping technique and require a certain a priori knowledge of the system parameters as well as the use of the output time derivatives for systems with high relative degree (see, for instance, [2], [8], [13], [14]). Recently, a growing interest has been given to the energy-based approach which takes its essence from the Lyapunov theory [3], [4], [5], [15], [16], [17], [18].

In this paper, we propose an iterative control strategy for the transient performance improvement of MRAC schemes. The proposed IMRAC scheme achieves a global asymptotic tracking along the time horizon at the first iteration, and a point-wise convergence of the tracking error to zero (in the case of systems with relative degree one) or to a prescribed small domain around zero (for systems with higher relative degree), over the whole finite time-interval, when the number of iterations tends to infinity. In fact, at the first iteration, we use a continuous-time integral-type parametric adaptation law, while for the subsequent iterations, we use a discrete integral-type parametric adaptation law along the iteration-axis, which is obtained in a straightforward manner from the continuous-time parametric adaptation law used in standard MRAC schemes. The proof is based upon the use of a Lyapunov-like sequence which is shown to be monotonically decreasing along the iterative process. Basically, the role of the discrete integral-type parametric adaptation law is to refine the transient response from iteration to iteration in order to achieve a 'perfect tracking' (in the case where the initial tracking error is zero) over a finite time horizon after an infinite number of iterations. In contrast to existing contraction mapping-based ILC schemes, the proposed control strategy does not require the use of the output time-derivatives for systems with a high relative degree.

II. Problem formulation

In this paper we consider SISO-LTI systems described by

$$y_k(t) = G_p(s)[u_k(t)] = k_p \frac{Z_p(s)}{R_p(s)}[u_k(t)], \qquad (1)$$

and operated repeatedly over a finite time-interval [0,T]. The nonnegative integer $k \in \mathbb{Z}_+$ denotes the iteration

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or trial number. The desired trajectory $y_d(t)$ is given by a reference model as follows

$$y_d(t) = G_m(s)[r_f(t)] = k_m \frac{Z_m(s)}{R_m(s)}[r_f(t)], \qquad (2)$$

where $r_f(t)$ is a bounded reference input.

Assuming that the system parameters are unknown (except the sign of the high-frequency-gain k_p), our objective is to design an adaptive iterative learning controller guaranteeing the boundedness of the tracking error $\forall t \in [0,T]$ and $\forall k \in \mathbb{Z}_+$, and the convergence of the tracking error to zero $\forall t \in [0,T]$ when k tends to infinity. To this end, we will assume that $y_d(0) = y_k(0)$ and without any loss of generality we will assume that $y_d(0) = y_k(0) = 0$. We will use the $\mathcal{L}_{pe}$ norm defined as follows

$$\|x(t)\|_{pe} \triangleq \begin{cases} \left(\int_0^t \|x(\tau)\|^p d\tau\right)^{1/p} & if \quad p \in [0,\infty) \\ \sup_{0\le\tau\le t} \|x(\tau)\| \quad if \quad p = \infty \end{cases}$$

where $\|x\|$ denotes any norm of x, and t belongs to the finite interval $[0,T]$. We say that $x \in \mathcal{L}_{pe}$ when $\|x\|_{pe}$ exists (i.e., when $\|x\|_{pe}$ is finite).

We will also make the following classical assumptions related to the MRAC technique:

$B1)$ Z_p is a monic Hurwitz polynomial of degree m_p

$B2)$ An upper bound n of the degree n_p of $R_p(s)$ is available

$B3)$ The relative degree $r = n_p - m_p$ of G_p is known

$B4)$ The sign of the high frequency gain k_p is known

$B5)$ Z_m and R_m are monic Hurwitz polynomial of degree m_m and n_m respectively, with $n_m \le n$

$B6)$ The relative degree $r_m = n_m - m_m$ of G_m is the same as that of G_p.

