First, observe that when $L_m = 0$ the ellipsoid $\overline{\mathcal{F}}_N$ collapses onto a segment with extremal points $\overline{h} = h_c \pm L_M v_1$. Then, one has $\|z^* - \overline{h}\|^2 = L_M^2 + d^2$, and hence from (28) and (29)$E[h_{cp}]/E[h_{cc}] \geq L_M + d/\sqrt{L_M^2 + d^2}$ as stated in the upper part of (18).

Now, let us examine the case $L_m > 0$. A generic point on the boundary of $\overline{\mathcal{F}}_N$ can be written as $\overline{h} = h_c + \alpha_1 v_1 + \cdots + \alpha_N v_N$, where

$$\frac{\alpha_1^2}{L_M^2} + \frac{1}{L_m^2}\sum_{i=2}^{N}\alpha_i^2 = 1. \tag{30}$$

Then

$$\begin{aligned}\|z^* - \overline{h}\|^2 =& \|h_c - d\,v_N - h_c - \alpha_1\,v_1 - \cdots - \alpha_N\,v_N\|^2 \\ =& \alpha_1^2 + \alpha_2^2 + \cdots + (\alpha_N + d)^2 \\ =& \alpha_1^2\left[\frac{L_M^2 - L_m^2}{L_M^2}\right] + 2\alpha_N\,d + L_m^2 + d^2\end{aligned}$$

where the last equality has been obtained by using (30). Exploiting the aforementioned expression, the maximization of $\|z^* - \overline{h}\|$ with respect to $\overline{h} \in \overline{\mathcal{F}}_N$ is a straightforward exercise that leads to

$$\sup_{\overline{h}\in\overline{\mathcal{F}}_N} \|z^* - \overline{h}\| = \begin{cases} L_M\sqrt{1 + \frac{d^2}{L_M^2 - L_m^2}}, & \text{if } d < \frac{L_M^2 - L_m^2}{L_m} \\ L_m + d, & \text{if } d \geq \frac{L_M^2 - L_m^2}{L_m}. \end{cases} \tag{31}$$

Then, (18) is an immediate consequence of (28), (29), and (31).

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Robust Iterative Learning Control Design is Straightforward for Uncertain LTI Systems Satisfying the Robust Performance Condition

A. Tayebi and M. B. Zaremba

***Abstract*—This note demonstrates that the design of a robust iterative learning control is straightforward for uncertain linear time-invariant systems satisfying the robust performance condition. It is shown that once a controller is designed to satisfy the well-known robust performance condition, a convergent updating rule involving the performance weighting function can be directly obtained. It is also shown that for a particular choice of this weighting function, one can achieve a perfect tracking. In the case where this choice is not allowable, a sufficient condition ensuring that the least upper bound of the $\mathcal{L}_2$-norm of the final tracking error is less than the least upper bound of the $\mathcal{L}_2$-norm of the initial tracking error is provided. This sufficient condition also guarantees that the infinity-norm of the final tracking error is less than the infinity-norm of the initial tracking error.**

***Index Terms*—Linear time-invariant (LTI) systems, robust performance, robust iterative learning control (ILC).**

I. INTRODUCTION

Iterative learning control (ILC) has been recently generating a considerable amount of interest in the automatic control community. A more detailed discussion about this control technique, which applies to systems that operate repeatedly, can be found in the survey papers [13] and [14]. The main idea behind ILC techniques is to take advantage of the previous operations in order to adjust the control signal to be applied to the system in the upcoming operations. This allows the controller to perform progressively better with every new operation in order to achieve accurate tracking after a certain number of iterations. The ILC control scheme was initially developed as a feedforward action applied directly to the open-loop system (see, for example, [1], [3], [5], and [11]). However, this control scheme may generate harmful effects if the open-loop system is unstable or an inappropriate initial control law is chosen. To overcome this drawback, several feedback-based ILC and learning feedforward control (LFFC) algorithms have been proposed in the literature, e.g., [4], [6]–[8], [10], and [12]. To the best of our knowledge, all of the existing feedback-based ILC schemes in the literature are based upon the design of the ILC filters and the feedback controller separately.

Manuscript received October 29, 2001; revised January 14, 2002 and May 10, 2002. Recommended by Associate Editor C. Wen. This work was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada under Grant RGPIN/238701-2001.