III. Preliminaries

Let us define $\Lambda(s) = \Lambda_0(s)Z_m(s)$, which is a monic Hurwitz polynomial of degree $n-1$. Define also $\alpha(s)$ as follows

$$\alpha(s) = \begin{cases} [s^{n-2}, s^{n-1}, \ldots, s, 1]^T & \text{for } n \ge 2 \\ 0 & \text{for } n = 1 \end{cases}$$

As shown in [6], [9], there exists a set of parameters $c_0^* \in \mathbb{R}$, $\theta_3^* \in \mathbb{R}$, $\theta_1^* \in \mathbb{R}^{n-1}$ and $\theta_2^* \in \mathbb{R}^{n-1}$ such that the following control law

$$u_k = \theta^{*T}\Omega_k,$$

where $\theta^* = [\theta_1^{*T}, \theta_2^{*T}, \theta_3^*, c_0^*]^T$, $\Omega_k = [w_{1,k}^T, w_{2,k}^T, y_k, r_f]^T$, $w_{1,k} = \frac{\alpha(s)}{\Lambda(s)}[u_k]$ and $w_{2,k} = \frac{\alpha(s)}{\Lambda(s)}[y_k]$, leads to

$$y_k = G_m(s)[r_f] = k_m \frac{Z_m}{R_m}[r_f].$$

The parameters can be obtained from the following relationships

$$c_0^* = \frac{k_m}{k_p},$$

$$(\Lambda - \theta_1^{*T}\alpha)R_p - k_p Z_p(\theta_2^{*T}\alpha + \theta_3^*\Lambda) = Z_p\Lambda_0 R_m.$$

The signals $w_{1,k}$ and $w_{2,k}$ are the outputs of the following systems

$$\begin{array}{rcll} \dot{w}_{1,k} & = & Fw_{1,k} + gu_k & w_{1,k}(0) = 0 \\ \dot{w}_{2,k} & = & Fw_{2,k} + gy_k & w_{2,k}(0) = 0, \end{array} \qquad (3)$$

where (F, g) is a state space realization of $\frac{\alpha(s)}{\Lambda(s)}$.

The state space representation of the overall closed-loop system is given by the following nonminimal realization

$$\begin{array}{rcl} \dot{Y}_{c,k} & = & A_c Y_{c,k} + B_c c_0^* r_f \\ y_k & = & C_c Y_{c,k}, \end{array} \qquad (4)$$

with $Y_{c,k} = [x_k^T, w_{1,k}^T, w_{2,k}^T]^T \in \mathbb{R}^{n_p+2n-2}$, where x_k denotes the state vector associated with system (1), and

$$A_c = \begin{bmatrix} A + B\theta_3^* C & B\theta_1^{*T} & B\theta_2^{*T} \\ g\theta_3^* C & F + g\theta_1^{*T} & g\theta_2^{*T} \\ gC & 0 & F \end{bmatrix}, \qquad (5)$$
$$B_c = \begin{bmatrix} B \\ g \\ 0 \end{bmatrix}, C_c = [C, 0, 0].$$

Hence, the transfer function from r_f to y_k is given by

$$\frac{y_k}{r_f} = G_m(s) = C_c(sI - A_c)^{-1}B_c c_0^*.$$

Therefore, the reference model can also be described by

$$\begin{array}{rcl} \dot{Y}_m & = & A_c Y_m + B_c c_0^* r_f \\ y_d & = & C_c Y_m, \end{array}$$

Note that A_c is a stable matrix, since $det(sI - A_c) = \Lambda(s)Z_p(s)\Lambda_0(s)R_m(s)$.

Let $e_k = Y_{c,k} - Y_m$ be the state error and $e_{1,k} = y_k - y_d$ be the output tracking error. It follows that

$$\begin{array}{rcl} \dot{e}_k & = & A_c e_k \\ e_{1,k} & = & C_c e_k, \end{array} \qquad (6)$$

which shows that the tracking error converges exponentially to zero.