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Digital Object Identifier 10.1109/TAC.2002.806659

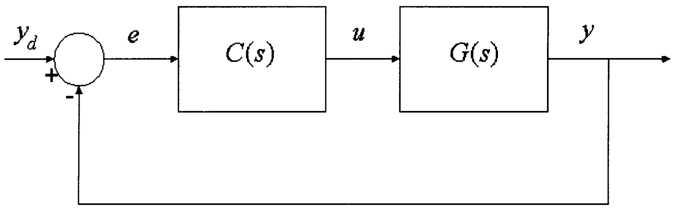

Fig. 1. Feedback system.

In this note, we show that once a feedback controller $C(s)$ is designed to guarantee the robust performance condition, there is no need to design the ILC filters, and a convergent updating rule involving the performance weighting function $W_1(s)$ can be directly obtained. It is also shown that for a particular choice of this performance weighting function, one can achieve a perfect tracking. In the case where this choice is not allowable, a sufficient condition ensuring that the final tracking error is most likely to be less than the initial tracking error—obtained with the feedback controller alone—is provided.

In this approach, we are simultaneously benefiting from the robust performance at the first iteration—when the ILC is not effective—and guaranteeing the convergence of the iterative process. Another important advantage of this approach is that it allows to establish the connection between the ILC convergence condition and the well known robust performance condition. This fact permits to the ILC designer to benefit from the wide range of tools from robust control theory, such as loop shaping, model matching, H_∞, and μ-synthesis approaches [2], [9], [16], to solve ILC problems. For the sake of simplicity, single-input single-output plants are considered, but the results can be generalized to multivariable systems. Finally, two illustrative examples are provided to demonstrate the effectiveness of the proposed ILC scheme.

II. Main Result

Consider the feedback system in Fig. 1, where the plant G is described in the following multiplicative uncertain form:

$$G = (1 + \Delta W_2)G_n \tag{1}$$

where G_n is the nominal plant model, W_2 is a known stable transfer function, and Δ is an unknown stable transfer function satisfying $\|\Delta\|_\infty \leq 1$. The reference signal $y_d(t)$ is assumed to be bounded within the tracking interval.

In the sequel, the Laplace variable s will be omitted when this does not lead to any confusion. To derive our results, we will need the following lemma [9].

Lemma 1: Consider the feedback system in Fig. 1, with G as described in (1). The robust performance condition is then

$$\|W_2T\|_\infty < 1 \text{ and } \left\|\frac{W_1S}{1+\Delta W_2T}\right\|_\infty < 1$$

which is equivalent to

$$\||W_1S| + |W_2T|\|_\infty < 1 \tag{2}$$

where W_1 and W_2 are known stable transfer functions, $S = (1/1+CG_n)$ is the sensitivity function and $T = 1 - S$ the complementary sensitivity function. □

If the system in Fig. 1 is operated repeatedly, the application of the same control input at every operation will lead to the same tracking error over and over again. The main idea in ILC techniques is to add another iteratively updated control input v_k to the feedback control variable u_k, as shown in Fig. 2, in order to ensure that the tracking error $e_k(t)$ converges to a small neighborhood of zero when k tends to infinity, for all t within a given time-interval. The subscript k is introduced to designate the variable at the kth operation.

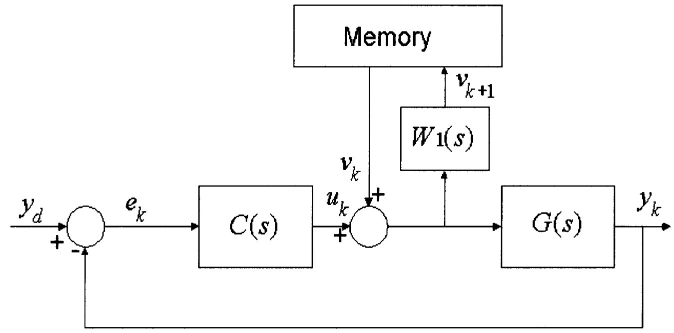

Fig. 2. Feedback-based ILC.

Throughout this note, we assume that $y_k(0) = y_d(0)$, and without any loss of generality, we consider that $y_k(0) = y_d(0) = 0$.

Our main result can be stated as follows: If one is able to design a feedback controller $C(s)$ guaranteeing the robust performance condition (2), then the design of the iterative updating rule for v_k is straightforward and is given by

$$\begin{aligned} V_{k+1}(s) &= W_1(s)\left(V_k(s) + C(s)E_k(s)\right) \\ &= W_1(s)\left(V_k(s) + U_k(s)\right) \end{aligned} \tag{3}$$

with $V_1(s) = 0$. Where $W_1(s)$ is the performance weighting function involved in the robust performance condition (2), and $E_k(s)$, $V_k(s)$, $U_k(s)$ are, respectively, the Laplace transforms of $e_k(t)$, $v_k(t)$ and $u_k(t)$.

The control scheme in Fig. 2 ensures the boundedness and the convergence, in the sense of the $\mathcal{L}_2$-norm, of the tracking error when k tends to infinity. Moreover, the tracking error converges to zero if $W_1 = 1$.

Summarizing, we have the following theorem.

Theorem 1: Consider the iterative control scheme in Fig. 2.

If there exists $C(s)$ such that the robust performance condition (2) is satisfied, then the tracking error is bounded for all $k \in \mathbb{N}$ and converges uniformly to

$$e_\infty(t) = \lim_{k\to\infty} e_k(t) = \mathcal{L}^{-1}\left(\frac{1-W_1}{1-W_1+CG_n(1+\Delta W_2)}Y_d\right) \tag{4}$$

when $k \to \infty$, in the sense of the $\mathcal{L}_2$-norm.

Proof: From Fig. 2, the tracking error at the kth iteration is given by

$$E_k(s) = Y_d(s) - Y_k(s) = \frac{Y_d(s)}{1+C(s)G(s)} - \frac{G(s)V_k(s)}{1+C(s)G(s)}. \tag{5}$$

Hence, the tracking error at the $(k+1)$th iteration is given by

$$E_{k+1} = \frac{Y_d}{1+CG} - \frac{GV_{k+1}}{1+CG}. \tag{6}$$

Using (3), (5), and (6), one has

$$E_{k+1} = \left(W_1 - \frac{CGW_1}{1+CG}\right)E_k + \frac{1-W_1}{1+CG}Y_d \tag{7}$$

which, in view of (1), becomes

$$E_{k+1} = \left(\frac{W_1}{1+CG_n(1+\Delta W_2)}\right)E_k + \frac{1-W_1}{1+CG_n(1+\Delta W_2)}Y_d. \tag{8}$$

Since $(W_1/1+CG_n(1+\Delta W_2)) = (W_1S/1+\Delta W_2T)$, (8) becomes

$$E_{k+1} = \left(\frac{W_1S}{1+\Delta W_2T}\right)E_k + \frac{1-W_1}{1+CG_n(1+\Delta W_2)}Y_d. \quad (9)$$

Hence

$$E_k = \left(\frac{W_1S}{1+\Delta W_2T}\right)E_{k-1} + \frac{1-W_1}{1+CG_n(1+\Delta W_2)}Y_d. \quad (10)$$

From (9) and (10), one has

$$E_{k+1} - E_k = \left(\frac{W_1S}{1+\Delta W_2T}\right)(E_k - E_{k-1}) \quad (11)$$

which leads to

$$\begin{aligned}\|E_{k+1}(s) - E_k(s)\|_2 =& \|e_{k+1}(t) - e_k(t)\|_2 \\ \leq& \left\|\frac{W_1S}{1+\Delta W_2T}\right\|_\infty^{k-1} \\ &\times \|e_2(t) - e_1(t)\|_2. \end{aligned} \quad (12)$$

Using (5) with $V_1 = 0$, one can easily conclude, under the robust performance condition (2) and the fact that y_d is bounded, that $e_1(t)$ is bounded. Therefore, since W_1 is stable, one can also conclude from (8) that $e_k(t)$ is bounded for all k. Hence, from (12), it is clear that if

$$\left\|\frac{W_1S}{1+\Delta W_2T}\right\|_\infty < 1 \quad (13)$$

the tracking error converges to $e_\infty(t) = \mathcal{L}^{-1}\{E_\infty(s)\}$ given in (4), when k tends to infinity, in the sense of the $\mathcal{L}_2$-norm. The limit $E_\infty(s)$ can be obtained from (8) by substituting E_{k+1} and E_k by E_∞. Finally, according to Lemma 1, (13) is guaranteed under the robust performance condition. □

Now, we would like to determine a condition under which one can ensure that the tracking error, when k tends to infinity, is most likely to be less than the initial tracking error. This condition is given in the following theorem.