Since the system parameters are unknown, the vector θ^* cannot be obtained and hence, the control law $u_k(t) = \theta^{*T}\Omega_k(t)$ cannot be applied. In this case, the MRAC technique consists of applying a control law of the form $u_k(t) = \theta_k^T(t)\Omega_k(t)$, where $\theta_k(t)$ is generated by an appropriate adaptive law.

In our approach, at the first iteration, i.e., for $k = 0$, the vector $\theta_0(t)$ is generated by a continuous-time integral-type adaptive law as in the usual MRAC framework, whereas for $k \ge 1$, the vector $\theta_k(t)$ is generated by a discrete integral-type adaptive law (iterative law along the iteration axis).

IV. Iterative-SPR-Lyapunov Lemma

In this section, we propose a modified version of the strictly positive real(SPR)-Lyapunov approach [6], [12], which is the key in the derivation of our IMRAC in the next section. Our result can be stated as follows:

Lemma 1: Let the signals $e_k(t)$ and $\gamma\Psi_k^T(t)v_k(t)$ be related by a strictly positive real (SPR) transfer function $H(s)$ as follows:

$$e_k(t) = H(s)[\gamma\Psi_k^T(t)v_k(t)], \tag{7}$$

where t belongs to the finite time-interval $[0,T]$, $e_k(t) \in \mathbb{R}$, γ is an unknown constant with known sign, $v_k(t) \in \mathbb{R}^m$ is a measurable vector. The vector $\Psi_k(t) \in \mathbb{R}^m$ is generated by

$$\Psi_k(t) = \Psi_{k-1}(t) - \Gamma v_k(t)e_k(t)sgn(\gamma) \quad for \quad k \geq 1, \tag{8}$$

and

$$\dot{\Psi}_0(t) = -\Gamma v_0(t)e_0(t)sgn(\gamma), \tag{9}$$

where $\Gamma \in \mathbb{R}^{m\times m}$ is a symmetric positive definite matrix. Then

- The state vector $\bar{X}_k \in \mathcal{L}_{\infty e}$, $e_k(t) \in \mathcal{L}_{\infty e}$ and $\Psi_k(t) \in \mathcal{L}_{2e}$, for all $k \in \mathbb{Z}_+$.
- $\lim_{k\to\infty} e_k(t) = 0$, for all $t \in [0,T]$.

Proof: Because of space limitation, we will skip some calculation details in the proof.
Let the state space representation of (7) be

$$\begin{aligned} \dot{\bar{X}}_k &= \bar{A}\bar{X}_k + \bar{B}(\gamma\Psi_k^T(t)v_k(t)), \quad \bar{X}_k(0) = 0 \\ e_k &= \bar{C}\bar{X}_k. \end{aligned}$$

Since $H(s)$ is SPR then, from Meyer-Kalman-Yakubovich (MKY) lemma [6], [12], for any given symmetric positive definite matrix L there exist a symmetric positive definite matrix P, a vector q and a strictly positive scalar ν such that

$$\begin{aligned} \bar{A}^T P + P\bar{A} &= -qq^T - \nu L \\ P\bar{B} &= \bar{C}^T. \end{aligned}$$

Consider the following Lyapunov-like function candidate

$$W_k(\bar{X}_k, \Psi_k) = \frac{1}{2}\bar{X}_k^T P\bar{X}_k + \frac{|\gamma|}{2}\int_0^t \Psi_k^T(\tau)\Gamma^{-1}\Psi_k(\tau)d\tau, \tag{10}$$

Using the fact that $H(s)$ is SPR, and using MKY-lemma, in view of (7) and (8), one can show that the difference of the Lyapunov-like function is given by

$$\begin{aligned} \Delta W_k &= W_k - W_{k-1} \\ &\leq -\frac{1}{2}\bar{X}_{k-1}^T P\bar{X}_{k-1} - \frac{|\gamma|}{2}\int_0^t \bar{\Psi}_k^T\Gamma^{-1}\bar{\Psi}_k d\tau \\ &\leq 0. \end{aligned} \tag{11}$$

where $\bar{\Psi}_k = \Psi_k - \Psi_{k-1}$. Hence $W_k(t)$ is nonincreasing and consequently $\bar{X}_k(t)$, $\int_0^t \Psi_k^T(\tau)\Gamma^{-1}\Psi_k(\tau)d\tau$ and $e_k(t)$ are bounded if $W_0(t)$ is bounded.