Theorem 2: Consider the iterative control scheme in Fig. 2. If W_1 is such that $\|1-W_1\|_\infty < 1/2$ and if there exists $C(s)$ such that

$$\||W_1^*S| + |W_2^*T|\|_\infty < 1 \quad (14)$$

with $W_1^* = (W_1/1-2\|1-W_1\|_\infty), W_2^* = (W_2/1-2\|1-W_1\|_\infty)$, then

i) the tracking error is bounded for all $k \in \mathbb{N}$ and converges uniformly to the value given in (4), when $k \to \infty$, in the sense of the $\mathcal{L}_2$-norm;
ii) the least upper bound of the $\mathcal{L}_2$-norm of the final tracking error is less than the least upper bound of the $\mathcal{L}_2$-norm of the initial tracking error, i.e., $\|e_\infty\|_2 \leq \alpha_1$, $\|e_1\|_2 \leq \alpha_2$, with $\alpha_1 < \alpha_2$;
iii) $\|E_\infty\|_\infty < \|E_1\|_\infty$.

Proof: If $W_1 = 1$, (14) is nothing else but the robust performance condition (2) and the proof of convergence of the tracking error to zero is exactly the same as the proof of Theorem 1.

Now, let us consider the case where $W_1 \neq 1$. Since $\|1-W_1\|_\infty < 1/2$, (14) implies that

$$\||W_1S| + |W_2T|\|_\infty < 1 - 2\|1-W_1\|_\infty \quad (15)$$

which also implies that the robust performance condition (2) is satisfied. Hence, according to Theorem 1, the tracking error is bounded for all k and the final tracking error is given by

$$\begin{aligned}E_\infty = \lim_{k\to\infty} E_k =& \frac{1-W_1}{1-W_1+CG_n(1+\Delta W_2)}Y_d \\ =& \frac{(1-W_1)S}{1-W_1S+\Delta W_2T}Y_d.\end{aligned} \quad (16)$$

The initial tracking error, obtained by setting $V_1 = 0$ in (5), is given as follows:

$$E_1 = \frac{1}{1+CG_n(1+\Delta W_2)}Y_d = \frac{S}{1+\Delta W_2T}Y_d \quad (17)$$

where S and T are defined in Lemma 1.

Hence, it is clear that $\|e_\infty\|_2 = \|E_\infty\|_2 \leq \alpha_1$, and $\|e_1\|_2 = \|E_1\|_2 \leq \alpha_2$, where α_1 and α_2 being the least upper bounds [9]

$$\alpha_1 = \left\|\frac{(1-W_1)S}{1-W_1S+\Delta W_2T}\right\|_\infty \|y_d\|_2 \quad (18)$$

and

$$\alpha_2 = \left\|\frac{S}{1+\Delta W_2T}\right\|_\infty \|y_d\|_2. \quad (19)$$

Condition (15) implies that

$$1 - |W_1S| - |W_2T| > 2|1-W_1| \qquad \forall\,\omega. \quad (20)$$

On the other hand, one has

$$\begin{aligned}1 =& |1 + W_1S + \Delta W_2T - W_1S - \Delta W_2T| \\ \leq& |W_1S| + |W_2T| + |1 - W_1S + \Delta W_2T|\end{aligned} \quad (21)$$

which leads to

$$1 - |W_1S| - |W_2T| \leq |1 - W_1S + \Delta W_2T| \qquad \forall\,\omega. \quad (22)$$

Hence, from (20) and (22), one has

$$|1 - W_1S + \Delta W_2T| > 2|1-W_1| \qquad \forall\,\omega. \quad (23)$$

Since (2) is satisfied, one has $|W_2T| < 1, \forall\,\omega$, which implies that $|1+\Delta W_2T| < 2, \forall\,\omega$. Consequently, (23) implies that

$$|1 - W_1S + \Delta W_2T| > |1+\Delta W_2T||1-W_1| \quad (24)$$

which implies that

$$\left\|\frac{(1-W_1)S}{1-W_1S+\Delta W_2T}\right\|_\infty < \left\|\frac{(1-W_1)S}{(1+\Delta W_2T)(1-W_1)}\right\|_\infty \quad (25)$$