Now, to prove the boundedness of $W_0(t)$, we use the following Lyapunov function

$$S_0(\bar{X}_0, \Psi_0) = \frac{1}{2}\bar{X}_0^T P\bar{X}_0 + \frac{|\gamma|}{2}\Psi_0^T\Gamma^{-1}\Psi_0, \tag{12}$$

whose time derivative along the system trajectories can be shown to be negative semi-definite, which means that $\bar{X}_0(t)$ and $\Psi_0(t)$ are globally bounded. Hence, $W_0(t)$ is bounded over the finite time interval $[0,T]$.
Moreover, one can show that

$$\sum_{j=1}^{j=k} \bar{X}_{j-1}^T(t)P\bar{X}_{j-1}(t) \leq 2(W_0(t) - W_k(t)) \leq 2W_0(t). \tag{13}$$

Since $W_0(t)$ and $\bar{X}_k(t)$ are bounded for all $k \in \mathbb{Z}_+$ and $t \in [0,T]$, one can conclude that $\lim_{k\to\infty} \bar{X}_k(t) = 0$ and consequently $\lim_{k\to\infty} e_k(t) = 0$, $\forall t \in [0,T]$. □

Remark 1: Note that by virtue of Barbalat lemma, and under the assumption that $v_0(t)$ is bounded for all $t \in \mathbb{R}^+$, one can easily show that $\lim_{t\to\infty} e_0(t) = 0$.

Remark 2: If $v_k(t) \in \mathcal{L}_{\infty e}$ for any finite non-negative integer k, one can show that $\Psi_k(t) \in \mathcal{L}_{\infty e}$ for any finite non-negative integer k.

V. IMRAC for systems with relative degree one

For systems with relative degree $r = 1$, the design of an IMRAC is straightforward from Lemma1 as stated in the following theorem

Theorem 1: Assume that (B1-B6) are satisfied and $G_m(s)$ is SPR. Consider system (1), with a relative degree $r = 1$, under the following control law

$$u_k(t) = \theta_k^T(t)\Omega_k(t), \quad for \quad k \geq 0 \tag{14}$$

with

$$\theta_k(t) = \theta_{k-1}(t) - \Gamma\Omega_k(t)e_{1,k}(t)sgn(\rho^*), \quad for \quad k \geq 1 \tag{15}$$

and

$$\dot{\theta}_0(t) = -\Gamma\Omega_0(t)e_{1,0}(t)sgn(\rho^*), \tag{16}$$

where $\rho^* = \frac{k_p}{k_m}$ and $\Gamma \in \mathbb{R}^{2n\times 2n}$ is a symmetric positive definite matrix. Then,

- The state vector $Y_{c,k} \in \mathcal{L}_{\infty e}$, $e_{1,k}(t) = (y_k(t) - y_d(t)) \in \mathcal{L}_{\infty e}$ and $\theta_k(t) \in \mathcal{L}_{2e}$, for all $k \in \mathbb{Z}_+$.
- $\lim_{k\to\infty} e_{1,k}(t) = 0$, for all $t \in [0,T]$.

Proof: Since (6) is obtained with $u_k(t) = \theta^{*T}\Omega_k(t)$, one has

$$\begin{aligned} \dot{e}_k &= A_c e_k + B_c(u_k - \theta^{*T}\Omega_k) \\ e_{1,k} &= C_c e_k, \end{aligned} \tag{17}$$

which under the control law (14) becomes

$$\begin{aligned} \dot{e}_k &= A_c e_k + B_c\tilde{\theta}_k^T\Omega_k \\ e_{1,k} &= C_c e_k, \end{aligned} \tag{18}$$

with $\tilde{\theta}_k(t) = \theta_k(t) - \theta^*$. Since $C_c(sI - A_c)^{-1}B_c c_0^* = G_m(s)$, system (18) leads to

$$e_{1,k} = G_m(s)[\rho^* \tilde{\theta}_k^T(t)\Omega_k(t)] \quad (19)$$

where $\rho^* = 1/c_0^*$. Finally, under the adaptive laws (15) and (16), the result follows directly from Lemma 1. □

Remark 3: Since $r_f, e_{1,k}, Y_{c,k} \in \mathcal{L}_{\infty e}\ \forall k \in \mathbb{Z}_+$, one can conclude that $\Omega_k \in \mathcal{L}_{\infty e}\ \forall k \in \mathbb{Z}_+$. Hence, one can show that $\theta_k(t) \in \mathcal{L}_{\infty e}$ for any finite non-negative integer k. Consequently, $u_k(t) \in \mathcal{L}_{\infty e}$ for any finite non-negative integer k.

Remark 4: Note that, for $k = 0$, the control scheme proposed in theorem 1 is nothing else but a standard MRAC. It turns out that the second term of the right hand side of the discrete-type adaptation law (15) is similar to the right hand side of the continuous-time adaptation law (16). For systems with a relative degree $r > 1$ direct application of the Iterative-SPR-Lyapunov lemma is not possible. Nevertheless, it is possible to obtain IMRAC schemes, in a straightforward manner, from the standard MRAC algorithms dealing with higher relative degrees (see for instance [6], [9] and references therein), by associating to each continuous-time integral-type adaption law a discrete integral-type adaption law with saturation, along the iteration axis as shown in the next section.

VI. IMRAC for systems with relative degree $r \geq 1$

In this section, we propose an IMRAC scheme for systems with an arbitrary relative degree $r \geq 1$. Our result is based on the extension of the MRAC schemes proposed in [6], [9].

Theorem 2: Assume that (B1-B6) are satisfied. Consider system (1), with a relative degree $r \geq 1$, under the following control law over $[0, T]$:

$$u_k(t) = \theta_k^T(t)\Omega_k(t), \quad for \quad k \geq 0 \quad (20)$$

with

$$\theta_k(t) = \begin{cases} \theta_{k-1}(t) - \Gamma\epsilon_k(t)\phi_k(t)sgn(\rho^*), \ if \ \sup_{t\in[0,T]} |\epsilon_{k-1}| > \sigma \\ \theta_{k-1}(T) \ if \ \sup_{t\in[0,T]} |\epsilon_{k-1}| \leq \sigma \end{cases} \quad (21)$$

$$\rho_k(t) = \begin{cases} \rho_{k-1}(t) + \gamma\epsilon_k(t)\xi_k(t), \ for \ \sup_{t\in[0,T]} |\epsilon_{k-1}(t)| > \sigma \\ \rho_{k-1}(T) \ for \ \sup_{t\in[0,T]} |\epsilon_{k-1}(t)| \leq \sigma \end{cases} \quad (22)$$

for $k \geq 1$, and

$$\dot{\theta}_0(t) = -\Gamma\epsilon_0(t)\phi_0(t)sgn(\rho^*), \quad (23)$$

$$\dot{\rho}_0(t) = \gamma\epsilon_0(t)\xi_0(t), \quad (24)$$

where, $\rho^* = \dfrac{k_p}{k_m}$, $\Gamma \in \mathbb{R}^{2n\times 2n}$ is a symmetric positive definite matrix and γ is a positive parameter. The signals ϕ_k, ϵ_k and ξ_k are evaluated for all $k \in \mathbb{Z}_+$ as follows:

$$\begin{aligned} \epsilon_k &= \frac{e_{1,k} - \hat{e}_{1,k}}{m_k^2} \\ \hat{e}_{1,k} &= \rho_k\xi_k \\ \xi_k &= \bar{u}_k - \theta_k^T\phi_k \\ \phi_k &= G_m(s)[\Omega_k] \\ \bar{u}_k &= G_m(s)[u_k] \\ m_k^2 &= \begin{cases} 1 + \bar{u}_k^2 + \phi_k^T\phi_k \ or \ 1 + \bar{\phi}_k^T\bar{\phi}_k \ for \ k = 0 \\ \kappa \quad for \quad k \geq 1 \end{cases} \end{aligned} \quad (25)$$

where κ is a positive parameter, $\bar{\phi}_k = G_m(s)[\bar{\Omega}_k]$, with $\bar{\Omega}_k = [w_{1,k}^T, w_{2,k}^T, y_k]^T$.

Then, all signals are bounded $\forall k \in \mathbb{Z}_+$, $\forall t \in [0, T]$, and $\lim_{k\to\infty} |e_{1,k}(t)| \leq \kappa\sigma$, $\forall t \in [0, T]$.

Proof: The proof is omitted for space limitation.

Remark 5: Note that, for $k = 0$, the control scheme proposed in theorem 2 reduces to the standard MRAC schemes proposed in [6], [9]. In fact, $m_0^2 = 1 + \bar{u}_0^2 + \phi_0^T\phi_0$ has been used in [6] and $m_0^2 = 1 + \bar{\phi}_0^T\bar{\phi}_0$ has been used in [9].

Remark 6: It is worth noting that the saturation used for θ_k is required for a technical reason in the proof. It allows to ensure that $\theta_k(t)$ becomes constant when the augmented tracking error ϵ_k is sufficiently small. In this case, the augmented tracking error becomes the real tracking error $e_{1,k}$ since $\xi_k(t) = 0$. On the other hand, the saturation used for ρ_k is not necessary. In fact, we stop the learning for ρ_k because it has no effect on the system behavior once θ_k is constant.

Remark 7: It is worth noting that a simple and interesting approach to adaptive iterative learning control has been proposed in [3], where a standard Lyapunov design is used to solve ILC problems. The idea consists to use a standard adaptive controller and to start the parameter estimates with their final values obtained at the preceding iteration. However, as in standard adaptive control, this technique requires the unknown system parameters to be constant. In contrast to the approach of [3], the ILC framework used in the present paper, although applied here in the case of unknown constant parameters, is able to handle systems with time-varying parameters [18]. On the other hand, our approach does not require an integration every iteration to obtain the parameters estimates. The parameters are adjusted point-wisely in an iterative manner with a tunable learning gain.

VII. Simulation results

In this section, we consider two examples.

Example 1:

$$G_p(s) = \frac{s+1}{s^2 - 10s + 1}, G_m(s) = \frac{1}{s+1},$$

with $r_f(t)$ being a unit step input. The auxiliary variables $w_{1,k}$ and $w_{2,k}$ are given by

$$w_{1,k} = \frac{1}{s+10}[u_k], \quad w_{2,k} = \frac{1}{s+10}[y_k].$$

The matrix Γ is chosen as a $\Gamma = 10I_{4\times4}$. The time interval is taken as $[0, 8s]$ and the initial conditions for the adaptive law at the first iteration are chosen to be zero.

Figure 1 shows the evolution of the Sup-norm of the tracking error with respect to the iteration number. Figure 2 shows the performance of the standard MRAC, for G_{p1}. Figures 3 and 4, show the transient performance improvement over the iterations.