that is

$$\left\|\frac{(1-W_1)S}{1-W_1S+\Delta W_2T}\right\|_\infty < \left\|\frac{S}{1+\Delta W_2T}\right\|_\infty \quad (26)$$

which means that $\alpha_1 < \alpha_2$. From (24), one can also conclude that

$$\left\|\frac{(1-W_1)SY_d}{1-W_1S+\Delta W_2T}\right\|_\infty < \left\|\frac{(1-W_1)SY_d}{(1+\Delta W_2T)(1-W_1)}\right\|_\infty \quad (27)$$

that is

$$\left\|\frac{(1-W_1)SY_d}{1-W_1S+\Delta W_2T}\right\|_\infty < \left\|\frac{SY_d}{1+\Delta W_2T}\right\|_\infty \quad (28)$$

which means that $\|E_\infty\|_\infty < \|E_1\|_\infty$. □

Remark 1: According to Theorem 1, it is appropriate to take $W_1 = 1$ to ensure zero tracking error when k tends to infinity, and design

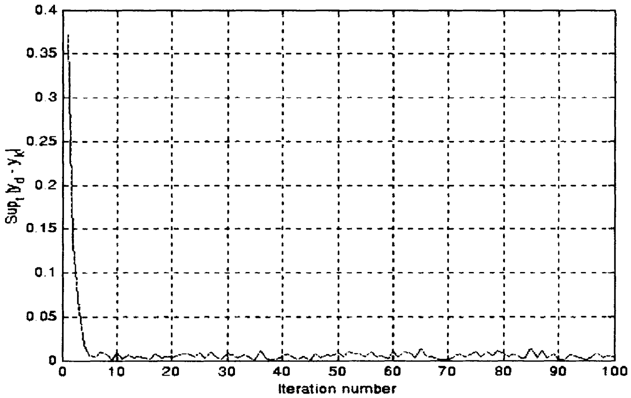

Fig. 3. Example 2, Sup-norm of the tracking error versus the number of iterations, with $\boldsymbol{\Delta}$ varying randomly between 0 and 1 along the iteration axis.

the controller $C(s)$ satisfying the robust performance condition (2) using the loop shaping, model matching methods [9], or the μ-synthesis approach [15], [16]. In this case (i.e., $W_1(s) = 1$), it is clear that the proposed control scheme is able to completely eliminate the effect of repetitive exogenous disturbances. If the problem is not solvable[1] with $W_1 = 1$, then according to Theorem 2, we have to take $W_1 \neq 1$, but close to one within the tracking bandwidth, such that $\|1 - W_1\|_\infty < 1/2$, and solve the modified robust performance condition (14) to determine the controller $C(s)$ guaranteeing that the final tracking error is less than the initial tracking error.

Remark 2: Generally, the problem of slow convergence occurs when the time weighted norm (or λ-norm) is used to prove the ILC convergence in time domain. In this paper, we prove the exponential convergence of the $\mathcal{L}_2$-norm—which is more effective than the λ-norm—of the tracking error. In the simulation results, one can see that the tracking error converges after a reasonable number of iterations.

Remark 3: This note deals with uncertain linear time-invariant (LTI) systems, where the system parameters are assumed to be unknown but constant. Since the system parameters are not affected by the time evolution, then it is more likely that these parameters will not be affected along the iteration axis. That is why we have assumed that $\Delta(s)$ is invariant from iteration to iteration. The case where $\Delta(s)$ is varying from iteration to iteration is a much more challenging problem which must be considered when dealing with time-varying systems. A nonrepetitive $\Delta(s)$ is an interesting and challenging case, which is out of the scope of this paper, and will be investigated in future research work. Nevertheless, without any theoretical support, we have performed a simulation for example 2, with Δ varying along the iteration axis as a random function taking its values between 0 and 1, and the results are shown in Fig. 3.

III. Simulation Results

In this section, we consider two illustrative examples.

[1]The problem is not always solvable as explained in [9, Ch. 6]. One necessary condition for robust performance is that $\min\{|W_1(j\omega)|, |W_2(j\omega)|\} < 1, \forall\, \omega$.

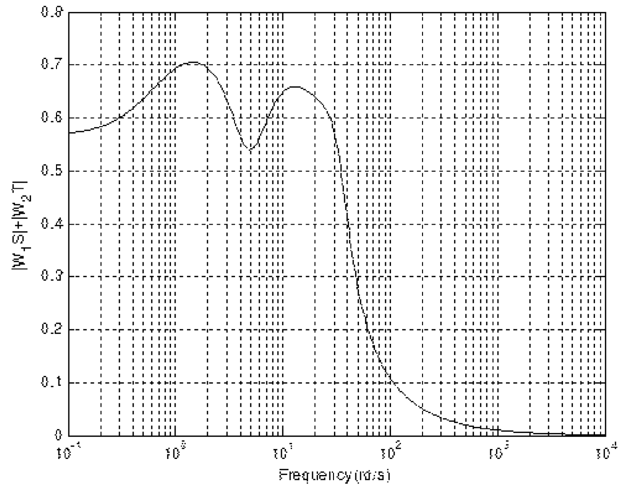

Fig. 4. Example 1, $|W_1S| + |W_2T|$ versus frequency.

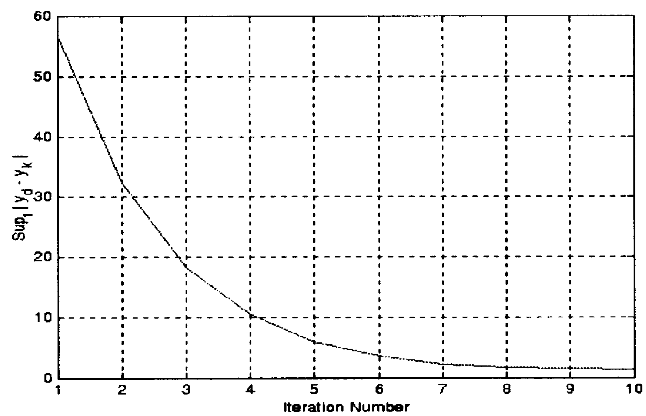

Fig. 5. Example 1, Sup-norm of the tracking error versus the number of iterations.

Example 1:

$$G_n(s) = \frac{s+1}{s^4 + 14s^3 + 71s^2 + 254s + 120}$$

$$W_1(s) = \frac{1}{0.1s+1} \quad W_2(s) = \frac{0.02s}{0.01s+1}.$$

Using the μ-Analysis and Synthesis Toolbox of Matlab [2], one can solve the robust performance condition to obtain the controller shown at the bottom of the page. This controller leads to

$$\||W_1S| + |W_2T|\|_\infty = 0.7052$$

as shown in Fig. 4.

We perform a simulation with $y_d(t) = 100\sin(0.1t), t \in [0, 20\pi]$. Fig. 5, shows the evolution of the tracking error with respect to the iteration number, and Fig. 6 shows the time evolution of the reference trajectory (star) and the output (solid) for $k = 1$, $k = 3$ and $k = 10$.

$$C(s) = \frac{2.50e-4s^7 + 3.53e5s^6 + 9.44e7s^5 + 1.09e9s^4 + 5.33e9s^3 + 1.33e10s^2 + 1.65e10s + 8.24e9}{s^7 + 197.86s^6 + 1.86e4s^5 + 6.98e5s^4 + 1.89e7s^3 + 9.70e7s^2 + 1.68e8s + 8.97e7}.$$

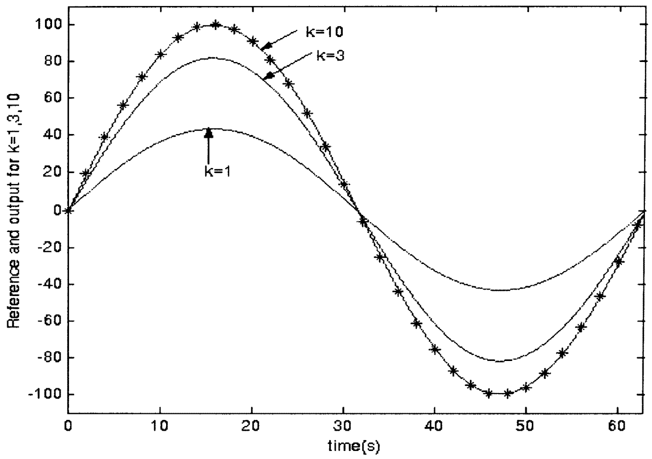

Fig. 6. Example 1, reference trajectory (star) and output (solid) for $k = 1, 3, 10$.