Example 2:

$$G_p(s) = \frac{2s+5}{s^3+6s^2+7s-4}, G_m(s) = \frac{2s+5}{s^3+6s^2+11s+6}$$

with $r_f(t)$ being a unit step input. The auxiliary variables $w_{1,k}$ and $w_{2,k}$ are given by

$$w_{1,k} = \left(\frac{s}{2s^2+15s+25}, \frac{1}{2s^2+15s+25}\right)^T [u_k],$$

$$w_{2,k} = \left(\frac{s}{2s^2+15s+25}, \frac{1}{2s^2+15s+25}\right)^T [y_k].$$

$\kappa = 0.1$, $\Gamma = 10I_{6\times6}$, $\gamma = 0.1$ and $\sigma = 0.05$. The time interval is taken as $[0, 8s]$ and the initial conditions for the adaptive law at the first iteration are chosen to be zero. At the first iteration, i.e., for $k = 0$, we use $m_0^2 = 1 + \bar{u}_0^2 + \phi_0^T \phi_0$

Figure 5 shows the evolution of the Sup-norm of the tracking error with respect to the iteration number using our approach.

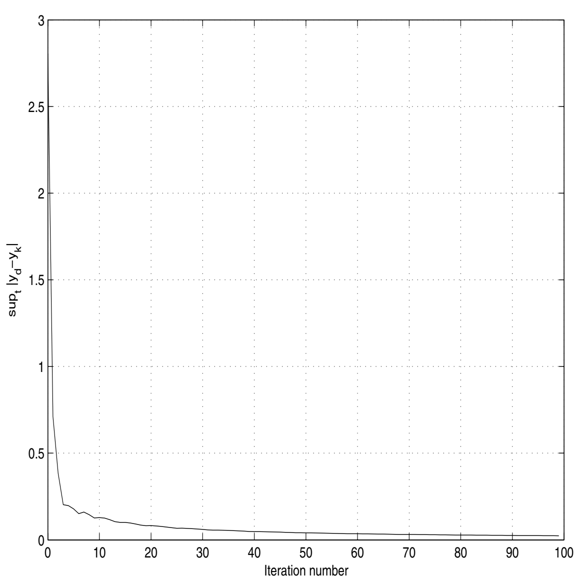

Fig. 1. *Example1* : $\sup_{t\in[0,8]} |y_d(t) - y_k(t)|$ with respect to the iteration number k.

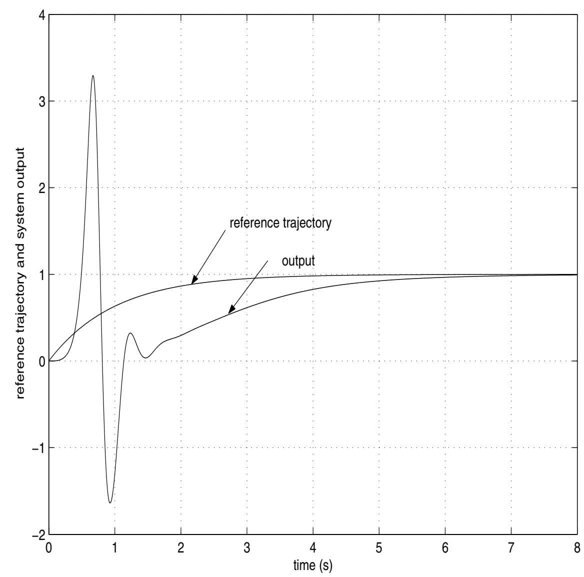

Fig. 2. Example1: Reference trajectory and system output with the MRAC (i.e., k=0).

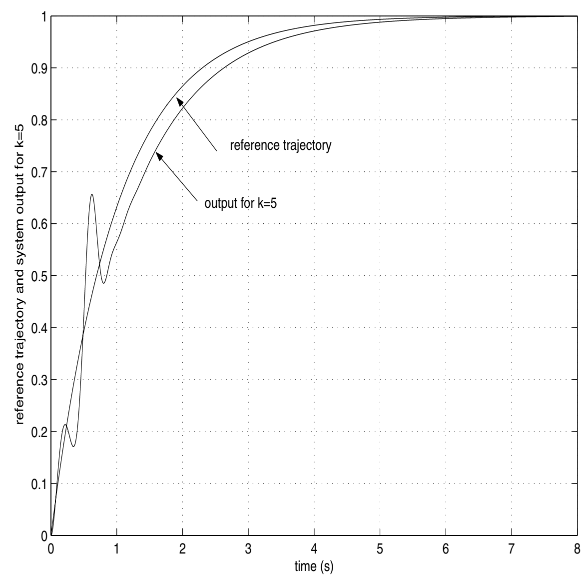

Fig. 3. Example1: Reference trajectory and system output at the 5th iteration.

VIII. Conclusion

In this paper, a two-dimensional adaptive control strategy has been proposed for SISO-LTI systems with arbitrary relative degree. Two different types of parametric adaptation laws are used. The first one is performed only at the first iteration, i.e., for $k = 0$, as a continuous-time integral-type adaptive law derived using a standard Lyapunov function in order to guarantee global boundedness and asymptotic convergence to zero of the tracking error in the time domain. The second one is a discrete integral-type adaptive law designed to make a Lyapunov-like function monotonically decreas-

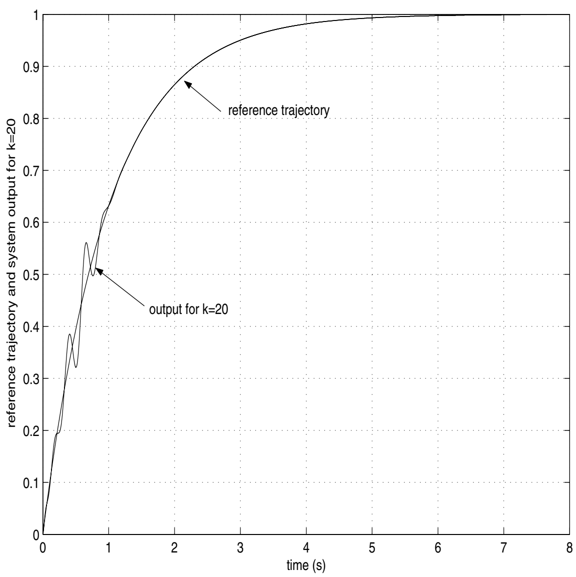

Fig. 4. Example1: Reference trajectory and system output at the 20th iteration.

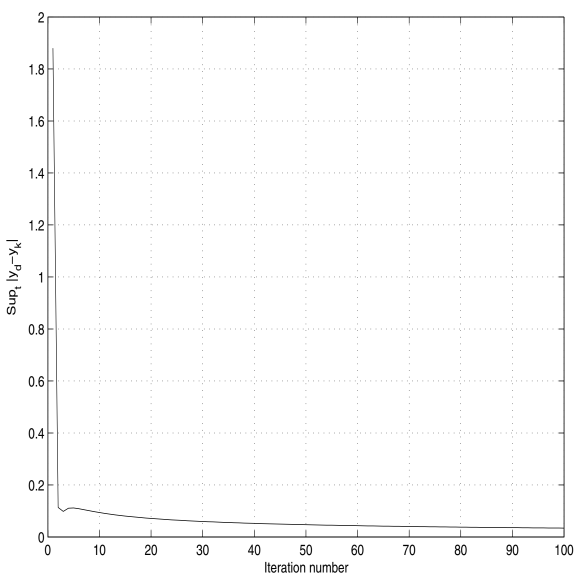

Fig. 5. $Example2:\ \sup_{t\in[0,8]} |y_d(t) - y_k(t)|$ with respect to the iteration number k.

ing along the iteration axis. Basically, the role of the later is to refine the output response from iteration to iteration in order to achieve a pointwise convergence of the tracking error to zero (in the case of systems with relative degree one), or to a prescribed small domain around zero (for systems with higher relative degree), over the whole finite time-interval, when the number of iterations tends to infinity. In contrast to existing contraction mapping-based ILC schemes, the proposed control strategy does not require the use of the output time-derivatives.

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