Example 2:

$$G_n(s) = \frac{24s + 70}{s^2 + 25s + 350} \quad W_1(s) = 1$$

$$W_2(s) = \frac{0.5s + 5}{s + 100}.$$

Using the μ-Analysis and Synthesis Toolbox of Matlab [2], one can solve the robust performance condition to obtain the following controller:

$$C(s) = \frac{4\ 10^6 s + 1.9912\ 10^8}{s + 2.4\ 10^7}$$

providing

$$\| |W_1 S| + |W_2 T| \|_\infty = 0.6$$

as shown if Fig. 7.

We perform a simulation with $y_d(t) = 1 - e^{-2t}, t \in [0, 10]$. Fig. 8, shows the evolution of the tracking error with respect to the iteration number, and Fig. 9 shows the time evolution of the reference trajectory (star) and the output (solid) for $k = 1$, $k = 3$ and $k = 6$. In this example one can see that the tracking error converges to zero since $W_1 = 1$.

IV. Conclusion

In this note, we have presented a straightforward derivation of a robust iterative learning controller for uncertain LTI systems satisfying the robust performance condition. It is shown that once a controller is designed to satisfy the well known robust performance condition, a convergent updating rule involving the performance weighting function W_1 can be directly obtained. Furthermore, a sufficient condition ensuring that the least upper bound of the $\mathcal{L}_2$-norm of the final tracking error is less than the least upper bound of the $\mathcal{L}_2$-norm of the initial tracking error is provided. The initial tracking error is obtained with the feedback controller alone, i.e., when $V_1 = 0$, whereas the final tracking error is obtained when the number of iterations tends to infinity.

One of the main objectives of this paper is to establish a connection between ILC and robust control theory. In fact, a relationship between the ILC convergence condition and the well known robust performance condition has been derived. This fact will allow the ILC designer to benefit from the wide range of tools from robust control theory to solve ILC problems. The ILC filter W_1 appearing in the robust performance condition can be set by the designer according to the ILC performance requirements, i.e., equal or close to one within the tracking bandwidth in order to minimize the tracking error when $k \to \infty$. Moreover, the proposed approach guarantees robust performance for the feedback system performing without ILC at the first iteration (i.e., when $V_1 = 0$). Consequently, with the design of a single controller $C(s)$, one can simultaneously guarantee robust performance at the first iteration and the convergence of the iterative process.

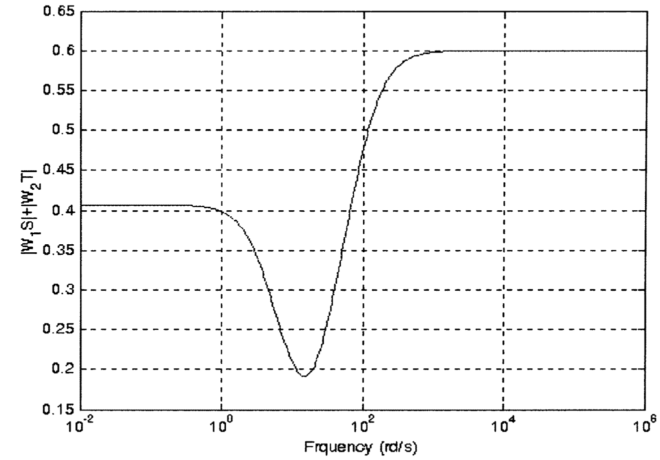

Fig. 7. Example 2, $|W_1 S| + |W_2 T|$ versus frequency.

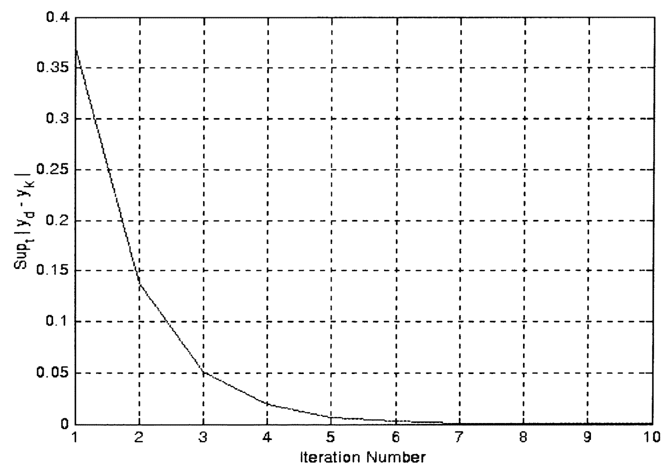

Fig. 8. Example 2, Sup-norm of the tracking error versus the number of iterations.

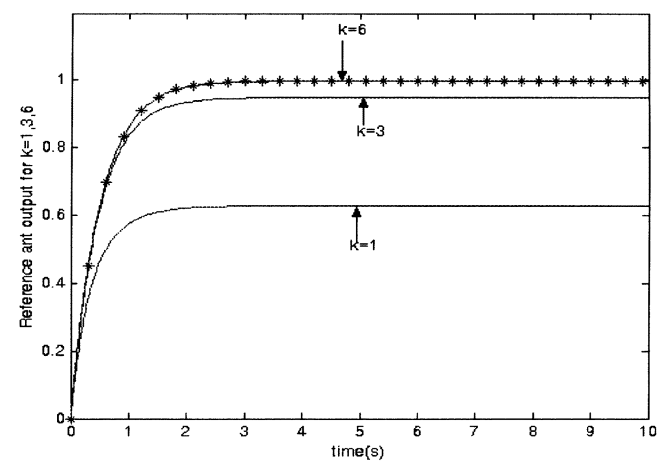

Fig. 9. Example 2, reference trajectory (star) and output (solid) for $k = 1, 3, 6$.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments.

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Exponentially Stabilizing Division Controllers for Dyadic Bilinear Systems

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***Abstract*—It is difficult to asymptotically stabilize a dyadic bilinear system with only multiplicative control inputs when the open-loop dynamics are unstable. The previous approach of cascading a division controller with a constant-size dead zone can only stabilize but *not* asymptotically stabilize the system. This note proposes a new control design which cascades a division controller with a modified dead zone whose size is proportional to the modulus of the system state. It is shown that this new division controller can globally and exponentially stabilize any open-loop unstable dyadic bilinear system as long as it is controllable.**

***Index Terms*—Asymptotic stability, dead zone, division controller, dyadic bilinear system, exponential stability.**

I. INTRODUCTION

A division controller is one whose control input is a quotient of two state functions

$$u = \frac{\beta(x)}{\alpha(x)}. \tag{1}$$

Such a control structure can be found in the feedback linearization control for nonlinear systems [1], and in the control for dyadic bilinear systems [2]. In the division controller (1), if $\alpha(x) = 0$ at some singular point x, the control signal becomes infinitely large at x. In the case of feedback linearization control, the singularity problem arises when the nonlinear system has no well-defined relative degree [3]. In the case of dyadic bilinear system control [2], the singularity problem is avoided by cascading the division controller (1) with a dead zone

$$u = \begin{cases} \frac{\beta(x)}{\alpha(x)}, & |\alpha(x)| > \epsilon \\ 0, & |\alpha(x)| \le \epsilon \end{cases} \tag{2}$$

where $\epsilon > 0$ is the size of the dead zone. The use of a dead zone is first proposed in the control [4] of a dyadic bilinear system whose control input is both multiplicative and additive

$$\dot{x} = Ax + b(y + d_0)u, \qquad y = cx. \tag{3}$$

where $x \in R^n$ is the state vector, $u \in R$ is a single control input, $A \in R^{n\times n}$ is a constant matrix, b and c^T are constant vectors, and d_0 is a nonzero constant. The division controller (2) becomes

$$u = \begin{cases} -\frac{kx}{y+d_0}, & |y + d_0| > \epsilon \\ 0, & |y + d_0| \le \epsilon \end{cases} \tag{4}$$

where $\epsilon > 0$ is the size of the dead zone, and the state feedback gain $k \in R^{1\times n}$ is chosen such that $A - bk$ is a stable matrix. It is proved that asymptotic stability is achieved by the division controller (4) if the open-loop trajectories satisfy a geometric condition [4].

Manuscript received February 25, 2002; revised July 5, 2002 and July 29, 2002. Recommended by Associate Editor C. Wen.
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Digital Object Identifier 10.1109/TAC.2002.806660